A study of school mathematics curriculum enacted by teachers in Singapore secondary schools

The "Enactment Project" is a Programmatic Research Project funded by the Ministry of Education, Singapore, and administered through the Office of Educational Research, National Institute of Education, Nanyang Technological University. The project began in 2016 and its aim is to study the enactment of the Singapore mathematics curriculum across the whole spectrum of secondary schools within the jurisdiction. Under this overarching goal, there are two supporting studies: Study 1 examines the classroom enactment by teachers in relation to the curriculum framework as organised in the Pentagon (Skills, Concepts, Attitudes, Processes, Metacognition, with Problem Solving at its centre); Study 2 focuses on the enactment as seen through the instructional materials designed by the teachers.

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Paper 1: Toh Tin Lam, Berinderjeet Kaur, Tay Eng Guan, Lee Ngan Hoe, & Leong Yew Hoong *A study of school mathematics curriculum enacted by teachers in Singapore secondary schools.*

This paper provides an overview of the study, which covers the background, the organisation into two supporting studies, the methodology, and the phases of the project.

Paper 2: Berinderjeet Kaur, Lee Ngan Hoe, Ng Kit Ee Dawn, Yeo Boon Wooi Joseph, Yeo Kai Kow Joseph, & Liyana Safii *Instructional Strategies Adopted by Experienced Secondary Teachers when Enacting the Singapore School Mathematics Curriculum.*

This paper presents preliminary findings of Study 1. In particular, it examines the instructional strategies adopted by teachers in the first phase of the project – where thirty competent teachers were selected for close study, which included video-recording of a suite of lessons and post-lesson interviews.

Paper 3: Leong Yew Hoong, Cheng Lu Pien, & Toh Wei Yeng Karen *Chronologically-grounded survey.*

This paper describes a methodological contribution by Study 2. From Phase 1 of the project, we obtained some characteristics of design utilised by competent teachers. To study the extent in which these characteristics capture the design work of teachers across Singapore secondary schools, we developed an instrument: Chronologically-grounded survey.

Paper 4: Tong Cherng Luen, Tay Eng Guan, Berinderjeet Kaur, Quek Khiok Seng, & Toh Tin Lam *Singapore Secondary Mathematics Pedagogy: The DSR DNA.*

This paper reports findings from a statistical analysis of a survey on 689 teachers in the second phase of the project. In particular, it analyses data from 32 items in one component of the survey regarding teacher moves in the classroom.

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Singapore Secondary Mathematics Pedagogy: The DSR DNA

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The Enactment Project seeks to find out in more detail what is happening in Singapore mathematics classrooms. In particular, the video data of 30 teachers suggests that there is a distinctive Singapore secondary mathematics pedagogy, almost like its DNA, which is characterized by cycles of Development, Seatwork and Review (DSR). This paper reports findings from the statistical analysis of a survey on a further 689 teachers regarding aspects of the DSR.

In this paper, we attempt to further explore the instructional core, comprising aspects related to three main components, Development, Seatwork and Review (Kaur, 2017). Our in-depth study of sequences of lessons of 30 competent secondary school mathematics teachers suggests that there is a distinctive Singapore secondary mathematics pedagogy, almost like its DNA. Teachers develop conceptual knowledge through a myriad of ways, before engaging students in seatwork to consolidate their learning which is followed by review of student work in class which allows errors to be springboards for deeper understanding of knowledge explored during the lesson. Each instructional cycle is guided by a micro-instructional objective. Through such cycles the objective of the lesson is carefully achieved, ensuring that conceptual understanding and procedural fluency are achieved through both teacher-centred and student centred activities.

Background

The study reported here is part of a bigger research enterprise known as the Enactment Project. Leong, Cheng and Toh (2019) in another paper in this symposium series explained the two-phase research method as follows: In-depth analyses of classroom instructional practices adopted by thirty exemplary teachers, from which common characteristics were derived, and a survey of 689 teachers to ascertain the extent of these common characteristics. The video data from the first phase suggests that there is a distinctive Singapore secondary mathematics pedagogy, almost like its DNA, which is characterized by cycles of Development, Seatwork and Review (DSR). This paper reports findings from the second phase regarding aspects of the DSR.

Participants and Instrument

The participants were Singapore secondary school teachers from 4 different academic courses: Integrated Programme (60), Express (388), Normal Academic (151), and Normal Technical (90). These four academic courses are broadly based on academic achievement in the Primary School Leaving Examination.

An on-line questionnaire was administered to these teachers with their written consent. The questionnaire consisted of 3 sections: pedagogical structure and student-teacher interaction (60 items); enactment of the different facets of the “pentagon framework” (MOE, 2012) undergirding the secondary mathematics curriculum (78 items); and instructional materials (226 items, of which a participant needs only respond according to one of the subjects Additional Mathematics, Elementary Mathematics, Normal Academic Mathematics).

