

# **STRONG STANDARDS:**

A Review of Changes to State Standards Since the Common Core



## **CONTENTS**

Introduction	2
English Language Arts	3
High-Level Findings for ELA	4
An Analysis of State ELA Standards	6
Foundational Skills	6
Reading Standards for Literature and Informational Texts	7
Evidence Drawn from Text	10
Academic Vocabulary Acquisition and Use	10
Writing (Specifically from Sources) and Research	11
Oral Communication and Collaboration	12
Grammar and Conventions	13
An Analysis of Text Complexity and Guidance	14
Standards are Necessary but Insufficient	18
Disciplinary Literacy	20
High-Level Findings on Disciplinary Literacy	20
An Analysis of Disciplinary Literacy Standards	21
Readiness Requires More than Standards	74

Mathematics	. 26
High-Level Findings for Mathematics	. 26
An Analysis of State Mathematics Standards	. 28
Structure	. 29
Mathematical Practices	. 30
Procedures, Conceptual Understandings, and Applications	. 32
Sequencing	. 34
Grades K–5	. 35
Grades 6–8	37
High School	. 38
Discussion	. 43
References	.44
Appendix A	.48
Appendix B	.48
Appendix C	.48
Appendix D	.49
Acknowledgments	50

## INTRODUCTION

It has been seven years since the Common Core State Standards (CCSS or Common Core) were developed and quickly adopted by 45 states and the District of Columbia. Since then, 24 of those states have reviewed and revised their English language arts (ELA) and mathematics standards. In most states, the review process was triggered in response to mounting political opposition to the Common Core or associated testing and accountability policies. As the political battles have largely subsided, it is time to take stock of the condition of standards across these states. At Achieve, we wanted to know whether the new standards are any good.

More specifically, we set out to learn how well these state-specific standards incorporate the best available evidence about the characteristics of high-quality college- and career-ready (CCR) standards — standards that, if met, would prepare young people to enter and succeed in entry-level, credit-bearing courses in postsecondary institutions and to have access to careers.

Achieve was founded more than 20 years ago for this very purpose: to evaluate and help states get their standards right. Because of our experience working with states on their standards over the past two decades, Achieve is uniquely positioned to evaluate them. Getting state CCR standards "right" is key to improving the performance of each state's K–12 system and the preparation of its young people. Academic standards are the cornerstone of a state's K–12 education system, and all other academic efforts in states and districts are predicated on getting the standards — academic expectations — correct. Aim too low, and high school graduates are unprepared for life after high school. It goes without saying that standards are just the beginning and that without solid implementation (educator professional development, aligned curricula and assessments, etc.), the promise of high expectations cannot be fully met.

Achieve pioneered the concept of CCR standards through our signature initiative, the American Diploma Project, a research and development project carried out in partnership with the Thomas B. Fordham Institute and The Education Trust to identify the mathematics and literacy skills essential for success in postsecondary education and training programs. In 2005, Achieve formed the American Diploma Project Network of 35 states committed to aligning their standards, assessments, and high school graduation requirements with the skill demands of college and careers. We worked with teams of K–12 and postsecondary educators and employers in 16 states to revise their high school standards in ELA and mathematics. By 2008, we found that when each state anchored its standards in the real-world evidence of what students need for postsecondary success, a common core of expectations emerged across the states. This finding provided an "existence proof" that states could work together to develop common standards and was a key part of the foundation for the CCSS initiative led by the Council of Chief State School Officers and the National Governors Association.

The CCSS initiative was also spurred by the growing recognition that state standards suffered from common weaknesses. For example, virtually none were intentionally aligned with the skills necessary for postsecondary success, and few were benchmarked to expectations in high-performing countries. Instead, state standards represented an agreement among content experts regarding what is desirable for students to learn, but they were not developed based on a careful analysis of evidence regarding the skills students must have by the time they complete high school to enter and succeed in two- and four-year colleges, career training programs, and the workplace.

Prior to the CCSS, there was little consistency across states in the content, clarity, or rigor of expectations from grade to grade in core reading and mathematics standards. Research by Andy Porter and his colleagues found as few as 20 percent of grade-level mathematics standards were common across states. Further, compared to high-performing countries, mathematics curricula and standards in U.S. states were "a mile wide and an inch deep" according to Professor Bill Schmidt at Michigan State University. Schmidt also noted that "[t]he U.S. curriculum as reflected in many of the states' standards and in our nation's textbooks tends to reflect an arbitrariness where topics appear somewhat haphazardly throughout the grades. For example, teachers are expected to introduce relatively advanced mathematics in the earliest grades before students have had an opportunity to master basic concepts and computational skills. Secondly, the curriculum continues to focus on basic computational skills through grade eight and perhaps beyond." <sup>3</sup>

State reading standards also had characteristic weaknesses. For example, standards for reading comprehension typically showed little or no progression from grade to grade. As a result, the expectations for students sometimes stagnated, which contributed to low levels of rigor. We learned from ACT's research that students in college and employees in the workforce are expected to read and understand texts that are significantly more challenging than the texts students typically read in high school.

The CCSS effectively addressed these weaknesses; so have the state standards reviewed in this study. Based on this analysis as well as even more detailed reviews Achieve has conducted for a number of states in this study, we can say with confidence that across the country and with very few exceptions, the quality of state standards is significantly higher now than prior to the development of the Common Core.

## **ENGLISH LANGUAGE ARTS**

The state English language arts (ELA) standards reviewed in this analysis almost universally reflect the key elements research has identified as necessary foundations for college and career readiness. These elements are supported by research and constitute a core body of knowledge and skills that students need for citizenship and to be successful in college and their chosen careers. Achieve reviewers looked specifically at seven key elements in ELA standards to determine if the standards meet the bar for college and career readiness:

- **Foundational Skills:** College- and career-ready (CCR) standards should address the skills of phonemic awareness, phonics, and fluency in the early grades and delineate the skills that will prime students for success in reading comprehension and writing.
- Reading Standards for Literary and Informational Texts: Reading standards should address a suite of skills that require students to make meaning from text. Research suggests that a key element of college and career readiness is students' ability to comprehend literature and informational texts, including literary nonfiction. The relationship between content knowledge and informational text reading is also well supported by research. Both findings point to the importance of standards including expectations around literature and informational text reading.
- Evidence Drawn from Text: Standards should delineate expectations that students defend ideas, beliefs, or claims with authoritative, relevant, and well-researched sources. Thus, CCR standards should state clear expectations that students refer to text(s) when researching a topic and presenting their ideas.

<sup>&</sup>lt;sup>1</sup> Porter, McMaken, Hwang, & Yang, 2011.

 $<sup>^2</sup>$  Schmidt, 2010.

<sup>3</sup> Ibid.

- **Academic Vocabulary Acquisition and Use:** CCR standards for ELA should state clear expectations around vocabulary, including a focus on vocabulary acquisition skills and on Tier II/academic and domain-specific words.
- Writing (Specifically from Sources) and Research: Writing standards should focus on process and product, with an emphasis on using text-based sources to develop a written idea. CCR research also suggests that students will need skills in producing texts across narrative, informational, and argumentative modes. In addition, CCR research points to the importance of students' skills in inquiry evaluating the credibility and reliability of sources, integrating across sources, and focusing research questions. Standards can encourage this work by specifying short- and long-term research projects.
- Oral Communication and Collaboration: Standards for communication and collaboration should state the expectation for students to acquire language through listening to others and verbally producing expressions, starting in the earliest years and continuing through high school. These twin skills help students in later years understand and produce written language. CCR research also suggests that students at all grade levels should develop skills in collaborative and cooperative learning and discussion, in making oral presentations and using effective techniques for delivery, and in analyzing and evaluating the messages and techniques of oral and multimedia messages.
- **Grammar and Conventions:** CCR standards should include a sequence of grammar and convention skills that build and can be reinforced across grade levels in increasingly sophisticated oral and written contexts.

Achieve analyzed the state standards themselves — those statements that articulate the specific expectations for student performance, knowledge, and/or skill — and how the state approaches text complexity. To evaluate states' text complexity guidelines, Achieve reviewed information often presented in accompanying state documents that are inextricably tied to student performance in ELA. Specifically, the reviewers:

- Evaluated each state's ELA standards against the key elements, listed above, indicated by CCR research; and
- Appraised each state's standards and guidelines around text complexity for steadily increasing expectations from kindergarten through grade 12 that prepare students for postsecondary reading expectations.

This section of the report presents and discusses the key findings around the ELA standards and their attendant text complexity guidelines.

## HIGH-LEVEL FINDINGS FOR ELA

This review of state ELA standards revealed the following trends and key findings:

• Twenty out of 24 states reviewed include or have retained each of the key elements required in ELA to prepare students for citizenship, college, and career.

In ELA, most states reviewed strongly include the seven research-backed elements of college and career readiness listed above and reflect an ongoing commitment to preparing students to meet a range of postsecondary literacy demands. As a result, whether students move from school to school, across districts, or over state lines, they will face similar rigorous expectations — to cite textual evidence in support of their claims and conclusions, to write arguments, to conduct research, and to analyze and discuss what they have read. The shared commitment to key CCR elements is promising for educators as well. In large part, educators can still share a common language, professional development, and curricular materials across state lines as well as collaborate with their peers on a whole range of academic issues.

## • States address text complexity in a variety of ways; some of the approaches threaten to undermine the state's CCR agenda.

The texts high school graduates are likely to encounter in college and on the job are complex and challenging. And, the ability to comprehend complex text — not the ability to distinguish between literal and inferential thinking, find the main idea, or identify supporting details — is the factor that most differentiates college-ready readers from those students who are not ready for the reading demands.<sup>4</sup>

Various elements affect the complexity of a text; these elements include vocabulary, the length of the sentences, the text's organization and structure (such as shifting perspectives or flashbacks in a literary text or the use of clear transitions in an informational text), and the knowledge required to understand a text (and the reader's familiarity with the topic). A text's complexity is determined through a combination of quantitative measures (using formulas that calculate sentence and word length and word frequency, for example) and qualitative elements (such as by an evaluation of the language, structure, and knowledge demands of the text). Research has shown that college, workforce training, and workplace reading materials are written at a level of difficulty that exceeds the levels of difficulty of the texts most students encounter in high school. As a matter of equity and excellence — and to ensure that all students are prepared for postsecondary demands — states need to set clear guidelines for text complexity (using both quantitative and qualitative measures) that steadily increase through the grades to college- and career-level reading.

Some states have adopted strong ELA standards and clear, explicit guidance on text complexity, resulting in a CCR-aligned system. One key concern of Achieve reviewers, however, is that this fundamental college and career readiness requirement — text complexity — is imperfectly represented in several states. Some states include standards that set an expectation for increasing text complexity but fail to provide — perhaps inadvertently — clear, explicit guidance on how to evaluate appropriate complexity grade to grade. As noted above, this lack of guidance creates serious equity concerns: Without clarity on text complexity, the state cannot assume parity in academic reading expectations among students across the state, especially for those students most vulnerable to low academic expectations.

A few states present conflicting or optional text complexity guidance, leaving these decisions to individual districts or schools even though a lack of clear state-level guidance could easily result in lower expectations for some students. These states stand in stark contrast with other states that provide clear guidance around text complexity so that districts, schools, and teachers across the state grasp the importance of consistent CCR expectations for text complexity.

In addition to providing clear, consistent, and comprehensive guidance, states can also nudge districts toward making effective CCR decisions around text complexity in other ways, including:

- Emphasizing text complexity in preservice education;
- Providing targeted and ongoing professional development;
- Encouraging use of resources such as the Council of Chief State School Officers (CCSSO) Navigating Text Complexity website<sup>5</sup> built by and for states; and
- Ensuring that the statewide assessment test passages align in complexity with the state's grade-level text complexity guidelines and that these parameters around text complexity on the statewide assessment system are made transparent to educators statewide.

<sup>&</sup>lt;sup>4</sup> ACT, 2006.

<sup>&</sup>lt;sup>5</sup> http://www.ccsso.org/Navigating\_Text\_Complexity.html

• While most state standards continue to address vital content and skills, some revisions have made sharing curricular or assessment materials more challenging for states.

As states have made revisions to reorder or renumber standards, add standards, remove standards, manipulate anchor standards, or remove clear expectations around text complexity, educators in different states will have more difficulty sharing curricular and assessment materials and noting alignment of materials. Educators aligning their new standards to curricular and assessment materials developed outside of their home state will have to pay careful attention to differences in language to ensure that their state standards are addressed.

## AN ANALYSIS OF STATE ELA STANDARDS

Aligning academic expectations to postsecondary demands is critical for states interested in preparing students for college, career, and civic life. By setting clear, rigorous K–12 educational standards, states can help ensure that the same high standards for academic excellence are shared across districts, schools, and classrooms statewide and that all students are prepared for the next grade level and beyond.

Each state was analyzed for each CCR element and rated on a three-point scale:

Rating	Explanation							
2: STRONG	The CCR element is clearly and fully addressed.							
1: MODERATE	The CCR element is not clearly or completely addressed.							
0: WEAK	The CCR element is weak/nonexistent.							

## **FOUNDATIONAL SKILLS**

Almost all states reviewed have foundational literacy standards that address phonemic awareness, phonics, and fluency.

2: Sī	2: STRONG						1: M	ODE	RATE			0: WEAK/ABSENT								
	CCR standards should address the skills of phonemic awareness, phonics, and fluency in the early grades and delineate the skills that will prime students for success in reading comprehension and writing.																			
	AZ												ND	ОН	ОК	PA	sc	TN	UT	wv

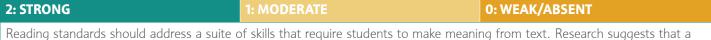
States should include clearly delineated foundational reading standards in the early grades. Early literacy is the foundation upon which more complex reading and writing skills are built. Students with weak phonics skills who are unable to decode fluently and automatically when reading texts of increasing grade-level complexity will struggle with reading and writing in later grades.

Some states revised their foundational skills standards in interesting and innovative ways that further support early literacy development. For example, Arizona, Oklahoma, and Tennessee added standards for foundational writing, which focus on applying grade-level phonics and word-analysis skills when encoding words and writing legibly in grades K to 5. Massachusetts includes expectations for prekindergarten literacy that segue nicely into the kindergarten expectations. For example, one of the prekindergarten Massachusetts standards requires that students "[s]how awareness of the rhythmic structure of a poem or song by clapping or through movement" as the grade-level expectation for an analysis of the structure of texts.

The importance of early literacy instruction cannot be overstated: It is key to preparing students to meet the reading and writing demands they will face in later grades. A significant element of that preparation is the development of fluency with grade-level complex texts, not texts at students' reading level. Fluency — the ability to read quickly, accurately, and with proper expression — is strongly intertwined with comprehension, and students should engage regularly in fluency practice with appropriately complex grade-level texts to build needed skills. Two states' text complexity guidance, North Dakota's Appendix C and Oklahoma's quantitative ranges, set levels that are below CCR levels, resulting in students practicing fluency on texts that are too low for them to develop the crucial early reading skills they need.

## READING STANDARDS FOR LITERATURE AND INFORMATIONAL TEXTS

Most reading standards reviewed were rated as 2 in terms of the CCR content; these states include the suite of skills that must be taught as part of a CCR reading curriculum and integrate expectations for both fiction and nonfiction texts into the standards.



Reading standards should address a suite of skills that require students to make meaning from text. Research suggests that a key element of college and career readiness is students' ability to comprehend literature and informational texts, including literary nonfiction. The relationship between content knowledge and informational text reading is also well supported by research. Both findings point to the importance of standards including expectations around literature and informational text reading.



When evaluating reading standards, Achieve reviewers looked specifically for how the standards address theme or central/main idea; the development of characters, events, or ideas; text structure; point of view and purpose of a text; argument and claims in a text; and cross-text comparisons and synthesis of themes or topics.

