Recognising Mathematical Giftedness

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This paper describes a task that was used, in a broader study on mathematical giftedness and mindsets, as part of a process to identify mathematical giftedness in primary school students (see Parish, 2019). A discussion of the task, together with an analysis of student responses, offers insights that could be used by classroom teachers for both recognising and understanding mathematical giftedness. It is important for all classroom teachers to be able to recognise exceptional mathematical aptitudes in young students, as mathematically gifted students require specific consideration for appropriate teaching and learning experiences.

Teachers generally do not have difficulty recognising students who do well in mathematics, but ‘doing well in mathematics’ does not automatically translate to being mathematical gifted. Even more importantly *not* doing well in mathematics at school does not necessarily translate to *not* being mathematically gifted (Parish, 2014). This paper attempts to address the issue of equipping teachers to effectively recognise mathematical giftedness, for the ultimate purpose of ensuring appropriate learning experiences for these, and other, students in regular classrooms. The first part of the paper briefly defines mathematical giftedness in the context of the study the paper is drawn from, followed by a discussion about the need for identification of mathematically gifted students. A description of two tasks completed by select primary school students in the study, together with an analysis of student responses to the tasks, is then presented. Finally, how such tasks could be developed and used as a tool for classroom teachers to recognise and understand mathematical giftedness, is considered. Note, the paper does *not* address profoundly gifted students (mathematical prodigies). These are exceptional students with special needs that go beyond inclusive classroom strategies (Gross, 2004).

Background and Rationale

According to the Australian Curriculum, Gagné’s (2003) *Differentiated Model of Giftedness and Talent [DMGT]* provides the most generally accepted definition of both giftedness and talent in Australian education (Australian Curriculum Assessment and Reporting Authority [ACARA], 2014). According to Gagné’s model, gifted students are those whose learning capabilities (in any one, or more domains) is greater than 90% of their aged peers. This definition suggests that, on average, there will be two or three gifted individuals in any regular class of 20 to 30 students (i.e. 10% of the class). This being the case, it is imperative that all classroom teachers are aware of the characteristics and needs of gifted students. Indeed, the Australian Curriculum suggests teachers need to be aware of 1) how gifted students’ learning processes differ, 2) the ways gifted students may demonstrate their learning uniquely, and 3) what differentiated content may be required (ACARA, 2014). However, within the domain of mathematics, a search for research-based information for teachers on identifying and understanding mathematical giftedness suggests that there is a dearth of information readily available. For example, a search of the *Australian Primary Mathematics Classroom* journal (from 1996 to 2018) for the terms ‘gifted’, ‘highly capable’ and ‘high achieving’, found only three articles, the most recent of which was published ten years ago. Practical classroom applications of research on mathematically gifted students seems to be an area of great need, which the study this paper is based on aims to address.

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The theoretical framework of mathematical giftedness used in this paper, is based on Gagné's *DMGT*, which differentiates between the gift (inherent aptitude) and talent (mastered abilities), with the transformation of gifts into talents requiring appropriate school support (ACARA, 2014; Gagné, 2003). With this theoretical underpinning, and the resultant implications, it is important that teachers can recognise and understand mathematical giftedness. As a brief definition, mathematically gifted are those students who are capable of constructing robust neural networks of mathematical understanding (Skemp, 1978) with fewer learning experiences than their age peers. These students utilise intuitive reasoning based on what Geake (2008) calls *fluid analogising*, which curtails the need for multiple learning experiences. Fluid analogising is a cognitive process that enables quick recognition of similarities between problem types that allows for ready generalisations (Krutetskii, 1976). This can make it appear that a student 'just knows' something without being taught, but it is vital to understand that gifted students still require appropriate learning experiences to ensure constructed concepts and generalisations are accurate and not *mis*conceptions.

Many current processes for identifying mathematical giftedness, both formal (testing) and informal (observational checklists) rely on student achievement – high achievement in assessments, perceived achievement in the classroom, outstanding achievement in mathematics competitions, et cetera. This is an issue if there are gifted students who are underachieving (Siegle, 2013), and if there are high achieving students who have received additional tuition in mathematics, or who are extremely motivated. High achievement may be a result of intensive learning rather than efficient learning, and high achieving students who are not necessarily 'gifted learners' may be at risk of burnout if high achievement becomes an ongoing expectation (whether that expectation be external or internal).