The 60 items in the first section were divided into two parts of 36 and 24 items respectively.

The first part had items which elicited responses about what the teacher did in class. Three sample items follow below:

1. I focus on mathematical processes (such as compare and contrast, logical reasoning) to facilitate the development of concepts or student understanding
2. I engage students in practising past exam papers
3. I provide collective feedback to whole class for common mistakes and misconceptions related to in-class work and homework

The second part had items which elicited responses about what the teacher wanted the student to do in class. Three sample items follow below:

1. I get my students to explain how their solutions or how their answers are obtained
2. I get my students to practise a similar problem after you have shown them how to do it on the board
3. I get my students to critique one another's work presented on board/screen so as to improve their understanding of concepts or elegance in their presentation/solution

The 60 items were also constructed around ‘teaching moves’ of Development, Student Seatwork and Review. The sample items above are, in order, of each type. Participants were required to respond on a Likert Scale of 1 (Never/Rarely) to 4 (Mostly/Always). In addition, the items in each type were designed to reflect two main teaching styles, namely direct teaching for fluency (Fluency) and student-centred teaching (Student-Centred).

In this paper, we shall only discuss findings based on the first part of the pedagogical structure and student-teacher interaction section. We removed 3 items which focused on whether the teachers used the textbook or customized worksheets as these could be interpreted both as Fluency and Student-Centred. We shall refer to the remaining 33 items as ‘the instrument’.

Data analysis

Generally, satisfactory reliability and validity of the instrument were established. Means, standard deviations, and Cronbach’s α of the three subscales are shown in Table 1. According to Hatcher and Stepanski (1994), a threshold of .55 level of Cronbach’s α can be used in exploratory research. As shown in Table 1, all six subscales demonstrated satisfactory internal consistency.

Table 1

Means, standard deviations and Cronbach's α

Scale	Item Mean	Item Mean Variance	Cronbach's α
Development Fluency (6 items)	3.248	0.049	.709
Development Student-Centred (9 items)	3.002	0.053	.883
Seatwork Fluency (5 items)	2.844	0.117	.722
Seatwork Student-Centred (2 items)	3.119	0.013	.559
Review Fluency (7 items)	3.023	0.036	.765
Review Student-Centred (4 items)	2.336	0.049	.753

A Principal Component Analysis with Varimax rotation method was performed to identify the factor structure of the instrument. Six factors were returned and they accounted for about 54% of total variance. One item was removed because it had loading of less than 0.5 and only on the 6th component, which had the least commonality. A Principal Component Analysis with Varimax rotation method was performed again. Five factors were returned this time and they accounted for about 52% of total variance. No further items were removed but the factor structure was further reduced to four factors because the fifth component was significantly less in communality than those preceding it (1.311 compared with 1.723, 2.072, 3.167, and 8.411).

Since it was reasonable to believe that there would be correlations between the factors, a Principal Component Analysis with Promax rotation was performed on four factors with the factor structure shown in the Appendix. The four factors were named *Student-centred in-class learning*, *Teaching and practice for fluency*, *Teacher-led conceptual learning*, and *Teacher-guided student self-directed learning*. Table 2 shows the correlations among the four factors.

Table 2

Component Correlation Matrix

	1	2	3
2	.252		
3	.507	.361	
4	.391	.127	.216

Discussion

Referring to Table 1, we see that 4 of the 6 subscales had item averages of at least 3, indicating that the teachers in general performed such moves in their classrooms. These moves were linked to development of mathematical concepts through teacher talk and student engagement, student seatwork and review for fluency. Moves which targeted practice (Seatwork Fluency (Item mean 2.844)) were surprisingly less common than teacher-student interaction during seatwork (Seatwork Student-Centred (Item mean 3.119)). However, review seemed to be more teacher-directed (Review Fluency (Item mean 3.023)) than student centred (Review Student-Centred (Item mean 2.336)).