Having knowledge about a topic is strongly correlated with reading comprehension; it has a greater impact on students' reading comprehension than does their generalized reading ability. The more students know about a topic, the more capably they read more complex texts and build new schema.<sup>6</sup> Students draw on their topical knowledge to learn and retain new information when reading more complex texts on that topic. Beginning in kindergarten and extending through grade 12, standards should clearly require that students build knowledge from texts, specifically from content-rich nonfiction. Wide reading of content-rich nonfiction is one of the most effective ways for students to develop skills and build schema and background knowledge. All states reviewed include expectations that students read both literature and informational text. ELA teachers should expose students to a wide range of literature, poetry, drama, and content-rich literary nonfiction in grades 6–12, which might include speeches; opinion pieces; journalism; historical, scientific, and economic accounts written for a broad audience; travel or nature writing; biography, autobiography, and memoirs; or personal essays on a range of issues. Teachers across the content areas, including science and history/social studies, should also encourage reading of content-rich informational text across all grade levels.

Most states reviewed have CCR reading standards, and several have revised these standards in innovative ways. For example, Massachusetts includes literary expectations for K–5 mathematics in the prekindergarten to grade 5 reading standards and provides educators with examples of narrative stories that include mathematics elements, reinforcing the importance of cross-disciplinary connections. South Carolina includes fluency standards that extend beyond foundational reading through grade 12, reflecting the connection between comprehension and fluency. Oklahoma and South Carolina include standards for sustained independent reading — an expectation that reinforces knowledge building.

<sup>&</sup>lt;sup>6</sup> Recht & Leslie, 1988; Willingham, 2006; Kintsch, 1998.

While most of the states' focused and targeted reading standards progressively build in sophistication and demand over the years, Oklahoma's and Missouri's reading standards often require students to identify a laundry list of literary terms, or they lack analytical or clear expectations, more frequently in the grades K-5 than in the grades 6-12 standards.

State Standards	Standard	Commentary
MISSOURI Learning Standards (2016)	Reading, Grade 5, 3.B  Read, infer, and draw conclusions to: a. evaluate if the author's purpose was achieved, identify reasons for the decision, and provide evidence to support the claim b. analyze multiple accounts of the same event or topic, noting important similarities and differences in the point of view they represent c. verify facts through established methods d. identify the author's viewpoint or position, supporting premises and evidence, and conclusion of a persuasive argument e. recognize exaggerated, contradictory, or misleading statements f. explain the type of evidence used to support a claim in a persuasive text g. use reasoning to determine the logic of an author's conclusion and provide evidence to support reasoning	This reading standard appears under 3 ("Develop and apply skills and strategies to comprehend, analyze, and evaluate nonfiction [e.g., narrative, information/explanatory, opinion, persuasive, argumentative] from a variety of cultures and times.") and B ("Literary Techniques 6-12 Correlation Reading Informational 1D, 2D, 2B, 2C"). However, how the expectations relate to literary techniques is not clear.  The standard provides a laundry list of overlapping expectations, and some are unclear. For example, what does it mean to "verify facts through established methods"? How would that substandard translate into instruction?
OKLAHOMA Academic Standards	Grade 5: 5.3.R.4 Students will evaluate literary devices to support interpretations of literary texts:  • simile  • metaphor  • personification  • onomatopoeia  • hyperbole  • imagery  • symbolism*  • tone*  *Students will find textual evidence when provided with examples.  Grade 8: 8.3.R.4 Students will evaluate literary devices to support interpretations of literary texts:  • simile  • metaphor  • personification  • onomatopoeia  • hyperbole  • imagery  • tone  • symbolism  • irony	These 5th- and 8th-grade standards are mainly long lists of isolated literary terms, with little increase in cognitive rigor from 5th to 8th grade; the only change in grade 8 is the addition of irony.  Ironically, these expectations appear under Standard 3: Critical Reading and Writing, which states that "[s]tudents will apply critical thinking skills to reading and writing." Identifying examples of similes, for example, is not a demonstration of critical reading in grade 8.  Additionally, presenting such a list moves away from students understanding how to interpret the effect of a literary device and how it is used and instead zeroes in on a memorizing a myopic set of terms.

Most states' reading standards clearly distinguish between informational text and literature, clearly articulating specific reading expectations for each type of text. Some states, including Missouri, Oklahoma, New York, and West Virginia, combine both types of reading into one list of standards, with varying degrees of clarity. West Virginia, for example, merges informational text and literature expectations into one strand but then provides clear and separate standards for informational text and literature. Consider this example from 5th grade:

West Virginia Standard	Explanation
ELA 5.3: Compare and contrast two or more characters, settings, or events in a story or drama, drawing on specific details in the literary text (e.g., how characters interact).	This standard, found in the Key Ideas and Details section of the Reading standards, clearly identifies literary text as the source by using the words story, drama, setting, characters, and literary text.
ELA 5.6: Using an informational text, explain the relationships or interactions between two or more individuals, events, ideas, or concepts in a historical, scientific, or technical text based on specific information in the text.	This standard, also found in the Key Ideas and Details section of the Reading standards, clearly identifies informational text as the standard's focus.

Some states — though rated as 2 overall — include an occasional standard that Achieve cautions against, such as those with a focus on making personal connections to text or using specific learning strategies.

Standards should be attainable for all students. Academic standards that require students to make personal connections or use specific learning strategies pose challenges to equity and measurability. A standard that requires students to make personal connections to texts means that students without the relevant experiences or background cannot meet that standard. Texts and students are diverse, and educators cannot (and should not) assume that all students have the relevant background experiences necessary to make the most germane connections — in short, some students will be able to and others will not, raising a host of equity issues.

Standards that describe learning strategies also pose measurability challenges. Students cannot be reliably and validly assessed on their use of internal strategies, and these strategies may not be applicable to every testing situation — expert readers use strategies to make meaning only on an as-needed basis. Standards need to describe measurable, observable outcomes — the what — not internal metacognitive strategies or teaching or learning strategies — the how.

Sometimes, states include standards requiring that students make personal connections to text or use specific learning strategies (e.g., think-pair-share); often, the proposed standards are dropped after the review period and before adoption. New York's and Ohio's adopted reading standards, however, include language requiring students to make personal connections to text; Ohio's reading standards also include language specifying the use of specific reading strategies (e.g., activate prior knowledge).

State	Standard
ОНЮ	<b>RL.8.10:</b> By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band independently and proficiently. Build background knowledge and activate prior knowledge in order to make text-to-self, text-to-text, and text-to-world connections that deepen understanding of the text.
NEW YORK	<b>8.R.9:</b> Choose and develop criteria to evaluate the quality of texts. Make connections to other texts, ideas, cultural perspectives, eras, and personal experiences.

While making personal connections to text can, on occasion, be a worthwhile classroom activity to foster engagement, it is limiting as a state academic standard. In addition to issues of equity, standards like these pose both vertical articulation and measurability challenges; deepening the rigor of standards built on reading strategies from year to year is unlikely, and students cannot be reliably and validly assessed on the strength of their text-to-self connections. Concomitantly, learning strategies are the means to the end, not the intended goal or outcome of learning and teaching, and thus do not qualify as standards. While developing students' strategies for approaching texts may be worthy of classroom activities or modeling to illustrate the practices of good readers, strategy development is not an outcome appropriate for an academic standard.

## **EVIDENCE DRAWN FROM TEXT**

All but one set of state ELA standards reviewed were rated as 2 in terms of drawing evidence from text.



Standards should delineate expectations that students defend ideas, beliefs, or claims with authoritative, relevant, and well-researched sources. Thus, CCR standards should state clear expectations that students refer to text(s) when researching a topic and presenting their ideas.



States have adopted standards that take the position that students should know how to find and use evidence from text to support their findings and inferences. The one outlier — Oklahoma — includes some expectations for students to use textual evidence, but the requirement is only inconsistently included and is not present across all grade levels. For example, standard 3.R.5 morphs throughout the grades and inconsistently addresses textual evidence. The standard requires students to "evaluate textual evidence to determine whether a claim is substantiated or unsubstantiated" (8.3.R.5) in grades 8 and up, but in grades 1–7, the same standard is focused on determining fact from opinion (e.g., 4.3.R.5 "Students will distinguish fact from opinion in a text and investigate facts for accuracy"). As concerning is an additional standard that requires students to make inferences about textual evidence rather than *make inferences* and support them with textual evidence.<sup>7</sup>

Similarly, standards 3.R.6 (structure of texts) and 3.R.7 (comparison of texts) inconsistently address textual evidence across grades K-12, resulting in an incoherent system.

## ACADEMIC VOCABULARY ACQUISITION AND USE

All but one set of state ELA standards reviewed were rated as 2 in terms of vocabulary.

## 2: STRONG 1: MODERATE 0: WEAK/ABSENT

CCR standards for ELA should state clear expectations around vocabulary, including a focus on vocabulary acquisition skills and on Tier II/academic and domain-specific words.



Nearly a century's worth of research has correlated vocabulary knowledge to reading comprehension. A rich and varied vocabulary is necessary for students to understand what they hear and read as well as for them to communicate clearly with others. Interacting with text to strengthen comprehension and build knowledge includes acquiring a steadily increasing vocabulary with an emphasis on Tier II<sup>8</sup> words and academic language — words that are likely to appear in a variety of texts and content areas. Students need to build strong academic vocabularies, and so it is essential that standards at each grade level delineate clear and specific expectations around vocabulary acquisition and use.

Considering vocabulary's undeniable connection to comprehension and communication, state standards should prioritize acquiring and using a rich and robust vocabulary. Many of the states address vocabulary in multiple standards. They signal vocabulary's importance in a variety of ways, most often through including vocabulary expectations in reading, writing, and language standards.

 $<sup>^7</sup> https://www.achieve.org/publications/review-oklahoma-january-2016-english-language-arts-and-mathematics-academic-standards$ 

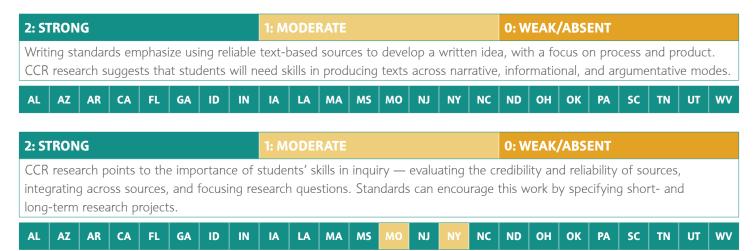
<sup>&</sup>lt;sup>8</sup> Tier II words are vital to comprehension, appear in many texts, and are frequently part of word families and semantic networks. These words may have multiple meanings depending on context, so familiarity with a traditional use of the word may not support a more specific or nuanced use.

For example, Arizona addresses vocabulary in Standard 4 for Reading Informational Text and Reading Literature as well as in Language Standards 4, 5, and 6. Indiana includes a separate section in the standards that is dedicated to vocabulary. Idaho not only addresses vocabulary in its reading and language sections of the standards but also includes vocabulary in the standard for writing arguments. While other states specify the use of precise and domain-specific vocabulary in writing in different modes, Idaho also includes an expectation for precise vocabulary use in students' written arguments: "Use precise language and domain-specific vocabulary to support the argument."

One state — Missouri — does not have a strong and consistent focus on vocabulary across grades K–12. While Missouri's K–5 standards address vocabulary comprehensively, standards in grades 6–12 do not. Missouri has only one standard for vocabulary in the reading section, and the standard itself is limited: "Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and content-specific meanings using context, affixes, or reference materials." Students are limited to merely determining the meaning of words and phrases; no reference is made to either academic vocabulary or how specific word choices shape meaning and tone. Moreover, the expectation for academic vocabulary use is not reinforced in the writing standards.

## WRITING (SPECIFICALLY FROM SOURCES) AND RESEARCH

All reviewed states' ELA standards were rated as 2 in terms of writing from sources. All but two ELA state standards were rated as 2 in terms of research.



Writing has been shown to strengthen reading comprehension and is a requirement in college and most salaried and blue-collar jobs. Thus, to be college and career ready, students should spend considerable time writing from sources for varied audiences and purposes. High-quality writing results from careful planning, drafting, and meaningful revision. The discipline used to create, reshape, and polish pieces of writing prepares students for occasions when they must write quickly and clearly on demand, whether in the workplace or in college classrooms. Research requires the ability to frame, analyze, and solve problems while building on the ideas and contributions of others. As future college students or employees, students will be asked to hone these essential skills with increasing sophistication. States with CCR standards should emphasize writing and research, and they should require students to use credible and reliably accurate sources to support their writing and conduct short and more sustained research regularly throughout the school year.

<sup>&</sup>lt;sup>9</sup> In a meta-analysis of writing instruction, Graham & Hebert (2010) found that having students respond to a text, including analyzing and interpreting the text, and teaching students the writing skills and processes that create text positively affect reading comprehension.

All states reviewed have standards for opinion/argument, informative/explanatory, and narrative writing modes.

Some standards reflect additional state priorities:

- Alabama includes standards specific to writing poetry in grades 1–2.
- Massachusetts emphasizes the link between reading, writing, research, and language by including instructional examples with multiple, integrated standards along with many of the writing standards. Consider this example from Writing Standard 2 in grade 6:

Massachusetts Writing Standard	Embedded Example
Write informative/explanatory texts (e.g., essays, oral reports, biographical feature articles) to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content.  a. Introduce a topic; organize ideas, concepts, and information in paragraphs and sections, using strategies such as definition, classification, comparison/contrast, and cause/effect; include text features (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension.  b. Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples.  c. Use appropriate transitions to clarify the relationships among ideas and concepts.  d. Use precise language and domain-specific vocabulary to inform about or explain the topic.  e. Establish and maintain a style appropriate to audience and purpose (e.g., formal for academic writing).  f. Provide a concluding statement or section that follows from the information or explanation presented.	After reading both historical fiction and non- fiction sources about the Salem witch trials, a student blends informational and narrative writing to present an individual character's actions and to explain the larger meaning of beliefs about guilt and innocence in 17th cen- tury Salem. See the Massachusetts Writing Standards in Action example, "The Salem Witch Trials." (W.6.2, W.6.3, W.6.4, W.6.8, W.6.9, RI.6.1, L.6.1, L.6.2, L.6.3, L.6.5, L.6.6)

Most states have requirements that students conduct both short- and long-term research. Missouri and New York, however, emphasize long-term, sustained research, which could lead to fewer opportunities for students to hone their research skills.

## ORAL COMMUNICATION AND COLLABORATION

All state standards reviewed for ELA were rated as 2 in terms of oral communication and collaboration.

2: S	TRONG						1: M	ODE	RATE						0: WEAK/ABSENT							
Stan	Standards for communication and collaboration should state the expectation for students to acquire language through																					
liste	listening to others and verbally producing expressions, starting in the earliest years and continuing through high school.																					
The	These twin skills help students in later years understand and produce written language. CCR research also suggests that																					
stud	lents at all	grade	levels	shou	ıld de	velop	skills	s in co	ollabo	rative	e and	соор	erativ	e lear	ning	and d	iscuss	ion, ii	n mal	king c	ral	
pres	entations a	and usi	ng ef	fectiv	e tec	hniqu	ies fo	r deli	very,	and in	n anal	yzing	and	evalua	ating	the m	iessag	es an	nd tec	hniqu	ies of	
oral	oral and multimedia messages.																					
													ND/	NG		011	01/	24				1407
AL	AZ AR	CA	FL	GA	TD	IN	IA	LA	MA	MS	MO	רא	NY	NC	ND	ОН	OK	PA	SC	IN	UT	WV

Oral language development is the primary source for written language, and children's oral language is predictive of their reading and writing skills. For the youngest learners, oral language is important and can predict how fast learners will acquire new language. Oral communication skills are not just critical for young learners: Older students, especially second language and low-income students, benefit from rich, robust oral communication. Concomitantly, communication skills are another requisite for college and career readiness. Employers and college professors cite strong communication skills and the ability to work in a team as two of the top three attributes they seek. Success in credit-bearing college coursework, whether in the humanities, sciences, or social sciences, depends heavily on effective communication about the concepts and detailed information contained within readings, lectures, and class discussions. Success in the workplace, whatever the profession, also depends heavily on one's ability to listen attentively to colleagues or customers, to express ideas clearly and persuasively, and to collaborate effectively. Considering the importance to colleges and employers, K–12 ELA standards should include expectations that students speak, listen, and collaborate effectively.

