TAKING MATHEMATICS INSTRUCTION TO TASK: APPLYING SECOND LANGUAGE ACQUISITION APPROACHES TO ANALYZE AND AMPLIFY LEARNING OPPORTUNITIES FOR ENGLISH LEARNERS

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Drawing upon task-based learning designs from second language acquisition, we critique how mathematics education has primarily conceived of tasks as problems to be solved. We extend the notion of communicative gaps (e.g., opinion and information) into a framework that considers the flows of information that tasks structure and facilitate. We then employ the analytic framework to examine collaborative mathematics tasks from three popular middle school curricula. Such a framework offers educators tools for assessing the extent to which existing curricula provide English learners with challenging and well-supported opportunities to communicate about mathematics. Based upon this initial survey of the field, in terms of mathematical and language development opportunities we offer next steps and alternatives for curriculum designers and teachers to consider as they create mathematical tasks for English learners.

Keywords: information gap, mathematics, second language acquisition, task-based learning, task design

As secondary English learners develop conceptual understandings of mathematics and engage in mathematical practices in increasingly sophisticated ways, they will need to use and develop language (Heritage, Walqui, & Linquanti, 2015). Expanding opportunities for English learners to engage in rigorous mathematics while simultaneously developing language is especially urgent given new standards that equally emphasize procedural fluency, conceptual understanding, and mathematical practices (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). To realize the promise of such ambitious mathematics instruction for English learners, educators will need to develop their expertise in designing instruction (Valdés, Kibler, & Walqui, 2014). How English learners respond will depend on whether they are adequately challenged and supported by the tasks they are offered in learning mathematics, and the extent to which those tasks include opportunities for meaningful communication with one another (Koelsch, Chu, & Bañuelos, 2014; Walqui & Heritage, 2018).

When viewed from the perspective of the communicative tasks that are typical of instructed second language acquisition, how rich are the opportunities to engage in talk about mathematics? In this article, we develop and apply a framework grounded in task-based learning in second language acquisition and specific to mathematics to critique dominant approaches to "task"* (i.e., the term mathematics educators use to describe a contextualized story problem) in eighth-grade mathematics curricula. This framework reflects broader concerns about lesson design in mathematics (Hamburger & Chu, 2019); a key consideration is whether tasks invite and support English learners to engage in quality peer interactions characterized by sustained and reciprocal talk about important mathematical ideas or practices (Chu & Hamburger, 2019; Walqui & Heritage, 2018). We then extend this framework to offer educators options for amplifying the communicative invitations offered to English learners in mathematics classes. Authentic opportunities to communicate with language are indispensable for all students to participate with full engagement in the discipline of mathematics.

Conceptual Framework

In this section, we first provide an overview of how mathematics education has framed tasks, as the term is implemented by educators as part of ambitious mathematics instruction. We then provide an alternative perspective from task-based language pedagogy in second language acquisition, drawing upon this more communicative approach to task design to develop an analytic framework that we then apply to a sample of math lessons on the same topics from three popular and widely used curricula.

