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Development and validation of a test instrument to measure pre-service mathematics teachers' content knowledge and pedagogical content knowledge

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Abstract. Teacher education in Indonesia aims to prepare pre-service teachers in becoming professional teaching personnel. Two important knowledge need to be acquired are content knowledge (CK) and pedagogical content knowledge (PCK). However, it is still a challenge to measure the pre-service mathematics teachers' CK and PCK. Therefore, the first aim of the present study is to develop and validate a test instrument to measure CK and PCK of pre-service mathematics teachers on high school level mathematics. Second, the present study aims to measure the CK and PCK of pre-service mathematics teachers by using the developed instrument. The present study employed developmental research method and used students' empirical data in investigating the validity, objectivity, and reliability of the instrument. The results of the study and its implications for future research and practice then will be presented in the paper.

Keywords: Content knowledge, Pedagogical content knowledge, Mathematical knowledge for teaching, Content knowledge for teaching mathematics, Teacher education.

1. Introduction

Teacher education in Indonesia aims to prepare pre-service teachers in becoming professional teaching personnel. However, concerns have been arisen with regard to the effectiveness of the program in equipping them with knowledge for teaching to provide high-quality teaching and learning for their prospective students. Two important knowledges needed by the pre-service teachers is content knowledge (CK) and pedagogical content knowledge (PCK) [1]. If those knowledges are possessed by the teachers, then they potentially can deliver high-quality instruction. As a result, their students will acquire high academic achievements [2, 3, 4].

CK and PCK need to be acquired by pre-service teachers in general, but also by pre-service mathematics teacher in particular. Within mathematics education, the concepts of CK and PCK are commonly known as mathematical knowledge for teaching (MKT) or content knowledge for teaching mathematics (CKTM) [5]. The MKT is critical component for pre-service mathematics teacher in their preparation and professional development. With this knowledge, for example, they can arrange the sequence through which their students learn mathematics effectively. In addition, with this knowledge,



they can present mathematics instruction in which suitable for the development and prior knowledges of their students. For these reasons, MKT become one of critical aspects for pre-service mathematics teachers in preparing mathematics teaching and learning. As an evident, Hatisaru and Erbas [6] found that MKT influence the teaching practices. As consequences, the teaching practices have positive role in student learning. The positive impact of MKT on teaching practices and student learning also has been found in recent studies [e.g., 7, 8, 9, 10].

Given the importance of CK, PCK, or MKT for pre-service mathematics teachers, it is necessary to measure their CK, PCK, or MKT. However, it still be a challenge to measure pre-service teachers' knowledge in teaching mathematics even though many studies have attempted to develop CK and PCK scale in mathematics [e.g., 11, 12]. Therefore, the aims of the present study are twofold. First, the present study aims to develop a reliable, objective, and valid instrument in measuring pre-service mathematics teachers' CK and PCK. Second, the present study also measures CK and PCK of the pre-service mathematics teachers using the developed instrument.

2. Literature review

2.1. CK, PCK, and MKT

The notion of CK and PCK were proposed by Shulman [1]. CK and PCK are two categories in Shulman's typologies for characterizing professional knowledge for teaching. However, the typologies are articulated variously across the literature. One of complete articulations is presented by Ball et al. [5]. Based on this articulation, there are seven main categories of teacher knowledge, namely (1) general pedagogical knowledge, (2) knowledge of students and their characteristics, (3) knowledge of educational context, (4) knowledge of educational ends, purposes, and values, and their philosophical and historical grounds, (5) content knowledge, (6) curriculum knowledge, (7) and pedagogical content knowledge. Two teacher knowledge categories are emphasized in this article, i.e., content knowledge and pedagogical content knowledge. Content knowledge referred to knowledge about the subject and its organizing structures. For pedagogical content knowledge, Shulman [1] identified two central components, namely knowledge about instructional methods and representations and knowledge regarding students' (mis)conceptions.

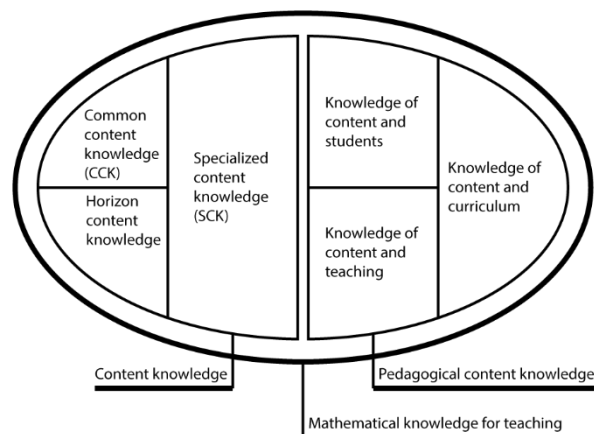


Figure 1. The relationship of CK, PCK, and MKT [13]