Some states have expanded their standards for speaking and listening to include additional, innovative expectations. Tennessee, for example, creates a Linking Standards category, which pairs each of the speaking and listening standards to specific reading and writing standards, reinforcing the relationships among reading, writing, speaking, and listening. Grade 6 Speaking and Listening Standard 1 reads, "Prepare for collaborative discussions on 6th grade level topics and texts; engage effectively with varied partners, building on others' ideas and expressing their own ideas clearly." In the standards document, this standard is linked to Reading — Literature Standards 1–7, 9, and 10; Reading — Informational Text Standards 1–10; and Writing Standards 5 and 6.

## GRAMMAR AND CONVENTIONS

All state standards reviewed for ELA were rated as 2 in terms of grammar and conventions.

2: S	TRON	G						1: M	ODE	RATE						0: WEAK/ABSENT								
					lude a isticat			_				ventic	n skil	ls tha	t buil	d and	l can	be rei	nforce	ed acı	ross g	grade		
AL	AZ	AR	CA	FL	GA	ID	IN	IA	LA	MA	MS	мо	NJ	NY	NC	ND	ОН	ОК	PA	sc	TN	UT	wv	

Though all states fared well with grammar and conventions, they certainly are not all one and the same. States' approaches to grammar and conventions in their respective standards vary:

- Alabama emphasizes subject—verb agreement throughout the grades.
- **Indiana** focuses on the same key skills (e.g., K–5: nouns/pronouns, verbs, adjectives/adverbs, prepositions, usage; 6–12: phrases and clauses, usage) across grade levels; at each grade, the expectation for the skill deepens, reinforcing the iterative nature of language learning.
- Massachusetts includes a new category, Sentence Structure, Variety, and Meaning, with standards specific to sentence structure, emphasizing the notion that sentences and sentence structure build in complexity across grades K–12.

<sup>&</sup>lt;sup>10</sup> Hart & Risely, 2003.

<sup>&</sup>lt;sup>11</sup> National Association of Colleges and Employers, 2016.

## AN ANALYSIS OF TEXT COMPLEXITY AND GUIDANCE

How well students comprehend, analyze, and respond to grade-appropriate complex literary and informational texts is the bedrock of their preparation for citizenship, college, and career. As a result, clear guidance on increasing grade-level text complexity is a necessary element of CCR standards for reading.

Data indicate that students in college and employees in the workforce are expected to make meaning of texts that are significantly more challenging than the texts students typically read in high school. A 2008 analysis of average text readability scores for grades 11/12 through to the freshman year of college concluded that the gap between the two is four years. Researchers found that the average 11th- to 12th-grade text had an average Lexile<sup>12</sup> score of 1123 but the average workplace, community college, and university text had average Lexile scores of 1248, 1292, and 1383, respectively.<sup>13</sup> This finding is echoed in a 2006 report from ACT Inc.: The biggest differentiator between students who were considered college ready and those who were not is whether students were able to read and understand complex text.<sup>14</sup> Reading levels are just as important in the workforce and skilled trades. Welders, cutters, and welder fitters encounter texts<sup>15</sup> with a Lexile score between 1150 and 1200; the Lexile measure of automotive service technicians and mechanics' workplace texts increases to between 1400 and 1500. The Pioneer Institute echoes these findings and identifies reading as an important workforce skill, indicating that professional plumbers, major appliance repair technicians, and automobile mechanics encounter workplace texts that are written at levels up to grade 14. Thus, to ensure readiness for what comes next (citizenship, college, career), as students advance through the grades, it is imperative that their reading abilities advance too.

Standards are the primary lever states can pull to ensure educational equity for all students; they are also one place states can take a strong stance about the expected reading levels for students statewide. Ensuring that students are prepared by high school graduation to meet the reading demands of college and the workplace requires providing them with access to texts during the school year that fall within an appropriately challenging quantitative range of grade-band complexity. Reading grade-appropriate complex text across ELA, science, and history/social studies, with the support of a teacher, provides students opportunities to develop the skills necessary to navigate the texts they will encounter in college and the workplace. Students who are not exposed to increasingly complex texts as they move up the grades risk never acquiring the skills they need to meet the reading demands they will encounter after graduation.

Because text complexity's importance to students' futures is undeniable, it should occupy a prominent place in state standards and/ or related resources (e.g., an appendix, an introduction, or a link on the state's website). To reflect the CCR research, states should include both grade-band complexity expectations and guidance for teachers about how to evaluate the complexity of texts. Therefore, in this review, Achieve analyzed each state from two perspectives: the standards themselves (whether the state includes standards that expect students to read grade-appropriate text independently and proficiently) and the state's guidance on how to determine grade-appropriate complexity.

Text complexity in state standards was rated on a three-point scale with more than half of the states rated as 2 (strong), though two states, Massachusetts and North Dakota, include guidance that may create confusion or result in lowered expectations statewide.

https://lexile.com/wp-content/uploads/2017/08/What-does-the-LM-mean.pdf

<sup>&</sup>lt;sup>12</sup> Lexile, a product of MetaMetrics, refers to one scientific approach to text measurement. The Lexile text measure identifies a text's difficulty on the Lexile scale that provides a quantitative analysis of text, examining the semantic and syntactic features. Quantitative dimensions of text refer to those aspects of text complexity, such as word length or frequency, sentence length, and text cohesion, that are difficult if not impossible for a human reader to evaluate efficiently, especially in long texts, and are thus typically measured by computer software.

<sup>&</sup>lt;sup>13</sup> Williamson, 2008, p. 617.

<sup>&</sup>lt;sup>14</sup> ACT, 2006.

<sup>&</sup>lt;sup>15</sup> Texts include reading materials encountered and/or used by workers in job training or the first year of employment. These texts include, but are not limited to, texts in a degree or certification program required for career entry, texts used by the field's professional organizations, texts on websites of career-related professional organizations, recruitment materials, and commonly used manuals or references associated with on-the-job performance during the first year of employment.

<sup>&</sup>lt;sup>16</sup> Williamson & Baker, 2013.

<sup>&</sup>lt;sup>17</sup> Fraser, 2008.

#### 2: STRONG

A standard for text complexity is included, and guidance is comprehensive.

#### 1: MODERATE

A standard for text complexity is included, but guidance defining complexity is not comprehensive

## **0: WEAK/ABSENT**

A standard for text complexity is not included, and/or guidance defines reading levels too low for college and career readiness.

Text complexity is the cornerstone of CCR standards for ELA. Text complexity should steadily increase through grade 12, strengthening the likelihood that students are prepared to meet the reading demands of college and the workplace.



State standards earned a rating of 2 in one of two ways. First, state standards could earn a full score when they address, at a minimum, both the quantitative and qualitative factors by providing quantitative ranges by grade band and a description of the qualitative factors that should be considered when determining the complexity of a text.

Quantitative measures are the sole objective measures for states to ensure that students have access to grade band-appropriate text. They are anchored in college and career readiness and use technology to measure dimensions of text complexity (e.g., word frequency and difficulty, sentence length, and text cohesion) that are difficult for a human reader to evaluate. On the other hand, while quantitative analyses can accurately place a text within a text complexity grade band, qualitative measures applied by teachers are needed to select the appropriateness of specific texts for students' grade level. For example, a quantitative analysis can situate a text in the grades 6–8 text complexity band; a qualitative analysis (conducted by educators) then can determine if the text is better suited for grade 6 rather than grade 8 students. The qualitative measures give educators considerable professional authority to select individual classroom text selections. Using quantitative and qualitative measures in tandem, combined with ongoing professional development, reduces the likelihood of students spending time reading texts that do not challenge them appropriately and ensures that they are prepared for the next grade level or their postsecondary experiences.

Second, states (e.g., Indiana) could earn a rating of 2 by including a detailed recommended reading list that grows in complexity by grade level or grade band through grade 12. Ideally, states would also discuss the factors involved in text selection, so educators could replicate the selection process in their own classrooms and choose additional titles comparable with the state's expectations. This option may be attractive to states because it provides educators very tangible and immediate options for appropriate texts to use in their classrooms. On the other hand, developing such a reading list is often politically charged, and it is essential that the list include a balance of literature and informational texts and that selected texts prepare students for college- and career-level reading.

The state has an important role to play in setting guidance around determining grade-appropriate text complexity because of text complexity's considerable impact on students' futures. Full transparency is another reason for states to set clear guidance: To guide their own text selection, teachers have a right to know the level of complexity of texts, quantitatively, that will appear in the statewide assessment system.

<sup>&</sup>lt;sup>18</sup> Massachusetts' text complexity guidance may cause confusion or result in lowered expectations statewide.

<sup>&</sup>lt;sup>19</sup> North Carolina's text complexity guidance is under development.

<sup>&</sup>lt;sup>20</sup> North Dakota's text complexity guidance may cause confusion or result in lowered expectations statewide.

Fifteen of 24 states include both a standard for text complexity and guidance for how to determine grade-appropriate complexity. Their approaches vary. One state — North Carolina — was in the process of developing guidance at the time of this report.

- **California** includes a standard for grade-appropriate text complexity, an appendix that provides quantitative ranges, a description of the qualitative factors, and an appendix of text exemplars by grade band.
- **Florida** includes a standard for grade-appropriate text complexity and a video that explains the quantitative measure and ranges and the qualitative features of text.
- Indiana includes a standard for grade-appropriate text complexity and a "Sample Texts by Genre and Grade Span" list for K–12 that provides on the state department website samples of a wide range of texts of the appropriate level of complexity for each grade span in a variety of genres. This document is "intended to help educators select texts of a similar complexity range, complexity and quality to use in their own classrooms."
- **Tennessee** includes a standard for grade-appropriate text complexity and a chart at the end of the standards with quantitative ranges and qualitative features.
- New York is commended for its approach to text complexity, which considers the text, the student, and educator responsibilities. New York includes a statement at the beginning of each grade level that details the text complexity expectations for the grade. In these introductory statements, New York distinguishes between the complexity of a text and how students experience the complexity of a text. The state also distinguishes between the need to have regular access to grade-level complex text and the need for students to read a volume of texts at a range of complexities. The 7th-grade example below is representative of how New York attempts to reflect the specific nuances of text complexity at each grade level in K–12.

By the end of the school year, 7th grade students will read and comprehend literary and informational texts that are at or above grade level. Though less striking than in the earlier grades, there still needs to be a distinction between the complexity of the texts used for children to work on their independent comprehension skills, and the complexity of the texts used as part of teacher-led classroom instruction and units of study to build up their language and knowledge. Because each reader brings different skills and background knowledge to the act of reading, a text that is "complex" for one reader may be accessible to a peer in the same classroom. For this reason, educators should provide scaffolding and support as needed to allow all students to access grade-level texts.

New York makes clear that in practice educators must provide scaffolding and support, and how students *experience* complexity is different from how educators measure complexity or what educators should expect them to do with texts that a set of metrics have deemed appropriately complex.

**Massachusetts** and **North Dakota** were rated with an asterisk in addition to a 2, indicating that the guidance the state provides on text complexity may create confusion or result in inequitable expectations statewide.

**Massachusetts'** standards explain the quantitative and qualitative measures; include quantitative ranges that meet the bar for college and career readiness; and provide a comprehensive list of authors, illustrators, and works organized by time period and

genre that illustrates by grade band representative texts that exemplify the range, quality, and complexity of student reading. However, the state also includes a statement immediately after the quantitative text complexity guidance that "[e]ducators should follow district or school practices and guidelines in determining quantitative text complexity. There is no statewide policy in Massachusetts on determining quantitative text complexity," which threatens to undermine the state's detailed guidance and may encourage a lowering of quantitative expectations statewide.

Similarly, North Dakota's standards explain the quantitative and qualitative measures but include language and an alternate scale that may cause confusion and/or support local districts and individual teachers in making text complexity decisions below those suggested by CCR research. The North Dakota standards include two appendices, Appendix A and Appendix C, which address text complexity with conflicting information. Appendix A: Research and Content Knowledge Supporting Key Elements of the Standards includes clear and explicit guidance for educators and provides quantitative ranges, qualitative features, and reader and task considerations. Appendix C: Text Complexity includes a conversion guide for quantitative text complexity ratings by reader level (emergent to fluent) that has Lexile ranges significantly lower than those in Appendix A, stops at grade 9, and also prompts districts to set varying ranges for complexity: "This chart is a guide to help local school districts determine grade level expectations. Each local district is expected to set its grade level expectations." North Dakota's conflicting guidance provides two different interpretations of quantitative complexity while not clearly indicating that only one is grounded in CCR research.

Explicitly calling out the quantitative ranges — the sole independent measure of complexity correlated to the reading expectations of college and the workplace — as a matter best left to individual interpretation could increase the chances that students in the same grade level but in different classrooms or school districts encounter texts of different complexity levels, creating inequity in students' readiness for success in college, career, and citizenship. In both states, the language around text complexity could lead to varying interpretations, leaving some students vulnerable to low expectations.

**North Carolina** earned a double asterisk in addition to a 2 because at the time this report was written, the state was in the process of developing text complexity guidance.

## Six states were rated as 1 (moderate); their approaches to text complexity vary but are insufficient.

- Arizona includes standards for text complexity and a glossary that explains the quantitative measure, qualitative features, and reader and task considerations. The glossary stops short of setting quantitative expectations, however, and includes only this sentence: "Choosing a valid text-analyzer tool from second grade through high school will provide a scale by which to rate text complexity over a student's career, culminating in levels that match college and career readiness." The standards do not indicate what would be considered a reliable tool and, more importantly, do not provide quantitative ranges by grade band.
- **Georgia** and **New Jersey** include standards for text complexity but do not define text complexity or detail the factors that should be considered to determine grade-level text complexity.
- **Pennsylvania** includes a standard for text complexity and a state literacy plan that explains the quantitative and qualitative measures, but it does not provide quantitative ranges.
- **South Carolina's** standards inconsistently refer to grade-level text (only in grades 6–8 for literature and grades 6–12 for informational text). South Carolina does not define grade-level text or detail any of the factors that should be considered to determine grade-level complexity.



• **West Virginia** includes standards for text complexity and provides quantitative ranges but does not provide any information on the qualitative features that should be considered to determine grade-level appropriateness.

While most of the states rated as 1 have standards for text complexity, they fall short in providing sufficient guidance for educators to select texts for instruction that are of appropriate grade-level complexity. As noted above, effective state guidance should provide quantitative ranges for each grade band that increase from grade 2 through grades 11–12 to ensure that the texts students read by the end of high school prepare them for the texts they will encounter in their postsecondary endeavors. While quantitative measures are necessary, they are not sufficient: Qualitative measures are needed to place text in the appropriate grade. Determinations of qualitative complexity can be made only by experienced, capable human readers who consider text structure, text purpose, levels of meaning, and knowledge demands. Working in concert, both measures ensure that educators and assessment designers will determine the appropriate texts for students.

Two states were rated as 0; their standards and guidance around text complexity do not communicate clear expectations by grade level that would prepare students for the kinds of reading they will face in both college and career.

- **Missouri** does not prioritize text complexity in the standards. Missouri suggests that texts should be leveled to the student and not the grade: The K–5 standards say students should read text that is "developmentally appropriate," which may not be the same as grade appropriate; the reading standards in grades 6–12 indicate that students should read independently and proficiently but do not mention that what they read should be appropriately complex for the grade.
- **Oklahoma** includes inconsistent standards for text complexity, and the quantitative ranges are too low to prepare students for college or workplace reading.

The number of states — nine out of 24 — that were rated below a 2 and omit essential guidance on text complexity is concerning; these revisions threaten to derail states' CCR agendas.

## STANDARDS ARE NECESSARY BUT INSUFFICIENT

Standards and guidance for making text complexity decisions are necessary, but recent surveys tell us that teachers need additional supports if the promise of grade-appropriate reading is to be realized for all students. RAND found that both elementary and secondary teachers in most states — including states with comprehensive and clear guidance — selected texts for individual students based on their independent reading level instead of selecting complex texts for the whole class.<sup>21</sup> RAND also found that, at the secondary level, teachers were using abridged or adapted versions of complex texts for struggling readers, arguably leaving students insufficiently prepared to meet the reading challenges of the next grade.