An alternate identification process suggested for recognising mathematical giftedness is the use of problem-solving tasks (Niederer & Irwin, 2001), whereby teacher observations of how a student approaches the tasks provide insights into the way the student thinks and reasons mathematically. This may identify students who think 'differently', and can uncover hidden mathematical aptitudes in students who are not necessarily performing at a high level, however, this relies on teachers being able to recognise, and understand, significant differences in a student's way of thinking. This paper discusses how interpreting the responses to one particular type of problem task can be used to recognise the unique thinking of mathematically gifted students. It is hoped that tasks such as this could be developed into an assessment tool for classroom teachers to use, to recognise mathematically gifted aptitudes, to differentiate between high achievers and gifted learners, and to ultimately inform suitable teaching strategies for supporting individual students' ongoing learning.

Methodology and Analysis

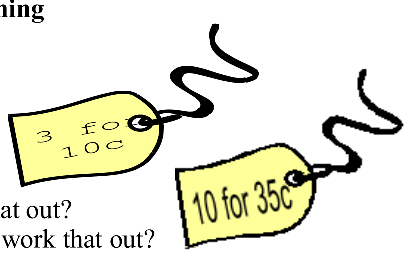
Any process for formally identifying mathematical giftedness needs to be multi-faceted (Reis, 2004). The tasks outlined below were one part of a task-based assessment interview, which was supplemented by a combination of other identification approaches, including teacher and parent perceptions, classroom observations, and archival mathematics assessment data that measured student mathematical learning over time.

A one-to-one task-based assessment was used so student approaches and thought processes in solving the task could be observed (Niederer & Irwin, 2001). A ratio task was used to assess the students' ability to reason proportionally, which has been shown to be a good indicator of overall mathematical ability (Lamon, 1999). Furthermore, ratio is not formally taught in primary schools in Australia, so students would need to employ their own intuitive mathematical understandings (Krutetskii, 1976) to solve the problem. There were two versions of the ratio task, *Lollies* (Parish, 2014) for early and middle primary students (see Figure 1), and *Oranges and Lemons* (Lamon, 1999) for upper primary students (see Figure 2).

Ratio task: Proportional reasoning

Lollies (for early and middle primary students): (Parish, 2010)

Place a 10c piece and a packet of three lollies on the table.
Show the price tag (3 for 10c).
Provide a piece of paper and pencil.



a) A shop sells a packet of three lollies for ten cents.
   i. How much would 12/15 lollies cost? How did you work that out?
   ii. How many lollies could I buy with 60c/80c? How did you work that out?

For middle primary students – Also show the student the price tag (10 for 35c).

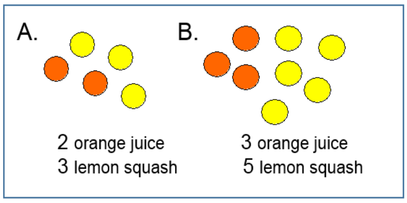
b) The same shop sells packets of ten lollies for 35c a packet.
   Which is the better value, three for ten cents, or then for 35c? How did you work that out?

Figure 1. Lollies Task, Grade 1 and Grade 3

Ratio task: Proportional reasoning

Oranges and Lemons (for upper primary students): (Lamon, 1999)

Show the student the orange juice/lemon squash card. Provide a sheet of paper and pencil.



This card shows the number of parts of orange juice mixed with lemon squash to make an orange and lemon punch. Which mixture will taste more orangey, A or B?

Figure 2. Oranges and Lemons Task, Grade 5

Participants

The task was used to assess students who had been nominated by their classroom teachers as being highly capable mathematically. The primary purpose of the assessment was to differentiate between those students who were mathematically gifted learners and those who were high achievers but not necessarily gifted. The use of tasks to uncover 'hidden' capabilities was not a main focus, although some students were nominated by their teachers on suspicion of higher mathematical ability than was apparent in class. Twenty-seven students from eight Grade 1, Grade 3 and Grade 5 classes in a private primary school in regional Victoria were nominated by their teachers as being mathematically highly capable. Interviews were conducted one-on-one during school hours, with responses recorded on a record sheet. An audio-recording of each interview was also taken to ensure authentic representation throughout the analysis (Merriam, 2009).

Analysis

Analysis of student responses to the ratio task was based on recognising Krutetskii's (1976) hallmarks of mathematical giftedness. The defining criteria being, 1) a tendency to look for the most elegant/efficient pathway to a solution, 2) the invention their own strategies for solving an unfamiliar mathematical problem, and 3) an ability to readily generalise prior knowledge to solve an unfamiliar type of mathematical problem. The analysis is presented as a narrative, using thick descriptions embedded with analytical, integrated interpretations to describe collected data (Merriam, 2009; Polkinghorne, 1995). This approach maximises the possibility of transferability for readers from a particular case study to other similar situations (Merriam, 2009).