Aside of Shulman's central contribution on conceptualizing teacher knowledge, it also has been criticized [13]. Therefore, many scholars develop alternative conceptualizations. In mathematics education, Ball et al. [5], Hill et al. [11, 14], and Hill, et al. [7] reconceptualized teachers' PCK as mathematical knowledge for teaching (MKT). This reconceptualization covers both CK and PCK. MKT covers six domains of mathematical knowledge of teaching, i.e. common content knowledge, horizon content knowledge, specialized content knowledge, knowledge of content and students, knowledge of

content and teaching, and knowledge of content and curriculum. The first three categories of MKT are related to teachers' CK whereas the remaining categories deal with teachers' PCK. The relationship of Shulman's [1] conceptualization of teacher knowledge and Ball et al.'s [5] MKT is shown in Figure 1.

2.2. Measuring CK and PCK

Section 2.1. shows that Ball et al.'s [5] MKT relates to Shulman's [1] conceptualization of CK and PCK. Therefore, in the rest of this article we will use the term CK and PCK.

There are various subscales which can be used to measure CK and PCK in literature. In present study, we use the CK and PCK subscales from the work of Krauss et al. [15]. The subscales are in line with our theoretical framework of PCK presented in Section 2.1. First PCK subscale is knowledge of mathematical tasks. The mathematical-task knowledge is included in PCK subscales because mathematical tasks are central to mathematics teaching and learning [16, 17]. Furthermore, there is evidence in literature [e.g. 18] showing that there is positive association between this knowledge and teachers' PCK. Second PCK subscale is knowledge of student misconceptions and difficulties. It is important for teachers to be aware of students' misconception because it can be used to acquire knowledge of students' thinking [19]. Knowing students' difficulties in mathematics is also critical because it can be used for teachers to help students so that they be able to pursue mathematics further [20]. Furthermore, the knowledge construction by students often succeeds with the guidance and support by teacher [21, 22]. Thus, the knowledge of mathematics-specific instructional strategies is third component of PCK subscales.

The subscale in measuring CK is background knowledge on the contents of the high school level mathematics curriculum. Pre-service mathematics teachers should master mathematics contents beyond the students should do. It means that the pre-service mathematics teachers must possess mathematical knowledge that much deeper than their prospective students.

3. Method

The developmental research method was employed in developing test items. The three parts of the study reported here are (1) conceptualization of test scales and subscales, (2) development of the CK and PCK items, and (3) validation and quantitative analysis of the developed CK and PCK items. From the step 1, we obtained three subscales in measuring pre-service mathematics teachers' PCK, namely knowledge of mathematical tasks (Task), knowledge of student misconceptions and difficulties (Student), and knowledge of mathematics-specific instructional strategies (Instruction). For CK, we conceived as a background knowledge on the contents of the high school level mathematics curriculum.

In step 2, we developed CK and PCK items based on the CK and PCK subscales, along with their scoring rubrics. We composed two items for each subscale. The mapping between subscales and items is shown in Table 1. In total, there were ten items. The items 1.1, 2.1, 3.1, 4.1, and 5.1 were put in first test instrument, whereas the remaining items were put in second test instrument. Example of one item in the items set is shown in Figure 2.

Table 1. Mapping between CK and PCK subscales and items

Subscale and indicator	Item
PCK Task	
Giving or evaluating mathematical explanations	1.1
Appraising and adapting the mathematical content of textbooks or other resources	1.2
PCK Student	
Giving examples and illustrations of possible student's misconceptions and difficulties in solving mathematical problem.	2.1 and 2.2
PCK Instruction	
Responding to students' "why" questions by using language that suitable with student's development	3.1 and 3.2
CK	

Solving non-routine mathematical problem

4.1, 5.1, 4.2,
and 5.2

The development of CK and PCK items were followed by the validation process. In this phase, the CK and PCK test instrument were distributed to two experts along with the validation sheet. The validation sheet contained four criteria which be used in assessing the CK and PCK items, i.e. goal-centered criteria, learner-centered criteria, context-centered criteria for assessments, and assessment-centered criteria [23]. Aside of these criteria, the expert validators were allowed to give comments on the validation sheet or directly on the test instrument.

Soal Nomor 3: Rerata Data Tunggal dan Berkelompok

Ada seorang siswa bertanya seperti ini: Bu, PR kemarin memberikan data seperti ini.