A failure to provide clear and sufficient guidelines around text complexity presents serious threats to reading equity. If students in some classrooms or schools are never challenged to wrestle with complex texts, these students will not be nearly as prepared as their peers who have been rightly challenged and have risen to that challenge. Students who read below grade level may never encounter the kinds of complex text that will prepare them for college or workplace reading.

Setting clear standards and guidance for text complexity affects more than an individual classroom or teacher. To have the greatest impact, states should approach text complexity from multiple angles: the standards themselves and accompanying guidance, ongoing teacher professional learning opportunities, assessment design, teacher preparation programs, and student and parent support. A multipronged approach is instrumental in realizing the promise of a system that prepares students to successfully meet future

<sup>&</sup>lt;sup>21</sup> https://www.rand.org/pubs/research\_reports/RR1529-1.html

### reading demands:

- States should make the necessary logistical, structural, and financial investments for ongoing systems of professional development that engage educators in planning and delivery and deepen their understanding of text complexity; its importance; how to make text complexity choices appropriately; and most importantly, how to instruct students and deepen their understanding of text complexity and its importance; and how to make text complexity choices appropriately. Tools and resources such as CCSSO's Navigating Text Complexity website can support these kinds of professional development efforts. This site built by and for member states provides a curated collection of text complexity resources, tools, and research aimed at supporting educators as they guide their students in learning to read and understand complex text.
- States should select texts for summative assessments by using quantitative CCR levels and stated qualitative guidelines to select text, whenever possible. State assessments that reflect state-determined grade-appropriate complexity levels ensure that teachers, parents, and students know what levels of texts need to be read and what levels of reading will be assessed, thus influencing classroom practices in a positive way and ensuring transparency.
- States should encourage open lines of communication among invested parties: education departments, policymakers, and the higher education community. The state should dialogue with teacher preparation programs to explain expectations for text complexity and the necessity of having teachers prepared to meet those expectations instructionally; the state should also listen to first- and second-year college professors and inquire about the reading demands students face in postsecondary coursework, not just in English but also in other common first- and second-year courses.
- Finally, the state should communicate with its most important stakeholders students and parents and explain not only the statewide expectations for reading and the connections between text complexity and college and career readiness but also most importantly how to achieve reading success on texts written at grade level.

Setting explicit and full guidance around text complexity, and providing support for educators to attend to this guidance using strategies like those detailed above, helps to ensure that all students will have equal prospects to be prepared for a range of postsecondary opportunities.

## **DISCIPLINARY LITERACY**

For college and career readiness, a wide body of research and literature points to the importance of students building content knowledge through texts. Most texts students will encounter, in both higher education and careers, will be discipline-specific texts.<sup>22</sup> All reading and writing is not the same: The kinds of literacy practices students are expected to master in the ELA classroom are similar to but also substantially different from the kind of literacy they need to be successful in science, history/social studies, the arts, and other technical subjects. Research suggests that expectations for disciplinary literacy should include a focus on reading, writing, and communicating in ways that reflect the unique demands of the discipline. To ensure that all students are fully prepared for the disciplinary reading demands of school, work, and citizenship and that they are prepared to engage in the demanding work of each discipline, states should adopt focused, specific, precise, and measurable standards for disciplinary literacy — and hold students accountable to them. Moreover, disciplinary literacy standards on their own will not be enough to support student literacy. Policies and other supports (e.g., professional learning) for standards implementation should work with other discipline-specific instructional improvement efforts, with the goal of an aligned system of standards, supports, and strategies working together to improve student literacy and learning outcomes in the disciplines.

In response to the substantive research on disciplinary literacy and its importance for college and career readiness, Achieve examined how the 24 states in this analysis address literacy in the content areas in their state standards. Achieve's review found that most clearly communicate and articulate the importance of disciplinary literacy by including specific standards for literacy (reading and writing) in history/social studies and science in grades 6-12.

## HIGH-LEVEL FINDINGS ON DISCIPLINARY LITERACY

This review of state disciplinary literacy standards revealed the following trends and key findings:

## • States address disciplinary literacy in a variety of ways.

Because literacy is a responsibility shared across content areas, states approach the inclusion of disciplinary literacy standards in different ways:

- Some states have adopted stand-alone literacy standards that are presented separately from their English language arts (ELA), mathematics, and other content-specific standards.
- Others offer standards for disciplinary literacy within or with their ELA standards.
- Still others embed expectations within their content-specific standards (such as those for science, history/social studies, mathematics, and so on).
- A few states acknowledge that disciplinary literacy is important, with some general statement to that effect, but do not delineate clear, grade-level expectations for reading and writing across the content areas.

## • All but one of the states reviewed address disciplinary literacy in some way.

While how states address disciplinary literacy varies, the majority of states recognize its importance. The key challenge for states will be how to track teachers' use of the standards, share best practices within and across content areas, and support teachers in each content area to implement disciplinary literacy expectations.

<sup>&</sup>lt;sup>22</sup> https://www.all4ed.org/wp-content/uploads/2007/06/LitCon.pdf

https://www.carnegie.org/media/filer\_public/88/05/880559fd-afb1-49ad-af0e-e10c8a94d366/ccny\_report\_2010\_tta\_lee.pdf

https://www.carnegie.org/media/filer\_public/b7/5f/b75fba81-16cb-422d-ab59-373a6a07eb74/ccny\_report\_2004\_reading.pdf

## AN ANALYSIS OF DISCIPLINARY LITERACY STANDARDS

Readers use discipline-specific approaches to comprehend and produce discipline-specific texts. Reading to acquire knowledge is essential for all students and particularly important for struggling readers.<sup>23</sup> For students to experience postsecondary success, evidence is clear that readers should be able to construct discipline-specific knowledge from texts.<sup>24</sup> Universities and community colleges typically mandate that first- and second-year students take courses to meet distribution requirements that reflect a wide range of disciplines (e.g., U.S. History, World History and Civilization, Psychology, Sociology, Economics). The average Lexile scores for university and community college texts for these disciplines are high (1383 and 1292, respectively);<sup>25</sup> if students are going to engage in college-level content-area reading competently and confidently, they need exposure to and instruction on reading complex disciplinary texts in middle and high school. To guide students in becoming fully literate in each discipline, educators must understand the reading demands of the discipline and consider each discipline's approach to content, vocabulary, language, style, forms, and structure.

Whatever their field, experts use specialized approaches to comprehend, analyze, and interpret important ideas in the discipline. Mathematicians, chemists, and historians, for example, each approach texts in ways unique to their respective disciplines and communicate their content by using specific vocabulary and discipline-specific forms and structures. The key differences in the literacy behaviors of experts in different disciplines support the need for disciplinary literacy and instruction, particularly in grades 6–12. By teaching students how to read and produce texts specific to the discipline, teachers in all content areas help students develop the reading, writing, and thinking skills they need for postsecondary success.

In this analysis, reviewers examined if and how a state addresses disciplinary literacy through its state standards — not the many other resources and tools that states may use to communicate expectations to classroom teachers and students. While literacy across the school day is important, this review is focused on literacy in science and history/social studies. Reviewers looked for standards clearly identified as disciplinary literacy standards (e.g., standards in a section titled Literacy in History/Social Studies). If no standards were clearly identified as disciplinary literacy standards, reviewers examined the state's science and social studies standards, looking specifically for standards that addressed literacy. As a result, some states that have separate disciplinary literacy standards may also have literacy standards embedded in the content areas, and those standards may not be noted in this review.

<sup>&</sup>lt;sup>23</sup> Cervetti, Jaynes, & Hiebert, 2009.

<sup>&</sup>lt;sup>24</sup> Chall, 1983; Shanahan & Shanahan, 2008.

<sup>&</sup>lt;sup>25</sup> Williamson, 2008.

<sup>&</sup>lt;sup>26</sup> Lee & Spratley, 2010; Shanahan & Shanahan, 2008; Shanahan, Shanahan, & Misischia, 2011.



Reviewers noted that states have taken a variety of approaches to disciplinary literacy and that those approaches could be categorized in the following ways:

Disciplinary Literacy Categories	Explanation	States
2	The state has standards for disciplinary	Oklahoma
Embeds disciplinary literacy standards into content-area standards	literacy that are included within the	Tennessee (Social Studies)
into content area standards	standards for the discipline.	West Virginia
		Alabama
		California
		ldaho
		lowa
		Louisiana
Provides disciplinary literacy standards	The state has standards for disciplinary	Massachusetts
within or with its ELA standards	literacy that are included within or with the standards for ELA.	Mississippi
	the standards for EE/ (.	New Jersey
		New York
		North Dakota
		Ohio
		Utah
		Arkansas
	The state has standards for disciplinary	Florida
Delineates stand-alone disciplinary	literacy that are not included within either the content standards or the	Georgia
literacy standards	ELA standards.	Indiana
		Pennsylvania
Acknowledges the importance of	The state has statements on the	North Carolina
disciplinary literacy but lacks clear		South Carolina
disciplinary literacy standards		Tennessee (Science)
Does not acknowledge the importance of and/or does not clearly include disciplinary literacy standards	The state does not have standards or documentation that support disciplinary literacy.	Missouri
*Is in the process of developing disciplinary literacy standards	ntertuey.	Arizona

**States that embed disciplinary literacy standards into content-area standards:** Some states embed literacy standards into their content-area standards, emphasizing how important literacy is to content acquisition. A strength of this approach is that content-area teachers see the standards embedded in the other expectations for instruction in their disciplines and may see the standards as more clearly integral to their role in fostering students' disciplinary knowledge and skills. A challenge may be that content-area teachers may need extensive and ongoing professional learning on pedagogical approaches to making disciplinary literacy a part of ongoing instructional practice. Three of the states reviewed, Tennessee (Social Studies), West Virginia and Oklahoma, do not include separate disciplinary literacy standards but instead embed the expectations for disciplinary literacy within the content-area standards.

**States that provide disciplinary literacy standards within or with ELA standards:** Some states include discipline-specific literacy standards in the same document or as a link on the same page as the ELA standards. A strength of this approach is that it may easily facilitate conversations and professional learning with the ELA teacher. Challenges to this approach are that disciplinary teachers may not know to look for the expectations or may mistakenly assume that these standards are the sole responsibility of ELA teachers. Twelve of the states reviewed, including California, Massachusetts, and Ohio, include disciplinary literacy standards within their ELA standards by grade or grade band, as a link beneath the ELA standards (online), or a separate section within the ELA standards (in print).

**States that delineate stand-alone disciplinary literacy standards:** Five of the states reviewed — Arkansas, Florida, Georgia, Indiana, and Pennsylvania — include standards for disciplinary literacy as a stand-alone set of standards, most often by providing a separate link for Literacy on the state's academic content standards site. A strength of this approach is that educators from different disciplines have easy access to the standards, clearly signaling that literacy is a multidiscipline responsibility. Two challenges to this approach may hinder its effectiveness: How the standards are meant to complement the content standards is not always clear, and history, social studies, and science teachers have another set of standards to reference, sometimes with separate coding systems, which may imply that the literacy standards are not part of the content standards.

**States that acknowledge the importance of disciplinary literacy but lack clear disciplinary literacy standards:** Some states include a statement that speaks to the importance of literacy in the content areas in their standards documents but have not adopted academic standards that address disciplinary literacy. The lack of standards may have two important implications: Students may not be held accountable for their ability to read grade-appropriate content, and the state cannot have a reasonable expectation that disciplinary literacy is addressed consistently, coherently, or equitably in classrooms statewide. Merely noting that disciplinary literacy is important is insufficient in promoting the disciplinary literacy instruction students need for success.

**South Carolina** and **Tennessee** (science), for example, include statements extolling the importance of disciplinary literacy but do not follow up with a set of clearly defined academic standards. The South Carolina Disciplinary Literacy practices, for example, consist of three broad-based recommendations — not sufficient detail to lead to effective instruction on disciplinary literacy:

- Read, write, and communicate using knowledge of a particular discipline.
- Integrate the Reading, Writing, and Communication Standards and the Inquiry-Based Literacy Standards to communicate and create understanding within content areas.
- Extend and deepen understanding of content through purposeful, authentic, real-world tasks to show understanding and integration of content within and across disciplines.

Furthermore, South Carolina explicitly states that the disciplinary practices "are not standards" and that they therefore should not be assessed. Expectations that are not assessed often take a back seat in instruction to content and skills that will be assessed.

On the other hand, the state does leave open the possibility that students will encounter grade-appropriate informational and technical texts on their ELA assessment. Two standards in ELA address technical and informational texts in high school:

- Standard 11: "Analyze and critique the reasoning in historical, scientific, technical, cultural, and influential argument writing"; and
- Standard 12: "Read and respond to grade level text to become self-directed, critical readers and thinkers." (It does not limit texts to literary nonfiction a subset of informational texts.)<sup>27</sup>

Tennessee's newly adopted science standards, for example, begin with a statement on literacy which notes that literacy should be integrated throughout all coursework, but stops short of academic standards with a literacy focus. "Effective communication within a scientific context requires students to apply literacy skills in reading, vocabulary, speaking and listening, and writing. Scientific information is presented in many formats with various tones and perspectives. Students must process and synthesize information effectively to generate new conclusions and ideas while avoiding the pitfalls of fallacious reasoning and bias."

North Carolina does not have standards for disciplinary literacy, but the state does embed literacy expectations in the North Carolina State Literacy Plan and in some exemplar lesson plans available on the state website. This approach is useful for creating an aligned standards-based system of standards, instruction, and assessment but is likely insufficient without a set of clearly delineated expectations for student learning as the foundation. It should not be assumed that educators will intuit how to include literacy in other lessons as a result.

**States that do not acknowledge the importance of and/or do not clearly include disciplinary literacy standards:** One state, Missouri, has neither statements nor standards for disciplinary literacy.

## READINESS REQUIRES MORE THAN STANDARDS

The primary responsibility of teachers is to teach the content of their disciplines — and teaching content requires that students read and write about the content under study to gain a deep and enduring understanding of disciplinary concepts, processes, and events. To achieve deep learning, students must approach reading, writing, and communicating in these fields in ways that are specific to each discipline. For this reason, articulating standards for literacy that are specific to each discipline provides important information to guide student learning and supports the state's ability to make claims about college and career readiness that extend beyond mathematics and ELA.

As Shanahan and Shanahan discuss in their preliminary findings around disciplinary literacy practices, effective disciplinary literacy instruction<sup>28</sup>

- Requires a clear set of expectations and a literacy curriculum;
- Necessitates conversations among disciplinary experts, literacy experts, middle and high school teachers, and teacher educators;
- Depends on close relationships among faculties across disciplines;
- Has implications for preservice educational curricula and professional development for content-area educators; and
- Requires sufficient resources to support the above elements.

Disciplinary literacy standards mark a step in the right direction for college and career readiness, but states must invest time and resources for successful implementation in the classroom if students are to experience the benefits of disciplinary literacy.

<sup>&</sup>lt;sup>27</sup> Both of these standards will allow the state to determine whether students are able to read and comprehend the types of texts they will most likely see in college and in the workplace.

<sup>&</sup>lt;sup>28</sup> Shanahan & Shanahan, 2008.

- States must find ways to share best practices with district and school leaders and communicate the importance of educators collaborating to foster students' disciplinary literacy.
- States should consider ways to measure success and hold teachers and students accountable for literacy in the disciplines.
- States should assess preservice requirements and invest in ongoing and substantive professional learning to not only prepare content-area teachers in best practices on disciplinary literacy instruction but also energize them so they are excited to be a part of a system that matriculates students who read and write academically and perceptively about a range of disciplines.
- States should encourage open lines of communication among invested parties: education departments, educators, and the higher education community. The state should dialogue with first- and second-year college professors and workforce professionals and inquire about the reading demands students face in postsecondary coursework, in other common first- and second-year courses, or on the job.
- States can provide guidance, resources, and support for schools and districts in tracking disciplinary literacy implementation, supporting educators in effectively integrating disciplinary literacy instruction in their teaching, and providing feedback to students so they know if they are on track for postsecondary, discipline-specific reading, writing, and communicating.
- Finally, states should communicate with their most important stakeholders students and parents to communicate the statewide expectations for language arts and literacy and explain their rationale: Literacy and learning across all disciplines is interconnected and essential for the high demands that today's students can expect to face in higher education and the changing workforce.