Results

Selected examples of student responses to the tasks are given in the following three tables and discussions. It is important to remember that all these students were nominated by their teachers as being mathematically highly capable.

Table 1.
Selected examples of Grade 1 responses to the Lollies task (see Figure 1)

	A shop sells a packet of three lollies for ten cents. How much would 12 lollies cost? [40c]	*How many lollies could I buy with 60c? [18 lollies]*
Brony	Three for ten, so that's six for 20 cents, and another six for 20c is 40c [so 12 lollies cost 40c]	I counted six on from 12 because if you added six more you would have 60c (sic) [so I could buy 18 lollies with 60c].
Elsa	I counted by 3s up to 12 and while I was counting I was counting by 10s as well [so 12 lollies cost 40c]	I was counting by 2s to 60 and while I was counting I was counting by 10s as well [so I could buy 102 lollies with 60c]
Hazel	I added three to the 10 and then another three and then another three [so 12 lollies cost 19c]	
Brett	I don't know how to work that out	

Table 1 shows a selection of Grade 1 responses. Brony was nominated by her teacher because, even though she was a quiet, seemingly average worker, her teacher believed there was more depth to her thinking than was immediately obvious. Brony's responses to the *Lollies* problem were quietly confident, and correct. Her reasoning was very sophisticated for her age, seeing the ratio '3 for 10c' as a composite unit, and building up from this (Lo & Watanabe, 1997; Parish 2010), as well as being able to elegantly contract this process (Krutetskii, 1976) by calculating how much for six lollies and then doubling the result. Elsa showed some proportional reasoning ability (simultaneously counting by 3s and by 10s), and was able to hold and manipulate these two pieces of information mentally to work out how much 12 lollies would cost. However, she confused herself trying to work backwards using this same strategy to work out how many lollies 60c would buy. Hazel was among the highest achievers on the Grade 1 formal mathematics assessment conducted at the beginning of the year, but she did not recognise the structure of the task and solved it using a simple, incorrect, counting approach. Her approach to the task nonetheless was equally as confident as Brony's. Brett had been receiving extra mathematics tuition (as a cultural norm) since he started school. He performed very well in mathematics classes, but had no strategies at all with which to approach this unfamiliar type of mathematical task.

Table 2.
Selected examples of Grade 3 responses to the Lollies task (see Figure 1)

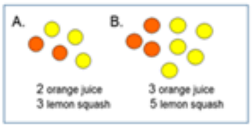
	A shop sells a packet of three lollies for ten cents. How much would 15 lollies cost? [50c]	*How many lollies could I buy with 80c? [24 lollies]*	*Which is better value, three for 10c, or ten for 35c? [3 for 10c]*
John	15 lollies cost 50c [used a composite-unit/build-up strategy (Lo & Watanabe, 1997; Parish 2010)]	I could buy 24 lollies with 80c [continued with the composite-unit/build-up strategy]	Three for 10c is better value than 10 for 35c. Three for 10c, so 9 is 30c and it would be less than 5c for one lolly [worked out it would be a bit more than 3c per lolly].
Janet	15 lollies cost 50c [used a composite-unit/build-up strategy]	I could buy 24 lollies with 80c [counted by threes keeping track with fingers]	10 for 35c is better value because you get more for five more cents.
Emma	15 lollies cost 50c. Each three is ten cents, so 10, 20, 30… [skip counted by 3s].	I could buy 72 lollies with 80c. Eight times ten is 80, and then I took away eight because I know three times three is nine.	

Table 2 shows a selection of Grade 3 data. John, like Brony, was a very quiet, very slow worker. He was cautiously nominated by his teacher who wondered if others may be more capable than he was. He sat very quietly for a long time, not giving any indication that he understood the *Lollies* question, let alone that he was solving the task. I had just decided he was stuck and was about to move on when he came out with the correct answer. He was then able to clearly explain his thinking by writing down what he had just done mentally (3=10c, 6=20c, 9=30c, etc.). His solution was correct and his explanation mathematically robust. He employed a mixture of the composite-unit/build-up strategy (Lo & Watanabe, 1997; Parish 2010) to determine how much and how many, and a mixture of this same strategy, plus calculating a unit value for each lolly (a bit more than 3c each), to determine best value. Janet was recognised as a *very* capable mathematics student. She was a quietly confident worker who enjoyed mathematics and loved the opportunity to share her successes and discoveries with the Principal, which was encouraged. Like John, she correctly used a composite-unit/build-up strategy to determine how much and how many, but did not negotiate the proportional difference between '3 for 10' and '10 for 35'. She understood that '3 for 10' was equivalent to '9 for 30', but then viewed the extra lolly as simply 'more'. Emma was another confident mathematics student with "very good number sense" according to her teacher. She was able to recognise the composite unit of '3 for 10' and build this up to '15 for 50', but was not able to reverse the process to calculate 'how many lollies for 80c'. Her reasoning was convoluted, "Eight times ten is 80, and then I took away eight because I know three times three is nine". She seemed confident in her thinking, but did not recognise the unreasonableness of her final answer.