70	49	57	48	46
53	60	39	40	54
52	68	47	56	35
33	56	35	46	51

Saya menghitung rerata data tersebut dengan rumus jumlah nilai dibagi dengan banyak nilai dan saya memperoleh hasil seperti berikut.

$$\bar{x} = \frac{\sum x}{n} = \frac{995}{20} = 49,75$$

Tetapi, teman saya menghitung rerata data tersebut dengan terlebih dahulu mengubahnya menjadi data berkelompok seperti ini.

Kelas	Frekuensi (f_i)	Nilai Tengah (x_i)	$f_i \times x_i$
31–40	5	35,5	177,5
41–50	5	45,5	227,5
51–60	8	55,5	444
61–70	2	65,5	131
	20		980

Dengan demikian, rerata data tersebut adalah

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{980}{20} = 49$$

Mengapa jawaban saya dan teman saya berbeda? Jawaban mana yang betul, Bu?
Bagaimana Anda menanggapi pertanyaan-pertanyaan siswa tersebut?

Figure 2. Example of an item from PCK Instruction subscale (in Indonesian)

The validation phase was then followed by the revision of items. The revision process was based on the validator's evaluation. After revision, the first test instrument then distributed to the participants to be answered and then the answer be assessed by three independent correctors. The three correctors were master students who were also serve as research assistant.

3.1. Participants

The participants of the present study were fifteen pre-service mathematics teachers in teacher education program of Universitas Sanata Dharma, Yogyakarta. Of the fifteen pre-service teachers, 14 (93.3%) are female and 1 (6.7%) is male. Their participation in the present study was one of their evaluation in the aforementioned program.

3.2. Data Analysis

The present study used participants' answers to analysis psychometric properties of CK and PCK test instrument as well as to evaluate psychometric items statistics. Psychometric properties of CK and PCK test instrument included internal consistency and distribution of test scores, whereas the psychometric items statistics contained item discrimination indices, item difficulties, and item variances. Furthermore, objectivity of the developed rubrics was evaluated by Kendall's coefficient of concordance [24].

The experts' validation data also were used in evaluating the validity of the test instrument. The result of the validation contains quantitative and qualitative data. The quantitative data were analysed by using percentage criteria, whereas the qualitative data were used to make revision on the items set.

4. Results

4.1. Psychometric Properties of CK and PCK Items Set

The reliability of the developed PCK and CK test instrument was determined by using Cronbach's alpha (α) values. The items set showed acceptable reliability, with Cronbach's alpha of $\alpha = .53$. The item-total correlations showed that the items determined adequately. The mean of the correlations is $M = .59$, and the correlations are ranging from $.28$ to $.72$. The intercorrelations of the items is given in Table 2. Based on the intercorrelations value, the PCK Task item is less reliable than the other items. The correlations between PCK Task and other items are quite small, ranging from $-.10$ to $.18$.

Table 2. Intercorrelations of pre-service mathematics teachers' CK and PCK

Correlations	Task	Student	Instruction	ck ₁	ck ₂
PCK					
Task					
Student	.18				
Instruction	-.10	.10			
CK					
ck ₁	-.03	.15	.36		
ck ₂	.01	.58	.12	.54	
M	2.47	3.16	1.60	1.16	.91
SD	.60	.73	1.00	.72	.65

A Kolmogorov-Smirnov test was performed in evaluating the distribution of CK and PCK total scores or the distribution of item difficulties. The Kolmogorov-Smirnov test confirmed that the total scores are from a normal distribution population ($p > .15$). This result shows that the CK and PCK test can distinguish between high, middle, and low achievers. Moreover, all items showed valid item difficulties (P_m) within the acceptable range, $.20 < P_m < .80$.

The objectivity of item scoring was evaluated by using Kendall's coefficient of concordance on the inter-rater agreement. The values of Kendall's coefficient of concordance are ranging from 17.87 to 33 and the p -values are ranging from .003 to .21. This result indicates that all raters are agree in their assessment, except on PCK Task item.

4.2. Results from Expert Validation

An expert validation was performed to evaluate the validity of the CK and PCK items. The quantitative data from this validation confirmed a satisfactory validity of the items. The mean scores of each criteria

of validity are 4 to 4.08 out of 5. Aside of quantitative data, this validation obtained comments from the validator. In summary, the comments were regard to the clarity of sentences structure used in each item and scoring rubrics, as well as the suggestions on alternative solution of each item. All these comments were used in revising the items draft.