## **MATHEMATICS**

Between 2010 and 2014 nearly all states adopted the Common Core State Standards for Mathematics (CCSS or Common Core). Since that initial adoption, 24 states (as of September 2017) have in some way revised their standards.<sup>29</sup> This section of the report examines the revised mathematics standards for the 24 states by focusing on the following components of high-quality mathematics standards:

☐ Structure: The K−5 standards should have an overall structure that emphasizes core ideas.	
☐ Mathematical Practices: The standards should emphasize the importance of mathematical practices, such as arguments and persevering in problem solving.	constructing
☐ Procedures, Conceptual Understandings, and Applications: The standards should emphasize procedure understandings, and applications in a balanced way.	s, conceptual
☐ Sequencing: The standards should be ordered in a way that makes mathematical sense.	
☐ <b>Grades K–5:</b> The K–5 standards should maintain a focus on arithmetic, providing a solid foundation for later rand expect students to know single-digit sums and products from memory.	nathematical studies,
☐ <b>Grades 6–8:</b> The grade 6–8 standards should include topics necessary for success in high school, including all pairs of simultaneous linear equations by the end of grade 8.	gebraically solving
☐ <b>High School:</b> States should have standards at least through the level of Algebra II (or a similar course of standards in the courses through Algebra II (or similar) should include an emphasis on the modeling procimportant topics in statistics.	, .

## HIGH-LEVEL FINDINGS FOR MATHEMATICS

This review of state mathematics standards revealed the following trends and key findings:

## ☐ Most states have maintained an emphasis on arithmetic in grades K-5.

Without a firm grounding in arithmetic in early grades, further studies in mathematics will be exceedingly difficult. The organizational structure of the standards can help to emphasize core ideas, such as arithmetic, while adding nonarithmetic standards can detract from that emphasis. Reviewers found that 20 of the 24 states use a high-level organizing structure that strongly emphasizes a focus on the main ideas of arithmetic. Eighteen of the states have avoided including topics that detract from a focus on arithmetic. Common examples of topics that take time and emphasis away from arithmetic include patterns (e.g., for shapes or sounds) and vocabulary (e.g., naming the days of the week), which are not related to numbers. In addition, out of the 24 states analyzed, just 17 require that students know single-digit addition and multiplication computations from memory.

<sup>&</sup>lt;sup>29</sup> Kansas is not included in this report. While the state adopted new standards in 2017, they were not publicly available as of September 2017.

## ☐ Important concepts that support the advanced study of functions, geometry, and statistics in high school are present in most of the states in grades 6–8.

What students learn in grades 6–8 is the foundation for the advanced mathematical studies of high school. If parts of that foundation are missing or weak, the student is likely to flounder. Reviewers found that 21 of the 24 states address a collection of topics that are found in the grade 8 National Assessment of Educational Progress (NAEP), a clear majority of state standards, and the CCSS and are critical for supporting high school studies of functions, geometry, and statistics.

To ensure that standards are keeping pace, reviewers selected one key requirement: Solve systems of linear equations with rational coefficients algebraically. Because this expectation brings together rational number operations and algebraic manipulation, it serves as an indicator in grade 8 that the pace of the K–8 standards is in keeping with high school prerequisites. In this regard, 17 of the 24 states meet this pacing indicator. Of the seven that fall short, only Oklahoma does not address solutions of linear systems at all until high school courses. Indiana, Missouri, North Carolina, Ohio, and Utah all require graphic, but not algebraic, solutions of linear systems in grade 8. New York restricts the work to equations with integer coefficients.

## ☐ Most states emphasize in a balanced way procedural skill, conceptual understanding, and application in real-world problems in grades K-8.

In the past, mathematics standards typically were a list of problem types that students needed to know how to do (e.g., "add two-digit numbers"). A high-quality set of standards, however, should also detail understandings of the concepts that underpin those procedures and expect students to apply the mathematics to real-word situations. Including these understandings encourages teachers and curriculum developers to emphasize the "hows" and "whys" for mathematical skills and the "what" of mathematical concepts. Nineteen of the 24 states address procedural skill, conceptual understanding, and application across a selection of eight key topics. Of the five states that are inconsistent in this regard, none are completely lacking in any one aspect. However, Missouri and Pennsylvania both show a tendency to be weaker in application, and in the Oklahoma standards, the weakest aspect is conceptual understanding.

## ☐ Most states include an emphasis on high school statistics and modeling.

Being able to understand and use statistics, as well as being able to model the real world with mathematics, are of increasing importance in the 21st century. In a data-driven society, knowledge of statistics is essential for everyday life but also for advanced studies, including the sciences and the social sciences. While all states have standards through the level of Algebra II, three states are missing one or two selected statistics topics addressed in NAEP for high school, many state standards, and the CCSS, while four states are missing three or more of the topics. In these states, the most commonly neglected topics are standard deviation, randomization, comparison of data sets, and conditional probability.

When students apply the modeling cycle to the world around them, they determine the relevant quantities in a situation, relate the quantities mathematically, and report how the mathematics can be used to explain the situation. Of the 24 states in this report, reviewers found just 15 that give modeling a special emphasis in high school, though it could be stronger in three of those states. Nine have no special emphasis on modeling at all.

## ■ Nearly all the states include practice standards for grades K-12.

Among others, the National Council of Teachers of Mathematics (NCTM), the Programme for International Student Assessment (PISA), and the SAT encourage expertise, such as practices, that are essential to mathematical success in the studies and work of the 21st century. Twenty of the 24 states in this report have heard the message and have included clear expectations related to mathematical practices. Reviewers were unable to find any evidence of official inclusion of practice standards in Arkansas, Florida, and Missouri.

Only half of the 24	states strongly meet 10 or 11 i	indicators that should be re	eflected in college- and o	areer-ready (CCR)
standards.				

Four states, California, Idaho, Iowa, and New Jersey, meet all 11 indicators at a strong level; another eight states, Alabama, Georgia, Louisiana, Massachusetts, Mississippi, North Dakota, Ohio, and West Virginia, strongly meet all but one indicator. Overall, 18 of the states have a strong or moderate match to at least 10 of the indicators. While no state is missing all 11 indicators, Indiana, Missouri, Oklahoma, Pennsylvania, and South Carolina have a strong match to five or fewer indicators that should be a part of a set of CCR standards.

## AN ANALYSIS OF STATE MATHEMATICS STANDARDS

For this analysis of the 24 states that originally adopted the CCSS and have since completed a revision process (as of September 2017), reviewers applied 11 indicators to examine at a high level certain key aspects of each state's standards. The code attached to each indicator serves as a reference throughout the report.

#### Structure:

For this indicator reviewers considered the high-level organizing structure of the standards to determine whether there is a special emphasis on core ideas specific to grade levels or whether the standards are instead organized under the same strands across all of the levels.

#### Mathematical Practices:

## ☐ M2: The standards emphasize the importance of the mathematical practices.

For this indicator, reviewers determined whether some form of mathematical practices exists in the state's standards documents, including such practices as constructing arguments and making sense of problems, and whether those practices are clearly labeled and described.

## Procedures, Conceptual Understandings, and Applications:

## ☐ M3: The standards emphasize procedures, conceptual understandings, and applications.

For this indicator, reviewers determined if the standards address all three aspects (procedures, understandings, and applications) for each of a selected group of key topics across the grade levels.

### Sequencing:

#### ☐ M4: The standards are sequenced in an order that makes mathematical sense.

For this indicator, reviewers looked for standards that are out of sequence. Standards that are out of sequence might include expectations that are out of sync with the requirements within a grade or a requirement to use a concept before it is introduced.

#### Grades K-5:

## ☐ M5: The K-5 standards maintain a focus on arithmetic.

For this indicator, reviewers looked for additional topics that detract from an emphasis on arithmetic.

## M6: The standards expect students to know single-digit sums and products from memory.

For this indicator, reviewers determined if the standards make a direct claim that students should know single-digit sums and products from memory.

#### Grades 6-8:

	M7: The standards address the critical topics in grades 6–8.  For this indicator, reviewers looked for standards that align to a particular, nonexhaustive list of topics related to the grade 8 NAEP objectives, a majority of state standards, and the CCSS.
	M8: By the end of grade 8, students algebraically solve pairs of simultaneous linear equations with rational coefficients. For this indicator of pacing, reviewers looked to see if the standards expect students to solve systems algebraically, without restrictions on the types of numbers used for coefficients.
Hi	igh School:
	M9 — The state includes standards through the level of Algebra II (or similar).  For this indicator, reviewers determined if the state includes standards through the level of Algebra II.
	M10 — The modeling process is given special emphasis in high school standards through Algebra II (or similar).  For this indicator, reviewers looked for a special emphasis on the modeling process in high school.
	M11 — Statistical topics are included in high school standards through Algebra II (or similar).
	For this indicator, reviewers looked for standards that align to a particular, nonexhaustive list of statistics and probability topics related to the grade 12 NAEP framework, a majority of state standards, and the CCSS.

## **STRUCTURE**

Standards should be organized around core ideas. Ideally, these core ideas are grouped by some intermediate level of organization, such as clusters of standards addressing related concepts. The use of clusters between the highest-level descriptive organizers and the standards helps to maintain coherence, ensures that standards are related, and discourages the inclusion of disconnected skills. In contrast, some standards are organized by high-level "strands" that appear every year, which promotes a typical problem in older standards in the United States:

We introduce topics early and then repeat them year after year. To make matters worse, very little depth is added each time the topic is addressed because each year we devote much of the time to reviewing the topic.<sup>30</sup>

Researcher Liping Ma, speaking of the strand model, indicates that the "damage this structure has caused to U.S. elementary mathematics education is the instability of content, discontinuity in instruction, and incoherence in concepts."<sup>31</sup> Adoption of a strand model tends to emphasize all topics as equal, with nothing central, and too easily allows additional, disconnected standards to be included. This situation promotes a curriculum that is a "mile wide and an inch deep."<sup>32</sup> Similarly, the 2008 The Final Report of the National Mathematics Advisory Panel recommends:

A focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics, should become the norm in elementary and middle school mathematics curricula. Any approach that continually revisits topics year after year without closure is to be avoided.<sup>33</sup>

<sup>30</sup> Schmidt, Houang, & Cogan, 2002, p. 3.

<sup>&</sup>lt;sup>31</sup> Ma, 2013, p. 1293.

<sup>32</sup> Schmidt, Houang, & Cogan, 2002, p.3.

<sup>&</sup>lt;sup>33</sup> National Mathematics Advisory Panel, 2008, p. 22.

Reviewers examined the high-level organization of the standards for the 24 states in this study to see if the standards for grades K–5 are organized around the core ideas of arithmetic or if they are instead presented under uniform strands across all grade levels. Reviewers also looked for an intermediate level of organization between the standards and the core ideas.

The indicators and rating descriptors are below:

	2: STRONG	1: MODERATE	0: WEAK/ABSENT
M1: The K-5 standards have an overall structure that emphasizes the core ideas of arithmetic.	High-level organization focuses on groups of related standards and includes intermediate levels of organization.	The standards focus on groups of related standards but lack intermediate levels of organization.	The standards use a strand model for the highest level of organization.

The table below uses color coding to show the ratings for each of the 24 states' mathematics standards. The table is followed with specific findings and examples:

2: STRONG	1: MODERATE	0: WEAK/ABSENT
M1: The K $-5$ standards have an overall str	ructure that emphasizes the core ideas of	arithmetic.
AL AZ AR CA FL GA ID IN	IA LA MA MS MO NJ NY NC	ND OH OK PA SC TN UT W
☐ Most of the states have structure that emp	phasizes the core ideas of arithmetic.	
,	in a way that detracts from the emphasis on ghest level of organization. Indiana and Soutl rds are clustered for specific purposes under	h Carolina lack an intermediate level of

## MATHEMATICAL PRACTICES

In 2001, the report *Adding It Up* by the National Research Council promoted a notion of mathematical proficiency that indicated proficiency requires more than performing a list of procedural skills well.<sup>34</sup> Proficiency, the Council claimed, consists of five components that are interdependent:

Conceptual understanding;
☐ Procedural fluency;
☐ Strategic competence;
☐ Adaptive reasoning; and
Productive disposition.

<sup>&</sup>lt;sup>34</sup> National Research Council, 2001.

Similarly, in 2000, NCTM released its process standards that expect, among other things, students to be problem solvers that reason about mathematics, make arguments, and communicate mathematically.<sup>35</sup> These standards helped to elevate expectations for what it means to learn mathematics well and served as the foundation in the development of the Standards for Mathematical Practice in the CCSS:<sup>36</sup>

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

Including practice standards, or something similar, helps to enforce the notion that learning mathematics requires more than performing procedures. In fact, these sorts of practice standards are also found in states that never adopted the Common Core. Texas has the "mathematical process standards," and Nebraska has the "mathematical processes." The new SAT exam now "covers all mathematical practices." Internationally, Singapore emphasizes mathematical processes by noting them as an expectation for all grade levels. Such practices are also reflected in the definition of mathematical literacy in the 2015 framework for PISA:

Mathematical literacy is an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens.<sup>39</sup>

Research has shown that college instructors recognize the importance of mathematical practices<sup>40</sup> and that middle school teachers see the practices as essential.<sup>41</sup> To be effective, though, practices should be easily referenced. Being easily referenced means that they are numbered and not just provided as a list so that teachers can call them by name in their lesson plans and in conversations with students, parents, and other teachers. To be useful for teachers, and to ensure that they will be used, the practices should also be explained and placed in the same document as the content standards.

<sup>35</sup> National Council of Teachers of Mathematics, n.d.

 $<sup>^{36}</sup>$  Common Core State Standards Initiative, 2010, p. 6.

<sup>&</sup>lt;sup>37</sup> College Board, 2015, p. 12.

<sup>&</sup>lt;sup>38</sup> Ministry of Education, Singapore, 2012.

<sup>&</sup>lt;sup>39</sup> Organisation for Economic Co-operation and Development, 2016, p. 4.

<sup>&</sup>lt;sup>40</sup> Conley, Drummond, de Gonzalez, Rooseboom, & Stout, 2011.

<sup>&</sup>lt;sup>41</sup> Davis, Choppin, McDuffie, & Drake, 2013.

For this analysis, reviewers considered the importance of mathematical practices in the standards, as described below:

	2: STRONG	1: MODERATE	0: WEAK/ABSENT
M2: The standards emphasize the importance of the mathematical practices.	The standards include something like the practices.	The standards include something like the practices but with less prominence or emphasis than should be present.	The standards severely de-emphasize or have nothing like the practices.

The table below uses color coding to show the ratings for each of the 24 states' mathematics standards. The table is followed with specific findings and examples:

2: STRONG	1: MODERATE	0: WEAK/ABSENT												
M2: The standards emphasize the importance of the mathematical practices.														
AL AZ AR CA FL GA ID IN	IA LA MA MS MO NJ NY NC	ND OH OK PA SC TN UT WV												
☐ Eighteen of the states place a strong emphasis on mathematical practices.														
☐ North Carolina, Oklahoma, and Pennsylvania do not number practices, which lessens the ability to reference them in discussion or to use them in assessment.														
☐ Florida and Missouri do not include pra	☐ Florida and Missouri do not include practices in their standards.													
☐ In Arkansas, practices are not included in	n the standards documents and are found onl	y as a "related file" on the state website.												