Table 3 shows a selection of Grade 5 data on the Oranges and Lemons task. Murray immediately recognised that he could approach the *Oranges and Lemon* ratio (part-part) problem as a fraction (part-whole) problem. He compared two-fifths with three-eighths

Table 3.
Selected examples of Grade 5 responses to the Oranges and Lemons task (see Figure 2)

Which mixture will taste more orangey, A or B?
[NB. A is more orangey]



Murray	Two parts out of five would be more orangey than three parts out of eight. Three eighths times two is six eighths [which leaves two eighths]. Two eighths equals 1/4 and 1/5 is less than 1/4 … So … actually B is more orangey because 1/4 is the bigger part … no, A is right because I was working out which part of lemon was bigger.
Amy	A is more orangey. A is five and B is eight, so the common number is 40. I made it into a common number to find out how many oranges.
Bruce	A is just under a half what there is of lemon…[hesitated]. They might be equal, the three and the five are sort of the same because they're both how many lemons, and the two and the three are how many oranges. So they're the same.
James	Two from three is one left over; three from five is two left over. Some people might say B because there is more orange, but I think it is the same.

using a residual thinking strategy (Clarke, Roche & Mitchell, 2011) after manipulating both fractions (multiplying by two) so that he would have two fractions that could be easily compared: four fifths is one fifth less than one; six eighths is equivalent to three quarters, which is one quarter less than one. One fifth and one quarter gave residual fractions which could be readily compared. The complexity of proportional reasoning is evidenced in his initial confusion – that the bigger residual fraction means less orangey, not more – but he was able to successfully renavigate this conundrum. Amy, like Brett in Grade 1, had received continuous mathematics tuition throughout her schooling, and was recognised as a very high achieving student. She also perceived the task as a fraction problem, and was able to correctly answer the question, but the use of a learned procedure (finding a common denominator) masked any further conceptual understanding. Bruce recognised that orange in A is just under half, demonstrating some conceptual understanding of fractions, but did not approach the problem using this knowledge. Instead he reverted to comparing whole numbers, and ultimately used a visual approximation to determine that the two mixtures were the same. James was another high achiever who had received extra mathematics tuition throughout his schooling. He had limited strategies with which to approach the problem, and also used a whole number explanation and a visual determination. He indicated that he thought it was a 'trick' question that would trip up other students, but seemed less than fully confident with his own answer.

Summarising the student responses considering the defining criteria, it can be seen that, at least with this task, not all students nominated by their teachers as being highly capable, showed evidence of the hallmarks of mathematical giftedness: of finding the most elegant/efficient pathway to a solution, inventing strategies, and/or generalising prior knowledge to solve an unfamiliar mathematical problem. Moreover, not all students with correct solutions showed evidence of these hallmarks of mathematical giftedness, and not all students with incorrect solutions lacked evidence of at least some hallmarks of mathematical giftedness. Interestingly, it was Brony (Grade 1) and John (Grade 3) who had been cautiously nominated by their teachers, who showed the most elegant responses, with Brony employing the 'cleanest, simplest and shortest' solution path by doubling her halfway result. Of the three students who received additional mathematics tuition, and were very high achievers in general mathematics assessments, two had no strategy with which to approach the task, and the third, Amy (Grade 5) used a learned procedure. While Amy's answer was correct, using a learned procedure did not provide evidence of the extent of her mathematical understanding, where a reasoned response may have. She may indeed be mathematically gifted, but has learnt to rely firstly on procedures she has been taught than on her own intuition.

Discussion

These ratio tasks assessed a student's ability to reason mathematically in devising a method for solving an unfamiliar type of mathematics problem. All students assessed as part of the research study had been nominated by their teachers as being mathematically highly capable. It can be seen from the results above, however, that responses to an unfamiliar type of problem varied greatly. Responses ranged from understanding with intuitive approaches, to attempted procedural approaches, to no understanding of how to approach the task.