We shall now consider the four factors that seems to underlie teacher moves in the Singapore classroom. Instead of bifurcating into student-centred versus teacher-directed learning, or fluency versus conceptual learning, we find that these aspects are mixed and matched into four amalgams. The first is student-centred in-class learning. Teachers are student-centred both in the development phase (they ask questions to encourage reasoning, and build on students' responses) as well as in the seatwork phase (they provide students with probing guidance (open ended questions), and walk around the class noting students' work that would be used to provide class feedback later). The second is teaching and practice for fluency. These all fall under items in Fluency subscales. Examples of these are using "I do, we do, you do" strategy during the development phase, and engaging students in practising past year exams. Next is teacher-led conceptual learning. Again these all fall under items in Fluency subscales but interestingly they are extracted under a different factor. Looking more closely at the items, we can understand why. Whereas the second factor emphasises fluency through thoughtful practice, this third factor emphasises fluency through conceptual understanding. Some items of this factor are focusing on mathematical vocabulary during the development phase, and helping students identify strategies during the review phase. The final factor is teacher-guided student self-directed learning. Indeed, students need guidance to revise on their own outside the classroom. Thus, moves such as getting students to set their own learning goals and working with their peers to make a plan for revision and correction of mistakes, are attempts by the teacher to ensure that learning takes place outside the classroom.

We chose Promax rotation because it allowed us to see the correlations between the factors. We had always felt that fluency learning and conceptual learning are not mutually exclusive, nor student-centred learning and teacher-directed learning. Indeed, from Table 2, the four factors are seen to correlate significantly. "Student-centred in-class learning" and "teacher-led conceptual learning" have a correlation of 0.507. In addition, "student-centred in-class learning" has reasonably high correlations with the other two factors as well.

The discussion above gives us a clearer picture of how the Development-Seatwork-Review cycle plays out in Singapore classrooms. Data shows that these moves within these phases are generally enacted in the classroom. Interestingly, the data also shows that underlying these moves are student-centred considerations towards fluency and conceptual understanding.

References

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Appendix

There are several approaches that we may adopt in our mathematics lessons. Reflecting on my lessons, I ...	Components 1	2	3	4
use "I do, We do, You do" strategy: Demonstrate how to apply a concept/carry out a skill on the board [I do] Demonstrate again using another similar example but with inputs from students [We do] Ask students to do a similar question by themselves [You do]		0.441		
emphasise basic facts/steps for students to memorise them		0.638		
provide students with sufficient questions from textbooks/workbooks/other sources to practise so as to develop procedural fluency		0.489		
use exposition (teacher at the front talking to whole class) to explain mathematical ideas, facts, generalisations		0.455	0.369	
focus on mathematical vocabulary (such as equations, expressions) to help students build mathematical concepts			0.795	
focus on mathematical vocabulary (such as factorise, solve) to help students adopt the correct skills needed to work on mathematical tasks			0.859	
ask students to recall past knowledge	0.486			
ask direct questions to stimulate students' recall of past knowledge/check for understanding of concepts being developed in the lesson	0.463	0.401		
ask questions to encourage reasoning and speculation, not just to elicit right answers	0.756			
use examples and non-examples to engage students in discussion to make sense of a concept	0.731			
focus on mathematical processes (such as compare and contrast, logical reasoning) to facilitate the development of concepts or student understanding	0.724			
lead whole class discussion (with guided questions) to facilitate the development of concepts	0.737			
exchange ideas with students on how to solve a problem	0.731			
ask students open-ended questions and allow them to build on one anotherâ€™s responses to develop concepts or clarify their understanding	0.846			
build on students' responses rather than merely receiving them	0.763			
get students to automatise steps leading to a solution through repetitive exercises		0.778		
engage students in practising past exam papers		0.681		
provide students with directed guidance (ask close-ended questions) when they face difficulty with a mathematical task they are doing, focusing them on the concept/skill necessary to do the task		0.703		
tell students how to do it when they face difficulty with a mathematical task they are doing		0.717	0.323	
walk around the class and provide students with between desk instruction (i.e. help them with their difficulties) when they are doing work at their desks		0.384		
provide students with probing guidance (open-ended questions about their thinking and why they are considering certain approaches) when they face difficulty with a mathematical task they are doing	0.462			
walk around the class noting students' work that I would draw on to provide the class feedback during whole class review when they are doing work at their desks	0.328			
explain what exemplary solutions of mathematics problems must contain (logical steps and clear statements and/or how marks are given for such work during examinations)			0.581	
help students identify strategies that would help them achieve their learning goals for mathematics			0.461	0.493
encourage students to show me their work and review their progress for mathematics			0.430	0.385
provide feedback to individuals for in-class work and homework to serve as information and diagnosis so that students can correct their errors or improve			0.588	
provide collective feedback to whole class for common mistakes and misconceptions related to in-class work and homework			0.617	
review student performance by providing the class detailed comments on tests and examinations			0.319	0.317
get students to set their own learning goals for mathematics at the beginning of each school term/semester				0.664
get students to make a plan to revise their work and correct their mistakes				0.724
get students to work with peers to make a plan for revision and correction of mistakes				0.734
get students to grade their own mathematics work (with the marking scheme/rubric provided and teach them how to use it)				0.621

Note: Grey rows indicate Fluency subscales.