## PROCEDURES, CONCEPTUAL UNDERSTANDINGS, AND APPLICATIONS

It is not enough for standards to focus only on procedural skills. *The Final Report of the National Mathematics Advisory Panel* explains the need for simultaneous emphasis on conceptual understanding and procedural skill and fluency:

To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem solving skills. Debates regarding the relative importance of these aspects of mathematical knowledge are misguided.<sup>42</sup>

 $<sup>^{\</sup>rm 42}\,\text{National}$  Mathematics Advisory Panel, 2008, p. xix.

Along with procedural skills and conceptual understandings, standards should also include an emphasis on applications of the mathematics. Students, after all, should see the usefulness of the mathematics and be able to apply it to real-life situations. The College Board claims that the revised SAT, for example, now stresses the importance of procedures, understandings, and applications:

To succeed on the redesigned SAT, students will need to exhibit command of mathematical practices, fluency with mathematical procedures, and conceptual understanding of mathematical ideas. In keeping with the evidence, the exam will also provide opportunities for richer applied problems.<sup>43</sup>

As an indicator for emphasis on procedures, understandings, and applications, reviewers considered a selection of eight K–8 topics, provided in Appendix A of this report, for which standards, or elements of standards, should address all three aspects. Specifically, reviewers looked to see if a state's standards expect procedural skill, evidence of understanding, and an application for each topic. For example, the standards should include an expectation that students can divide fractions (perform the procedure), show they understand what they are doing (or getting) when they divide (demonstrate understanding and connect to the concept of division), and divide fractions in a real-life setting (apply the concept and procedure). A state meets the target for a specific topic if the standards include expectations for all three aspects.

	2: STRONG	1: MODERATE	0: WEAK/ABSENT
M3: The standards emphasize	All three aspects are	All three aspects are	All three aspects are
procedures, conceptual	found for seven or eight	found for five or six of the	found in fewer than five
understandings, and applications.	of the selected topics.	selected topics.	of the selected topics.

The table below uses color coding to show the ratings for each of the 24 states' mathematics standards. The table is followed with specific findings and examples:

2: STRONG									ODE	RATE					0: WEAK/ABSENT										
M3:	The s	stand	ards e	emph	asize <sub>l</sub>	proce	dures	, cond	ceptu	al und	lerstandings, and applications.														
AL	AZ	AR	CA	FL	GA	ID	IN	IA	LA	МА	MS	мо	ИЛ	NY	NC	ND	ОН	ок	PA	sc	TN	UT	wv		
☐ Fo	■ For the sample of eight selected topics, 19 of the 24 states meet the indicator for all three aspects.																								
				,	lvania dersta			is lov	w due	e to a	weak	ness	in the	e area	of ap	oplica	tions.	In O	clahor	na, th	าe we	akest			

<sup>&</sup>lt;sup>43</sup> College Board, 2015, p. 38.

## **SEQUENCING**

Standards are interconnected and the web of relationships among them is complex.<sup>44</sup> A high-quality set of standards should always place mathematical content in the proper sequence across the grades. If Topic B *mathematically* depends on Topic A, then Topic B must follow Topic A. Otherwise, the student will experience the mathematics for Topic B as a disconnected skill. For example, if standards introduce a topic that requires working with decimal numbers (e.g., using dollars-and-cents notation) before the concept of decimals has been introduced or developed, students will have no reference for the meaning of the decimal point and will not be able to connect the work to mathematics they already know. This situation is a type of "missing ingredient" sequencing error.<sup>45</sup>

For this analysis, reviewers considered this sort of sequencing issue in the standards, as described below:

	2: STRONG	1: MODERATE	0: WEAK/ABSENT
M4: The standards are sequenced in an order that makes mathematical sense.	The standards have no observed issues with topics appearing out of sequence.	The standards have one or two topics out of sequence.	The standards have three or more sequencing issues.

The table below uses color coding to show the ratings for each of the 24 states' mathematics standards. The table is followed with specific findings and examples:

2: STRONG								1: M	ODE	RATE						0: W	/EAK	/ABS	ENT				
M4:	M4: The standards are sequenced in an order that makes mathematical sense.																						
AL	AZ	AR	CA	FL	GA	ID	IN	IA	LA	MA	MS	мо	NJ	NY	NC	ND	ОН	ок	PA	SC	TN	UT	wv

- ☐ Most states sequence standards in a way that avoids clear "missing ingredient" errors.
- Massachusetts and South Carolina expect work with angles in grade 3 before introducing and defining angles and their measures in grade 4. Some states, such as Arizona, Indiana, and Oklahoma, create sequencing issues by including standards addressing problems involving money that do not align with the arithmetic expectations of the grade. Pennsylvania references an understanding of fractions before fractions are introduced. In Tennessee similar triangles are used in grade 8 to define slope, but similarity is not addressed until high school.

<sup>&</sup>lt;sup>44</sup> See, for example, http:/achievethecore.org/coherence-map/.

<sup>&</sup>lt;sup>45</sup> Daro, Mosher, & Corcoran, 2011, p. 49.

### **GRADES K-5**

Historically, the U.S. curriculum has tended to be wide rather than deep, so much so that when compared to the rest of the world, one report found "there is no textbook in the world that has as many topics as our mathematics textbooks, bar none."<sup>46</sup> The 2008 *The Final Report of the National Mathematics Advisory Panel* even noted, "U.S. curricula typically include many topics at each grade level, with each receiving relatively limited development, while top-performing countries present fewer topics at each grade level but in greater depth."<sup>47</sup> In looking at pre-Common Core standards, the Panel also noted, based on the work of the Thomas B. Fordham Institute, that even in the six states with the highest-rated standards at that time, the states "provide an emphasis on fewer important topics per year than most states; but compared with the 'A+ countries' (Singapore, Japan, Korea, Hong Kong, Flemish Belgium, and the Czech Republic), they all spend a great deal of time in the primary grades on topics other than arithmetic." In its 2009 report *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, the National Research Council, recommends that "[m]athematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to the other topics."<sup>48</sup>

With an emphasis on arithmetic in grades K-5 in mind, reviewers looked to see if states have included particular nonarithmetic topics in their K-5 standards. Specifically, reviewers looked for the inclusion of probability, geometric transformation, measures of center, a strong emphasis on patterns, <sup>49</sup> currency unrelated to its value, or nonmathematical calendar topics.<sup>50</sup>

Additionally, it is essential that students know basic computations, such as sums of two one-digit numbers and single-digit multiplication from memory. This knowledge is particularly important because "[c]omputational proficiency with whole number operations is dependent on sufficient and appropriate practice to develop automatic recall of addition and related subtraction facts, and of multiplication and related division facts." Knowing these facts from memory is critical for solving complex problems.

The indicators and rating descriptors are below:

	2: STRONG	1: MODERATE	0: WEAK/ABSENT
M5: The K–5 standards maintain a focus on arithmetic.	The K–5 standards do not include any of the additional topics.	The standards include one, two, or three of the additional topics.	The standards include more than three of the additional topics.
M6: The standards expect students to know single-digit sums and products from memory.	The standards clearly require that students know single-digit sums and products from memory.	Either:  (a) The standards have an expectation to "know" single-digit sums and products but "from memory" is not explicitly included, or  (b) The standards expect either sums or products of single-digit numbers from memory but not both.	The standards have no clear expectation of knowing single-digit sums and products.

<sup>&</sup>lt;sup>46</sup> Schmidt, et al., 2002, p. 3.

<sup>&</sup>lt;sup>47</sup> National Mathematics Advisory Panel, 2008, p. 20.

<sup>&</sup>lt;sup>48</sup> National Research Council, 2009, p. 345.

<sup>&</sup>lt;sup>49</sup> Klein et al., 2005, found "[t]he attention given to patterns in state standards verges on the obsessive," p. 17.

<sup>&</sup>lt;sup>50</sup> See metric NN1A of http://achievethecore.org/page/1946/instructional-materials-evaluation-tool.

 $<sup>^{51}\,\</sup>text{National}$  Mathematics Advisory Panel, 2008, p. xix.

<sup>52</sup> Willingham, 2009.



The table below uses color coding to show the ratings for each of the 24 states' mathematics standards. The table is followed with specific findings and examples:

2: 5	TRON	IG						1: M	ODE	RATE						0: V	<b>VEAK</b>	/ABS	ENT				
M5:	The	K-5 s	standa	ards n	nainta	iin a 1	focus	on ar	ithme	etic													
AL	AZ	AR	CA	FL	GA	ID	IN	IA	LA	МА	MS	МО	ИЛ	NY	NC	ND	ОН	ок	PA	sc	TN	UT	wv
M6:	The	stand	ards e	expect	t stud	ents	to kn	ow sir	ngle-d	digit s	ums a	and p	roduc	ts fro	m m	emory	/. <sup>53</sup>						
AL	AZ	AR	CA	FL	GA	ID	IN	IA	LA	МА	MS	МО	ИЛ	NY	NC	ND	ОН	ОК	PA	sc	TN	UT	wv
m A arr so or "e ea to	few sond Sond Sond Sond Sond Sond Sond Sond S	natica states uth Ca ; othe gnizin e" coin se the metic een of specta	add of arolinater standing and ones. 54 - cese references.	ies. conce a add tes, ir nami Two s quirei 24 sta are n	pts th prima including co tates, ments ates re o long	at mi ary-gi ng India ins w India take equire ger cl	ight d rade s diana vith no ana ar time time	etract standa and No sam nd Ok and s stude identi	from ards r Missis e-gra lahon shift f ents k	n the elated sippi, ade cona, income to the constant of the co	emph d to p add onnec clude from both	nasis controlled the particular the particular single Misson	on arins that the color of the	thmethat are requented to the real representation of the real representation of the representation of the representation of the real representation of the r	tic in not controlled in the second in the s	grade conne nts, s milarl K–5, –5: a	s K—S cted to uch a y, Ne thou thou acquir	5. For to nur s nam w Yor gh bo ing kr	exammbers ning took exp oth do nowled	nple, (e.g. he da ects s so ir dge a	Oklah ,, shap ays of studen grac nd sk	oma bes and the vents to le 5.55 ills re	veek 5 In lated ades.
sta	andar	ds for	stud	ents t	0 W01	rk wit	h sta	ndard	algo	d that rithms plems	s. The	e lack	of th	is exp	ectat	ion a	lso ra	ises q	uestic				ner

<sup>53</sup> States that expect only that students "use mental strategies" for calculations do not meet this criterion. States that require automatic recall do meet the criterion.

<sup>&</sup>lt;sup>54</sup> See standard NY-K.MD.4.

<sup>&</sup>lt;sup>55</sup> Arkansas provides an example in the grade 5 standard 5.MD.B.8 that requires mean and median, but Achieve considers this a coherence issue in that mean and median are not introduced until later.

 $<sup>^{\</sup>rm 56}\,{\rm National}$  Assessment Governing Board, 2017.

### **GRADES 6-8**

The work in grades 6–8 serves as the foundation for much of the everyday mathematics we encounter in our lives. It also serves as the connection between earlier work in arithmetic and the mathematical demands of high school. For this review, reviewers looked for the inclusion of a short list of topics in grades 6–8 (see Appendix B) related to the grade 8 objectives in the 2017 mathematics framework for NAEP, a vast majority of state standards, and the CCSS. As an indication that students are on track for further mathematics, reviewers also looked to see if students are expected to solve pairs of simultaneous linear equations with rational coefficients algebraically by the end of grade 8. Because the number and algebra skills required are a critical foundation for the in-depth study of both linear and nonlinear equations and functions in high school courses, this expectation serves as a gauge for high school readiness.

The indicators and rating descriptors are below:

	2: STRONG	1: MODERATE	0: WEAK/ABSENT
M7: The standards address the critical topics in grades 6–8.	The standards address all the listed critical topics in grades 6–8.	The standards miss one or two of the listed critical topics.	The standards miss more than two of the listed critical topics.
M8: By the end of grade 8, students algebraically solve pairs of simultaneous linear equations with rational coefficients.	By the end of grade 8, the standards expect students to algebraically solve pairs of simultaneous linear equations with rational coefficients.	By the end of grade 8, the standards expect students to solve pairs of simultaneous linear equations but do not expect algebraic solutions with rational coefficients.	The standards do not expect students to solve pairs of simultaneous linear equations by the end of grade 8.

The table below uses color coding to show the ratings for each of the 24 states' mathematics standards. The table is followed with specific findings and examples:

2: 5	ΓRON	G						1: M	ODEI	RATE					0: WEAK/ABSENT								
M7:	M7: The standards address the critical topics in grades 6–8.  AL AZ AR CA FL GA ID IN IA LA MA MS MO NJ NY NC ND OH OK PA SC TN UT WV																						
AL	AZ	AR	CA	FL	GA	ID	IN	IA	LA	MA	MS	мо	ИJ	NY	NC	ND	ОН	ОК	PA	sc	TN	UT	wv
M8:	By th	ne en	d of g	grade	8, stu	udent	s alge	braic	ally so	olve p	airs o	f sim	ultane	eous l	inear	equat	tions	with r	ation	al coe	fficie	nts.	
AL	AZ	AR	CA	FL	GA	ID	IN	IA	LA	МА	MS	МО	NJ	NY	NC	ND	ОН	ОК	PA	sc	TN	UT	wv
□ Tv	venty-	one (	of the	24 s	tates	in thi	s reno	ort pr	esent	evide	ence d	of all	the se	electe	d ton	ics in	arade	es 6-	8.				

- ☐ In both Oklahoma and Tennessee, the grade 6 understanding of similarity and congruence, an important foundation for high school geometry, is omitted. Pennsylvania neglects problems about scaling geometric figures, foundational work with variables, informal comparison of data sets, and variability of data.
- Seventeen of the 24 states expect grade 8 students to solve a system of linear equations with rational coefficients algebraically.

<sup>&</sup>lt;sup>56</sup> National Assessment Governing Board, 2017.

☐ Five of the moderate ratings for the pacing indicator expect only graph-based, rather than algebraic, solutions for linear systems in grade 8. New York limits the work to integer coefficients rather than working with numbers appropriate to the grade, and Oklahoma does not require solutions of linear systems until high school.

#### **HIGH SCHOOL**

The issues around high school mathematics standards are complex. While standards in grades K–8 are articulated by grade levels across the states, the same uniformity is not true for the sets of high school standards. Some states present high school standards in courses, while others present the standards in categories based on concepts. Some states do both. All states have standards at the level of Algebra II (or similar), but not all states require Algebra II (or similar) to graduate,<sup>57</sup> indicating that not all high school standards are really expected of all students. Some states include standards for fourth-year courses; others do not. Some states require a fourth year of mathematics; others do not. Some states place an emphasis on modeling, and some have strong expectations for probability and statistics in the first three years of high school.

Historically, research such as the American Diploma Project clarified the importance of higher-level mathematics for all students, including through Algebra II, in high school.<sup>58</sup> The specific need for Algebra II was articulated in an extensive 1999 U.S. Department of Education study:

Of all pre-college curricula, the highest level of mathematics one studies in secondary school has the strongest continuing influence on bachelor's degree completion. Finishing a course beyond the level of Algebra 2 (for example, Trigonometry or Precalculus) more than doubles the odds that a student who enters postsecondary education will complete a bachelor's degree.<sup>59</sup>

A 2006 follow-up report reinforced the need for Algebra II.<sup>60</sup> In 2008, Achieve summarized the research on the importance of higher-level mathematics in high school and found:<sup>61</sup>

- Taking higher-level mathematics courses is key to accessing postsecondary education, especially for disadvantaged and minority students;
- Mathematics is critical for college success and degree completion;
- Students who take higher-level mathematics courses are better prepared for the workplace and earn higher salaries; and
- Higher-level mathematics skills are used in many kinds of work.

<sup>&</sup>lt;sup>57</sup> Achieve, n.d.

<sup>&</sup>lt;sup>58</sup> Achieve, 2004.

<sup>&</sup>lt;sup>59</sup> Adelman, 1999, p. vii.

<sup>60</sup> Adelman, 2006.

<sup>&</sup>lt;sup>61</sup> Achieve, 2008.