There are some important insights from these responses that teachers need to be aware of in recognising mathematical aptitudes, and for providing appropriate differentiation for mathematically gifted and mathematically high achieving students:

- The two terms, mathematically gifted and mathematically high achieving, are not synonymous. Mathematically gifted students are not always the highest achieving students in the classroom, and high achieving students are not necessarily gifted learners.
- Mathematical giftedness does not automatically translate into high mathematical output in the classroom. Teachers need to be on the lookout for students who process mathematical thoughts differently, and/or approach mathematical tasks uniquely. Regardless of whether they are 'fast' or 'slow' workers, teachers need to encourage individual approaches to tasks that develop mathematical inquiry and creativity.
- Mathematically gifted students still require appropriate learning experiences to construct sound mathematical understandings, but will not require the same repetition, nor the same amount of scaffolding as the standard curriculum suggests.

It is important to note that the interview results do not necessarily classify a student as mathematically gifted, or *not* mathematically gifted. Rather the interview enables teachers to recognise students who display mathematically gifted traits (whether a few or many), or not (where knowledge may be entirely procedural).

Conclusion

Tasks, such as the ratio problems discussed here, could be used by classroom teachers to reveal hallmarks of mathematical giftedness. Together with specific identifying criteria based on Krutetskii's observations of mathematical giftedness, a one-to-one assessment tool such as this could be used *by* classroom teachers for informal recognition of mathematical giftedness, and *for* classroom teachers as targeted professional learning on understanding common characteristics and learning needs of mathematically gifted students (Clarke, Roche & Mitchell, 2011). It is not an assessment tool to be used for selection to gifted programs, but for a teacher's understanding of individual student's learning needs within a regular classroom. The scope of this paper has not included considerations of the specific teaching and learning experiences that mathematically gifted students require, but that would be the next important step in practical classroom applications of a classroom teacher's knowledge and understanding of mathematical giftedness.

References

Australian Curriculum, Assessment and Reporting Authority (ACARA) (2014). *Gifted and talented students*. Retrieved from https://www.australiancurriculum.edu.au/resources/student-diversity/gifted-and-talented-students/#

Clarke, D., Roche, A. & Mitchell, A. (2011). One-to-one student interviews provide powerful insights and clear focus for the teaching of fractions in the Middle Years. In J. M. Bobis and J. Way, *Fractions: Teaching for understanding* (pp. 23-32). Adelaide: Australian Association of Mathematics Teachers.

Davis, R. B. (1984). *Learning Mathematics: The Cognitive Science Approach to Mathematics Education.* Norwood, NJ: Ablex Publishing Corporation.

Gagné, F. (2003). Transforming gifts into talents: The DMGT as a developmental theory. In N. Colangelo & G.A. Davis (Eds.), *Handbook of gifted education* (3rd ed., pp. 60-74). Boston, MA: Allyn and Bacon.

Geake, J. G. (2008). High abilities at fluid analogizing: A cognitive neuroscience construct of giftedness. *Roeper Review, 30*(3), 187-195.

Gross, M. U. M. (2004). *Exceptionally gifted children* (2nd ed.). London, England: Routledge Falmer.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in school children*. Chicago, IL: University of Chicago Press.

Lamon, S. (1999). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. Mahwah, NJ: Lawrence Erlbaum.

Lo, J. & Watanabe, T. (1997). Developing ratio and proportion schemes: A story of a fifth grader. *Journal for Research in Mathematics Education, 28* (2), 216-236.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation.* San Francisco, CA: Jossey-Bass.

Niederer, K., & Irwin, K. C. (2001). Using problem solving to identify mathematically gifted students. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th annual conference of the International Group for the Psychology of Mathematics Education* (pp. 431-438). Utrecht, The Netherlands: Freudenthal Institute.

Parish, L. (2010). Facilitating the development of proportional reasoning through teaching ratio. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia, pp. 469-476). Fremantle, Australia: MERGA.

Parish, L. (2014). Defining mathematical giftedness. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). *Curriculum in focus: Research guided practice* (Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia, pp. 509–516). Sydney: MERGA.

Parish, L. (2019). *Changing self-limiting mindsets of young mathematically gifted students to assist talent development* (Unpublished doctoral thesis). Australian Catholic University, Ballarat, Australia.

Polkinghorne, D. E. (1995). Narrative configuration in qualitative analysis. *Qualitative Studies in Education, 8*(1), 5-23.

Reis, S. M. (2004). Series introduction. In J. S. Renzulli (Ed.), *Identification of students for gifted and talented programs* (pp. ix-xxi). Thousand Oaks, CA: Corwin Press.

Siegle, D. (2013). *The underachieving gifted child: Recognizing, understanding, and reversing underachievement*. Waco, TX: Prufrock Press.