4.3. Pre-Service Mathematics Teachers' CK and PCK

In evaluating pre-service mathematics teachers' CK and PCK, mean and standard deviation of their score are determined. The mean and standard deviation of the score are $M_{CK} = 1.03$, $SD_{CK} = .60$, $M_{PCK} = 2.41$, and $SD_{PCK} = .48$. The maximum possible score is 4. The results indicate that, in general, the pre-service mathematics teachers' CK score are lower than the PCK score.

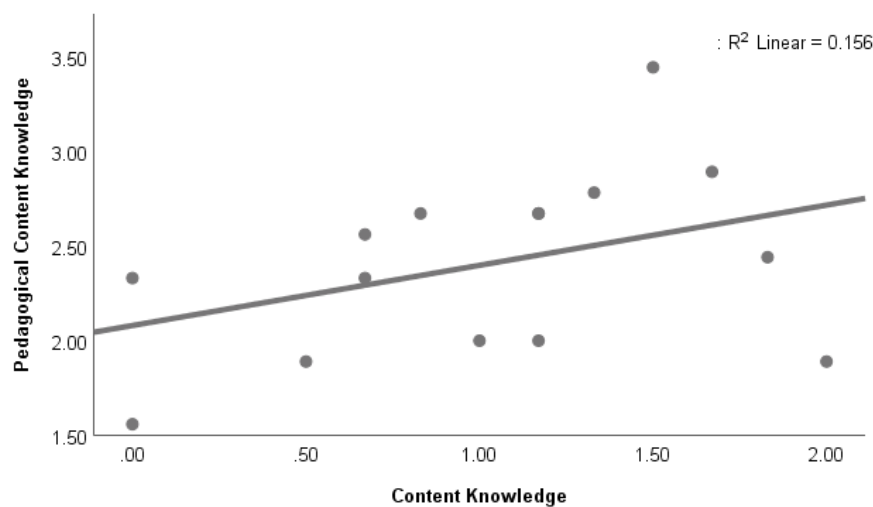


Figure 3. Scatter plot of participants' CK and PCK

Figure 3 shows the scatter plot of participants' CK and PCK. The CK score is the average of ck_1 and ck_2 items, whereas the PCK score is the average of PCK Task, PCK Student, and PCK Instruction. From the correlation analysis, it is obtained that $r = .40$ and $p = .15$. Even though the correlation value is moderate, but the p -value shows that the correlation is not statistically significant.

5. Discussion

The present study is a respond for the need of development of test instrument to measure pre-service mathematics teachers' CK and PCK [e.g., 25]. Three major steps have been conducted in developing such instrument, from deriving CK and PCK subscales from literature, constructing test items, to validating the items set. The first aim of the present study is to develop valid, objective, and reliable CK and PCK test instrument. The validity of the test instrument was evaluated by using the item-total correlations, Kolmogorov-Smirnov test of the scores' distribution, and item difficulties. Based on these analyses, it is found that, in general, the validity of the test instrument is acceptable. The objectivity of the scoring rubrics is found to be adequate. It also found that the test instrument is reliable.

From the result of pre-service mathematics teachers' CK and PCK measurement, it is shown that the PCK score is higher than the CK score. It indicates that the pre-service teachers who participated in present study have more knowledge in mathematical task for teaching, students' misconceptions and difficulties, and mathematics instruction than the knowledge about mathematics. This finding should be concern for mathematics educators on preparing future mathematics teachers. Shulman [1] emphasized that the teachers should possess content knowledge to in conducting teaching and learning.

Due to the small size of our sample, it not shown that there is statistically significant association between pre-service mathematics teachers' CK and PCK. This result not in line with many studies in literature [e.g. 26, 27]. Those studies found the connection between mathematical knowledge and

pedagogical content knowledge. Therefore, the pre-service mathematics teachers should be educated both from content knowledge and pedagogical content knowledge aspects, as stated by Shulman [1], “mere content knowledge is likely to be as useless pedagogically as content-free skill.”

6. Conclusion

The aims of the present study are to develop valid, objective, and reliable test instrument to measure pre-service mathematics teachers' CK and PCK, and to measure pre-service mathematics teachers' CK and PCK with the developed instrument. Therefore, the present study has been produced the test instrument that can used to quantify pre-service mathematics teachers CK and PCK. Aside of that, the present study also suggests that mathematics educators should educate the mathematics teacher candidate so that they possess mathematical knowledge and pedagogical content knowledge, even though the need is more on mathematical knowledge.

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Data availability statement

The data that support the findings of this study are openly available at DOI: [10.6084/m9.figshare.11418993](https://doi.org/10.6084/m9.figshare.11418993).

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