High school standards should also place a special emphasis on modeling, the process by which mathematics is used to describe the world around us. Students should be able to determine the relevant quantities in a situation, relate the quantities mathematically, and report how the mathematics can be used to explain the situation. The 2016 *Guidelines for Assessment and Instruction in Mathematical Modeling Education* clearly describe the importance of modeling:

In modeling, students see how mathematics can help them make predictions and provide insight into the real world. Further, the real world can help them with mathematics; they can identify mathematical mistakes if their model does not produce results that are consistent with the real world. This back-and-forth helps elevate the status of mathematics; perhaps for the first time, students are able to see how math relates to the real world and can provide insight into many aspects of their lives.<sup>62</sup>

According to the National Research Council's report The Mathematical Sciences in 2025, the building of mathematical models is "[c]entral to much of science, engineering, and society today."<sup>63</sup>The 2015 Common Vision report from the Mathematical Association of America (MAA) calls for entry-level college courses that include modeling as a focus.<sup>64</sup> Additionally, from an international perspective, "[t]he modelling cycle is a central aspect of the PISA conception of students as active problem solvers."<sup>65</sup> Given the importance of the modeling process, the reviewers looked to see if states stress that importance in high school.

Beyond modeling, in this era of big data, <sup>66</sup> states should be increasing, rather than decreasing, the emphasis on statistical literacy. The importance of understanding statistics is nicely described in the 2007 *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report:* 

Our lives are governed by numbers. Every high school graduate should be able to use sound statistical reasoning to intelligently cope with the requirements of citizenship, employment, and family and to be prepared for a healthy, happy, and productive life.<sup>67</sup>

Indeed, the MAA's 2015 Common Vision report notes the importance of increasing statistics education as "[d]ata-driven science is reshaping the processes of discovery and learning in the 21st century." The report continues to note that the "current attention to big data and the demand for college graduates with data skills should prompt changes in our entry-level courses which result in students being better prepared for jobs requiring computational and statistical skills." States should be equipping all high school students with statistical skills and understandings.

In this context, reviewers focused the review of high school mathematics standards on the inclusion of standards through the level of Algebra II and on the inclusion of an emphasis on modeling and a selection of statistics topics in the coursework through Algebra II (or similar).

PRESENT											ABSENT												
M9:	The :	state	includ	des st	andar	ds th	rough	ı Alge	bra II	(or s	imilar	·).											
AL	AZ	AR	CA	FL	GA	ID	IN	IA	LA	МА	MS	мо	ИJ	NY	NC	ND	ОН	ок	PA	sc	TN	UT	wv

<sup>&</sup>lt;sup>62</sup> Garfunkel & Montgomery, 2016, p. 46.

 $<sup>^{63}</sup>$  National Research Council, 2013.

<sup>&</sup>lt;sup>64</sup> Saxe & Braddy, 2015.

<sup>&</sup>lt;sup>65</sup> Organisation for Economic Co-operation and Development, 2016, p. 65.

<sup>&</sup>lt;sup>66</sup> Lohr, 2012.

<sup>&</sup>lt;sup>67</sup> Franklin et al., 2007, p. 1.

<sup>&</sup>lt;sup>68</sup> Saxe & Braddy, 2015, p. 16.

The findings indicate that all of the states in this review include standards through the level of Algebra II (or similar):

With respect to modeling and statistics, in some cases, states may provide related statistics standards in fourth-year courses, but such courses are typically optional and are not expected for all students. Additionally, fourth-year statistics courses present a problem of access, as only 68 percent of 12th graders in lower-income schools are offered a statistics course in their school curriculum.<sup>69</sup> The selected topics are listed in Appendix C and are all related to objectives in the 2017 grade 12 NAEP framework, a majority of state standards, and the CCSS. Some grade 12 objectives in the NAEP framework are designated as "content beyond that typically taught in a standard 3-year course of study (the equivalent of 1 year of geometry and 2 years of algebra)."<sup>70</sup> However, all of the selected topics match objectives that do *not* carry that designation and are therefore expected in a typical three-year course of study.

	2: STRONG	1: MODERATE	0: WEAK/ABSENT
M10: The modeling process is given special emphasis in high school standards through Algebra II (or similar).	The high school standards include special emphasis on the modeling process and make clear connections to appropriate standards.	The high school standards provide some special emphasis on modeling, but it could be stronger. For example, the connection to particular standards is not clearly explained or indicated.	The high school standards may include some that imply modeling, but there is no special emphasis to draw attention to that implication.
M11: Statistical topics are included in high school standards through Algebra II (or similar).	The standards address all the listed topics before the end of Algebra II (or similar).	The standards miss one or two of the listed topics before the end of Algebra II (or similar).	The standards miss more than two of the listed topics before the end of Algebra II (or similar).

The indicators and rating descriptors are below:

The table below uses color coding to show the ratings for each of the 24 states' mathematics standards. The table is followed with specific findings and examples:

2: ST	ron	G						1: MODERATE								0: WEAK/ABSENT							
M10	M10: The modeling process is given special emphasis in high school standards through Algebra II (or similar).																						
AL	AZ	AR	CA	FL	GA	ID	IN	IA	LA	MA	MS	мо	NJ	NY	NC	ND	ОН	ОК	PA	sc	TN	UT	wv
M11:	: Stat	istical	topic	s are	includ	ded ir	n high	scho	ol sta	indard	ds thr	ough	Algeb	ora II	(or sii	milar)							
AL	AZ	AR	CA	FL <sup>71</sup>	GA	ID	IN	IA	LA	MA	MS	мо	NJ	NY	NC	ND	ОН	ОК	PA	sc	TN	UT	wv

<sup>&</sup>lt;sup>69</sup> Change the Equation, 2016.

<sup>70</sup> National Assessment Governing Board, 2017, p. 7.

 $<sup>^{71}</sup>$  This analysis cannot be determined as Florida only lists its standards in a single, four-year format.

- Half of the 24 states place a strong and special emphasis on the modeling process in high school. Nine states have no special high school emphasis on modeling.
- Most of the states continue to emphasize statistics topics in coursework through Algebra II.
- Several statistics topics are absent from the high school standards through Algebra II in Oklahoma and Pennsylvania. In Alabama and South Carolina some of the statistics topics appear in high school but only in a fourth-year course, which de-emphasizes the importance of statistics for all students.

Adding to the complexities of the high school standards, the mathematics students need when they finish high school is more fluid now than in the past. States that will be revising their standards in the foreseeable future must carefully establish what students should understand and be able to do to be college and career ready. This effort requires working especially closely with faculty and other representatives from postsecondary education and with state workforce development agencies and employers so that high school mathematics standards and curricular pathways are carefully aligned with new and emerging pathways through postsecondary education and into careers. For example, higher education is beginning to explore alternative pathways to better meet the needs and postsecondary aspirations of students, as explained in a 2015 report from the Charles A. Dana Center at The University of Texas at Austin:

For students majoring in programs such as social or behavioral sciences, the most important mathematics is statistics, not algebra. For liberal arts students, who typically need to take one core math course to graduate in their majors, quantitative reasoning is likely to be more relevant to their future lives and careers.<sup>72</sup>

Similarly, the MAA's Common Vision report calls for entry-level options to College Algebra for students who are not heading to Calculus. Such courses should focus on "problem solving, modeling, statistics, and applications." The *Mathematical Sciences in 2025* indicates that Calculus is not the appropriate gateway for all students and that other college pathways should be developed. Finilarly, the *INGenIOuS Project* and *Transforming Post-Secondary Education in Mathematics (TPSE)* call for new entry-level pathways for college students.

These alternative postsecondary pathways exist and are gaining traction. Ohio, through its Articulation & Transfer Network,<sup>77</sup> and the University System of Georgia<sup>78</sup> now offer different mathematics pathways to better meet student needs. Additionally, pathways programs like Carnegie Foundation's Math Pathways (Statway and Quantway)<sup>79</sup> and the Dana Center's Mathematics Pathways<sup>80</sup> seek to move students more quickly through the remediation process. Students in the Statway program, for example, may enter the program even though their mathematics knowledge may be well below college level. Within one year, students learn the mathematics they need and earn college-level credit in statistics.<sup>81</sup> This process contrasts with the typical, yet ineffective,<sup>82</sup> remediation pathway that forces students through a multisemester developmental algebra pipeline.

<sup>72</sup> Kazis & Cullinane, 2015.

<sup>&</sup>lt;sup>73</sup> Saxe & Braddy, 2015, p. 16.

<sup>&</sup>lt;sup>74</sup> National Research Council, 2013, pp. 10–11.

<sup>&</sup>lt;sup>75</sup> Zorn et al., p. 3.

<sup>&</sup>lt;sup>76</sup> See the printable pdf at http://www.tpsemath.org/meeting1.

 $<sup>^{77}\,\</sup>mbox{Ohio}$  Department of Higher Education, 2016.

<sup>&</sup>lt;sup>78</sup> See http://completega.org/math-pathways.

<sup>&</sup>lt;sup>79</sup> Carnegie Foundation, n.d.-a.

<sup>80</sup> See https://dcmathpathways.org/

<sup>&</sup>lt;sup>81</sup> Carnegie Foundation, n.d.-b.

<sup>82</sup> Bailey, Jeong, & Cho, 2010.



Indeed, important conversations need to happen, such as the one summarized in Learning Works' Degrees of Freedom:

The notion of eliminating Algebra 2 in high school appears to give educators the greatest pause. Years of effort have been expended ensuring that more students have access to the courses that are correlated with higher education opportunities. Access to algebra, in fact, has been considered a civil rights issue. This history leaves many cautious that alternative pathways would create separate but unequal tracks. "It's an interesting idea, fraught with danger," says [Alan] Schoenfeld. "In the language of the 1989 math standards, the so-called 'non-college-intending' tracks all turned out to be dead ends."

UCLA's [Mark] Green is only slightly less wary ... "You have a danger of people being limited throughout their lives by what math they got early on—or didn't. There's a lot of stuff that uses Algebra 2, and students who don't take it may be unaware that they are limiting their options later on."

"On the other hand," he acknowledges, "it's much better to have someone who genuinely understands modeling and quantitative reasoning and has a feeling for statistics than someone who took an Algebra 2 class but is totally bewildered by it." He believes that it's incumbent on schools offering alternatives to inform students that such courses won't prepare them for various scientific fields. <sup>83</sup>

What implication these shifts may have on standards and curriculum in high school is not clear yet, though all students should be expected to learn advanced algebra, modeling, and statistics. States that meet these criteria will provide students with a strong foundation for further learning and careers and will ensure that all students have a range of options and pathways beyond high school.

## DISCUSSION

In this report, reviewers considered a collection of indicators to explore state-level mathematics standards for those states that at one point adopted the CCSS but have made modifications to their standards since that adoption. While many states continue to attend to many of these indicators, a few states fall far short.<sup>84</sup> Overall, the revisions seen in the states in this report reinforce the notion that writing a high-quality set of standards is complex and difficult work and that many aspects of high-quality standards could be missed during the revision process.

Standards need to be easily usable, or they may not be used at all. The most convenient presentations for standards are clear, single-document versions that allow teachers to look at their own grade-level expectations, along with other grades, and refer to specific standards with ease. Some states post their standards in multiple files, adding layers of complexity and making seeing the whole set of standards at once impossible. Tracking a progression for a specific concept through various files is particularly cumbersome.

While making small changes may be easy, determining the ripple effect of those changes is altogether more challenging. All modifications to standards, however trivial, must involve conversations across multiple grades. Tools like the Coherence Map can help to reinforce the necessary interconnectedness of a quality set of standards as well as the fragility and importance of connections and common language.<sup>85</sup> The removal of standards can affect coherence and should be done with caution.

Agreeing upon key mathematical definitions before changes are made is also important. A glossary can serve this purpose, but too often the glossary is an afterthought rather than an organizer of key definitions. The reviewers have seen significant mathematical issues in the glossaries. In some cases, terms are defined circularly, definitions of terms contradict other terms, terms simply do not align to the intention in the standards, or terms and/or their definitions lack precision. Precise standards require precise definitions.

The indicators in this report are high level, but a quality set of standards will also have to attend to finer points. Achieve has historically reviewed standards based on the criteria of focus, coherence, rigor, clarity, specificity, and measurability. Standards should **focus** on the most important aspects at each grade level, and they should lack repetition. They should be *fully* **coherent** within and between grade levels. Every standard should fit into some progression of ideas. They should evidence **rigor** throughout by balancing conceptual understanding, procedural skills and fluencies, and applications. States should maintain strong expectations for students to create arguments, prove, and justify. Standards (and examples) should be **clear**; lack ambiguity; and be free from mathematical, typographical, and grammatical errors. They should be **specific** and neither too vague nor too atomistic. And they should be **measurable**, using performance verbs that call for students to demonstrate knowledge and skills.<sup>86</sup>

 $<sup>^{\</sup>rm 84}\,\text{See}$  the full table in Appendix D.

 $<sup>^{85}\,\</sup>mbox{See}$  http://achievethecore.org/coherence-map/.

<sup>86</sup> Some states now have standards in which students "explore" or "discover," which are difficult, if not impossible, to measure.

## **REFERENCES**

Achieve. (2004). *Ready or not: Creating a high school diploma that counts.* Washington, DC: Author. Retrieved from https://www.achieve.org/files/ReadyorNot.pdf

Achieve. (2008). *The building blocks of success: Higher-level math for all*. Washington, DC: Author. Retrieved from https://www.achieve.org/BuildingBlocksofSuccess

Achieve. (2015). Rising to the challenge: Views on high school graduates' preparedness for college and careers. Washington, DC: Author. Retrieved June 20, 2017, from https://www.achieve.org/employers-and-college-faculty-report-gaps-recent-graduates%E2%80%99-preparedness-new-national-survey

Achieve. (n.d.). *State-by-state graduation requirements (class of 2015)*. Washington, DC: Author. Retrieved from http://www.achieve.org/publications/state-state-graduation-requirements-class-2015

Achieve. (2016). A review of the Oklahoma January 2016 English language arts and mathematics academic standards. Washington, D.C.: Author. Retrieved October 31, 2017 from https://www.achieve.org/publications/review-oklahoma-january-2016-english-language-arts-and-mathematics-academic-standards

ACT. (2006). Reading between the lines: What the ACT reveals about college readiness in reading. Iowa City, IA: Author.

ACT. (2015). *The condition of college & career readiness 2015*. Author. Retrieved June 20, 2017, from http://www.act.org/readiness/2015

Adams, M. J. (2011). Advancing our students' language and literacy: The challenge of complex texts. *American Educator*, 34(4), 3, 3–11, 53.

Adelman, C. (1999). Answers in the tool box. Academic intensity, attendance patterns, and bachelor's degree attainment. Washington, DC: U.S. Department of Education. p. vii. Available from http://files.eric.ed.gov/fulltext/ED431363.pdf

Adelman, C. (2006). *The toolbox revisited: Paths to degree completion from high school through college.* Washington, DC: U.S. Department of Education. p. xix. Available from https://www2.ed.gov/rschstat/research/pubs/toolboxrevisit/toolbox.pdf

Annenberg Learner. (2017). *Reading and writing in the disciplines*. WGBH Educational Foundation and Annenberg Foundation. Retrieved June 20, 2017, from https://www.learner.org/courses/readwrite/disciplinary-literacy/what-is-disciplinary-literacy/1.html

Bailey, T., Jeong, D. W., & Cho, S. W. (2010). Student progression through developmental sequences in community colleges (CCRC Brief No. 45). Community College Research Center, Columbia University. Retrieved from http://ccrc.tc.columbia.edu/media/k2/attachments/student-progression-through-developmental-sequences-brief.pdf

Beck, I. L., McKeown, M. G., & Kucan, L. (2008). *Creating robust vocabulary: Frequently asked questions and extended examples.*New York: Guilford.

Burdman, P. (2015). Degrees of freedom: Diversifying math requirements for college readiness and graduation. Oakland, CA: LearningWorks. Retrieved from http://edpolicyinca.org/sites/default/files/PACE%201%2008-2015.pdf

Carnegie Foundation. (n.d.-a). *Carnegie math pathways*. Retrieved September 11, 2017, from https://www.carnegiefoundation.org/in-action/carnegie-math-pathways/

Carnegie Foundation. (n.d.-b). *Carnegie math pathways* — Statway. Retrieved September 11, 2017, from https://www.carnegiefoundation.org/wp-content/uploads/2017/06/sw\_one\_pager\_June17.pdf

Cervetti, G., Jaynes, C., & Hiebert, E. (2009). Increasing opportunities to acquire knowledge through reading. In *Reading More, Reading Better.* (pp. 79–100). New York: The Guilford Press.

Chall, J. (1983). Stages of reading development. New York: McGraw Hill.

Change the Equation. (2016). New data: High schoolers lack access to statistics courses. Retrieved September 11, 2017, from http://changetheequation.org/blog/new-data-high-schoolers-lack-access-statistics-courses

College Board. (2015). *Test specifications for the redesigned SAT.* New York: Author. Retrieved from https://collegereadiness.collegeboard.org/pdf/test-specifications-redesigned-sat-1.pdf

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. Available from http://www.corestandards.org/wp-content/uploads/Math\_Standards1.pdf

Conley, D. T., Drummond, K. V., de Gonzalez, A., Rooseboom, J., & Stout, O. (2011). *Reaching the goal: The applicability and importance of the common core state standards to college and career readiness.* Educational Policy Improvement Center (NJ1). Retrieved from https://www.epiconline.org/download/36907/

Daro, P., Mosher, F., & Corcoran, T. (2011). Learning trajectories in mathematics: A foundation for standards, curriculum, assessment, and instruction (CPRE Research Report #RR-68). Philadelphia: Consortium for Policy Research in Education. doi: 10.12698/cpre.2011.rr68

Davis, J., Choppin, J., McDuffie, A. R., & Drake, C. (2013). *Common core state standards for mathematics: Middle school mathematics teachers' perceptions.* Rochester, NY: The Warner Center for Professional Development and Education Reform: University of Rochester. Retrieved from https://www.warner.rochester.edu/files/warnercenter/docs/commoncoremathreport.pdf

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report.* Alexandria, VA: American Statistical Association. Retrieved from http://www.amstat.org/asa/files/pdfs/GAISE/GAISEPreK-12\_Full.pdf

Fraser, A. L. (2008). Vocational-technical education in Massachusetts (Pioneer Institute White Paper No. 42). Pioneer Institute: Public Policy Research.

Garfunkel, S., & Montgomery, M. (2016). *Guidelines for assessment and instruction in mathematical modeling education* (GAIMME) report. Boston/Philadelphia: Consortium for Mathematics and Its Applications/Society for Industrial and Applied Mathematics. Retrieved from http://www.siam.org/reports/gaimme-full\_color\_for\_online\_viewing.pdf

Graham, S., & Hebert, M. (2010). Writing to read: Evidence for how writing can improve reading (Carnegie Corporation Time to Act Report). Washington, DC: Alliance for Excellent Education. Retrieved July 1, 2017, from https://www.carnegie.org/media/filer\_public/9d/e2/9de20604-a055-42da-bc00-77da949b29d7/ccny\_report\_2010\_writing.pdf

Hart, B., & Risley, T. R. (2003). The early catastrophe: The 30 million word gap by age 3. American Educator, 27(1), 4-9.

Hawkins, J., Ginty, E., Kurzman, K., Leddy, D., & Miller, J. (2008). Writing for understanding: Using backward design to help all students write effectively. South Strafford, VT: Vermont Writing Collaborative.

Intersegmental Committee of the Academic Senates of the California Community Colleges, the California State University, and the University of California. (2002). *Academic literacy: A statement of competencies expected of students entering California's public colleges and universities.* Sacramento, CA: Author.

Kazis, R., & Cullinane, J. (2015). *Modernizing mathematics pathways at Texas Universities: Insights from the New Mathways Project transfer champions*. Austin, TX: The Charles A. Dana Center at The University of Texas at Austin. Retrieved from http://www.utdanacenter.org/wp-content/uploads/modernizing\_mathematics\_pathways\_at\_texas\_universities.pdf

Kintsch, W. (1998). Comprehension: A paradigm for cognition. Cambridge University Press.

Klein, D., Braams, B. J., Parker, T., Quirk, W., Schmid, W., & Wilson, W. S. (2005). *The state of state math standards, 2005*. Thomas B. Fordham Institute. Retrieved from https://edexcellence.net/publications/sosmath05.html

Lee, C. D., & Spratley, L. A. (2010). *Reading in the disciplines: The challenges of adolescent literacy.* New York, NY: Carnegie Corporation of New York.

Lohr, S. (2012). The age of big data. *New York Times*, 11(2012). Retrieved from http://www.nytimes.com/2012/02/12/sunday-review/big-datas-impact-in-the-world.html

Ma, L. (2013). A critique of the structure of US elementary school mathematics. *Notices of the AMS, 60*(10), 1282–1296. Retrieved from http://www.ams.org/notices/201310/fea-ma.pdf

McKeown, M. G., Beck, I. L., & Blake, R. (2009). Rethinking reading comprehension instruction: A comparison of instruction for strategies and content approaches. *Reading Research Quarterly*, 44(3), 218–253.

Metametrics. (n.d.) What does the Lexile measure mean? The Lexile framework for reading. Author. Retrieved 10/31/17 from https://lexile.com/wp-content/uploads/2017/08/What-does-the-LM-mean.pdf

Ministry of Education, Singapore. (2012). *Mathematics syllabus primary one to five: Implementation starting with 2013 primary one cohort.* Retrieved September 11, 2017, from https://www.moe.gov.sg/docs/default-source/document/education/syllabuses/sciences/files/primary\_mathematics\_syllabus\_pri1\_to\_pri5.pdf

National Assessment Governing Board. (2017). *Mathematics framework for the 2017 National Assessment of Educational Progress*. Washington, DC: U.S. Department of Education. Retrieved from https://nagb.gov/content/nagb/assets/documents/publications/frameworks/mathematics/2017-math-framework.pdf

National Association of Colleges and Employers. (2016). *Job outlook 2016: The attributes employers want to see on new college graduates' resumes.* Bethlehem, PA: Author. Retrieved July 1, 2017, from http://www.naceweb.org/career-development/trends-and-predictions/job-outlook-2016-attributes-employers-want-to-see-on-new-college-graduates-resumes/

National Center for Education Statistics. (2012). *The nation's report card: Vocabulary results from the 2009 and 2011 NAEP reading assessments.* Washington, DC: Institute of Education Sciences, U.S. Department of Education.

National Council of Teachers of Mathematics. (n.d.). *Executive summary: Principles and standards for school mathematics*. Retrieved September 11, 2017, from https://www.nctm.org/uploadedFiles/Standards\_and\_Positions/PSSM\_ExecutiveSummary.pdf

National Mathematics Advisory Panel. (2008). Foundations for success: The final report of the National Mathematics Advisory Panel. Washington, DC: U.S. Department of Education.

Retrieved from https://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

National Reading Panel & National Institute of Child Health and Human Development. (2000). Report of the National Reading Panel: Teaching children to read: An evidence-based assessment of the scientific research literature on reading and its implications for reading instruction: Reports of the subgroups. Washington, DC: National Institute of Child Health and Human Development, National Institutes of Health.

Retrieved July 1, 2017, from https://www.nichd.nih.gov/publications/pubs/nrp/Documents/report.pdf

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: The National Academies Press. Retrieved from https://doi.org/10.17226/9822

National Research Council. (2009). *Mathematics learning in early childhood: Paths toward excellence and equity.* Washington, DC: The National Academies Press. Retrieved from https://doi.org/10.17226/12519

National Research Council. (2013). *The mathematical sciences in 2025. Washington, DC: The National Academies Press.* Retrieved from https://doi.org/10.17226/15269

Neuman, S. B. (2006). How we neglect knowledge — and why. American Educator, 30(1), 24.

Ohio Department of Higher Education. (2016, February). *Mathematics pathways: Alternative learning pathways now available to Ohio postsecondary students*. Math Matters, 1–2. Retrieved October 20, 2017, from https://www.ohiohighered.org/files/uploads/math/newsletters/Newsletter%203%20Pathways%20FORMATTED%202.26%20FINAL.pdf

Opfer, V. D., Kaufman, J. H., & Thompson, L. E. (2017). *Implementation of K–12 state standards for mathematics and English language arts and literacy: Findings from the American Teacher Panel*. Santa Monica, CA: RAND Corporation. Retrieved September 5, 2017, from https://www.rand.org/pubs/research\_reports/RR1529-1.html

Organisation for Economic Co-operation and Development. (2016). *PISA 2015 assessment and analytical framework: Science, reading, mathematic and financial literacy*. Paris: OECD Publishing. Retrieved from http://dx.doi.org/10.1787/9789264255425-en

Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Common core standards: The new U.S. intended curriculum. *American Educational Research Association*, 40(103), 2011.

Retrieved from http://edr.sagepub.com/content/40/3/103.full.pdf+html?ijkey=HF55LMNdgnKmw&keytype=ref&siteid=spedr.facc

Recht, D. R., & Leslie L. (1988). Effect of prior knowledge on good and poor readers' memory of text. *Journal of Educational Psychology*, 80(1), 16–20.

Saxe, K., & Braddy, L. (2015). *A common vision for undergraduate mathematical sciences programs in 2025.* Washington, DC: Mathematical Association of America. Retrieved from https://www.maa.org/sites/default/files/pdf/CommonVisionFinal.pdf

Schmidt, W. H. (2010, September 14). *Michigan State University: Papers and presentations, mathematics and science initiative. U.S. Department of Education.* Retrieved from http://www2.ed.gov/rschstat/research/progs/mathscience/schmidt.html

Schmidt, W., Houang, R., & Cogan, L. (2002). A coherent curriculum: The case of mathematics. *American Educator 26*(2), 10–26. Retrieved from https://www.aft.org/sites/default/files/periodicals/curriculum.pdf

Shanahan, C., Shanahan, T., & Misischia, C. (2011). *Analysis of expert readers in three disciplines: History, mathematics, and chemistry.* Journal of Literacy Research, 43(4), 393–429.

Shanahan, T., & Shanahan, C. (2008). Teaching disciplinary literacy to adolescents: Rethinking content-area literacy. Harvard Educational Review, 78(1), 40–59. Retrieved June 20, 2017, from

http://hepg.org/her-home/issues/harvard-educational-review-volume-78-issue-1/herarticle/rethinking-content-area-literacy\_640

Stanovich, K. E. (1986). *Matthew effects in reading: Some consequences of individual differences in the acquisition of literacy.* Reading Research Quarterly, 21(4), 360–407.

Williamson, G. L. (2008). A text readability continuum for postsecondary readiness. Journal of Advanced Academics, 19(4), 602–632.

Williamson, G. L., & Baker, R. F. (2013). Enriching the concept of career preparedness by examining text complexity associated with bright outlook occupations (MetaMetrics Research Brief).

Willingham, D. T. (2006). How knowledge helps: It speeds and strengthens reading comprehension, learning — and thinking. American Educator, Spring.

Retrieved July 1, 2017, from https://www.aft.org/periodical/american-educator/spring-2006/how-knowledge-helps

Willingham, D. T. (2009). Is it true that some people just can't do math? *American Educator, 33*(4), 14–19. Retrieved from https://www.aft.org/sites/default/files/periodicals/willingham.pdf

Zorn, P., Bailer, J., Braddy, L., Carpenter, J., Jaco, W., & Turner, P. (2014). *The INGenIOuS project: Mathematics, statistics, and preparing the 21st century workforce.* Washington, DC: Mathematical Association of America. Retrieved from https://www.maa.org/sites/default/files/pdf/ingenious/INGenIOuS-report.pdf

# **APPENDIX A**

Topics used for the analysis in M3:

- ☐ Addition and subtraction within 20
- ☐ Addition and subtraction within 1,000
- ☐ Multiplication and division within 100
- Angle measure
- ☐ Volume measure (geometric)
- ☐ Division of fractions
- $\square$  Equations of form x + p = q and px = q
- Pythagorean Theorem

## **APPENDIX B**

Select topics for grades 6–8:

- Ratio
- Proportional relationships
- Operations with rational numbers
- ☐ Expressions and equations
  - Representing situations with variables
  - Equivalent forms
  - Linear equations (representations and solving)
- ☐ Introduction to functions
- Statistics
  - Measures of center
  - Measures of variability
  - Comparing two data sets informally
  - Line fit to bivariate data

- Geometry
  - Scale drawings
  - Surface area and volume
  - Similarity and congruence
  - Pythagorean Theorem

## **APPENDIX C**

Statistics and probability topics for high school:

- Standard deviation of a data set
- ☐ Two-way tables
- ☐ Fitting a function to data
- ☐ The correlation coefficient
- Correlation and causation
- Random sampling
- Comparing two data sets
- Conditional probability and independence

## **APPENDIX D**

Summary table of all states by indicator:

Summary	M1	M2	М3	M4	M5	M6	M7	M8	М9	M10	M11
AL	2	2	2	2	2	2	2	2	2	2	0
AZ	2	2	2	1	2	2	2	2	2		
AR	2	0	2	1	2	2	2	2	2		
CA	2	2	2	2	2	2	2	2	2	2	2
FL	2	0	2	2	2	2	2	2	2	0	
GA	2	2	2	2	2	2	2	2	2	0	2
ID	2	2	2	2	2	2	2	2	2	2	2
IN	1	2	1	0	0	0	2	1	2	0	2
IA	2	2	2	2	2	2	2	2	2	2	2
LA	2	2	2	1	2	2	2	2	2	2	2
MA	2	2	2	1	2	2	2	2	2	2	2
MS	2	2	2	2	1	2	2	2	2	2	2
МО	2	0	0	2	1	0	2	1	2	0	2
NJ	2	2	2	2	2	2	2	2	2	2	2
NY	2	2	2	2	1	2	2		2	2	1
NC	2	1	1	2	2	1	2	1	2	0	2
ND	2	2	2	2	2	0	2	2	2	2	2
ОН	2	2	2	2	2	2	2	1	2	2	2
OK	0	1	0	0	1	0	1	0	2	0	0
PA	0	1	0	1	2	0	0	2	2	0	0
SC	1	2	2	1	1	0	2	2	2	0	0
TN	2	2	2	1	2	2	1	2	2	2	2
UT	2	2	2	2	2	2	2	1	2	0	2
WV	2	2	2	2	2	2	2	2	2	1	2

**M1:** The K-5 standards have an overall structure that emphasizes the core ideas of arithmetic.

**M2:** The standards emphasize the importance of the mathematical practices.

**M3:** The standards emphasize procedures, conceptual understandings, and applications.

**M4:** The standards are sequenced in an order that makes mathematical sense.

**M5:** The K-5 standards maintain a focus on arithmetic.

**M6:** The standards expect students to know single-digit sums and products from memory.

M7: The standards address the critical topics in grades 6-8.

**M8:** By the end of grade 8, students algebraically solve pairs of simultaneous linear equations with rational coefficients.

M9: The state includes standards through Algebra II (or similar).

**M10:** The modeling process is given special emphasis in high school standards through Algebra II (or similar).

**M11:** Statistical topics are included in high school standards through Algebra II (or similar).

## **ACKNOWLEDGMENTS**

Achieve would like to thank the individuals and organizations who contributed to this report. We would like to thank Ted Coe, director of mathematics at Achieve, and Miah Daughtery, director of English language arts & literacy at Achieve, for serving as the lead mathematics and English language arts (ELA)/literacy reviewers and writers of this report. We are also grateful to Mimi Alkire for her work as a mathematics standards reviewer and Elizabeth Heydel, Sue Pimentel, and Judson O'Dell for their work as ELA/literacy standards reviewers.

Sandy Boyd, chief operating officer of Achieve, provided leadership and essential feedback in shaping the report. Jeremy Thomas and Kelly Van Beveren provided feedback and editing in the review of the final version of this report. We would also like to thank Kathy Ames at Next Chapter Communications for her editorial contributions and Rings Leighton for their design work.

Michael Cohen President Achieve