Tasks in Mathematics Education

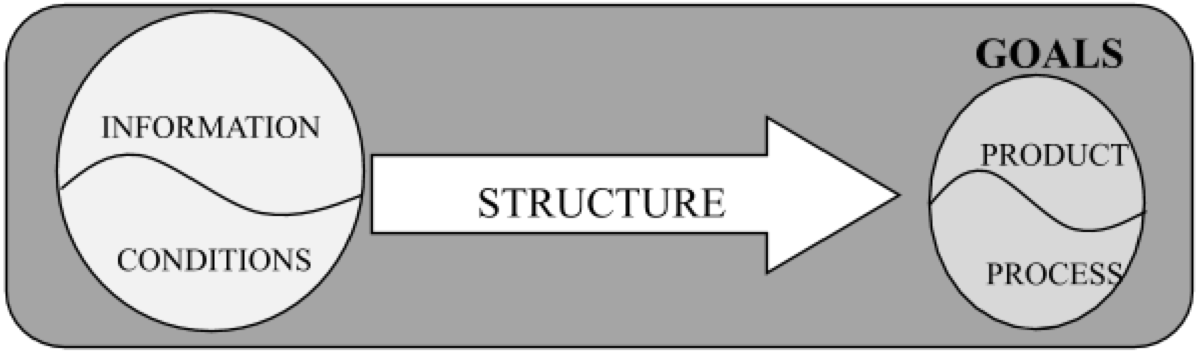
Reform-oriented, Standards-based mathematics education has focused on using mathematical problems as the basis for more ambitious, student-centered instruction (Stein, Smith, Henningsen, & Silver, 2009). The expectation is that if these problems are sufficiently rich in terms of their being about interesting and relevant real-world contexts and have the potential to develop important mathematical ideas, students will deepen their understanding beyond becoming fluent in standard mathematical procedures (e.g., multiplication of two-digit numbers). In order to reach this level of conceptual understanding, teachers must enact instructional practices orchestrating productive, whole-class discussions that go beyond "show and tell" (Stein, Engle, Smith, & Hughes, 2008, p. 313). To orchestrate such discussions requires that teachers *anticipate*** possible approaches, *monitor* what students are doing, *select* key approaches to be presented at large, *sequence* the approaches to build on each other, and *connect* the approaches in facilitating the discussion. Because effectively conducting these whole-class discussions for summarizing understanding requires substantive pedagogical expertise, novice teachers may not be able to sustain a focus on the "mathematical point" of the problem amid sharing different solutions for the sake of variety (Sleep, 2012, p. 936). For teachers of English learners in particular, an enduring dilemma is how to facilitate equitable student access to solving contextualized problems when knowledge of language and problem contexts may be a barrier (Martiniello, 2008).

Tasks in Second Language Acquisition

In the field of second language acquisition, there has not been a uniform way to define and design communicative tasks (Candlin, 1987; Ellis, 2003; Prabhu, 1987). Such tasks require interaction in the target language, moving beyond basic performances or displays of language where the emphasis is on grammatical correctness. Ellis (2003), however, identified six schema for analyzing tasks; based upon those schema, we developed a framework with different labels (information, construction, structure, goals, product, and process) to show the flow and connections between the components of an interactive task—specifically, to achieve the goals of a task (including both product and process), participants will need to draw upon the information they are given under certain conditions and follow the structure that is provided for them to take turns in talking with one another (see Figure 1). In previous research, we have found Ellis's schema helpful to amplify the design of mathematical tasks to include an explicit communicative focus (Chu, 2013).

We introduce each of these schema by considering a popular type of task often encountered in mathematics classes: a sorting task in which a small group of students is trying to come up with categories of parabolas that they are given, printed on individual cards (cf. Swan, 2007).

Figure 1. Framework for Analysis and Design of Conversational Tasks



Clearly articulating *what* students are meant to achieve may offer multiple options for *how* to reach that destination in terms of the other dimensions. We begin with *information* (what Ellis calls "input" [Ellis, 2003, p. 22]), the elements within a task consisting of the data, representations, texts, or prompts given to students (e.g., the carefully selected parabolas that may elicit categories). Furthermore, within instructed settings, a task provides certain *conditions* for how students gain access to the information. Ellis (2003, p. 21) identified two of the conditions as "shared" and "split." In the shared condition, students are looking at the same thing (e.g., looking at the same parabola and describing what they see). In the split condition, students are looking at different things and need to verbally explain what they see to make sense of what they have (e.g., looking at two different parabolas and trying to decide how they are similar or different). In our development of new tasks for mathematics classrooms, we were inspired by the split instructional design to create engaging tasks that require students to talk to one another about mathematical ideas and objects. We therefore have applied Ellis's categories—using our labels—as a way of amplifying the design of mathematical activities for English learners.

To reach its desired task goals, a task may include a well-defined *structure* or series of steps (what Ellis called "procedures" [Ellis, 2003, p. 21]) that students undertake as they complete a task together. In Figure 1, we refer to these well-defined steps or turns of a task as "structure" instead of "procedures" because the latter term has a particular and prominent meaning in mathematics (i.e., that of the algorithm). This structure often takes the form of a script, but it heightens productivity to offer students more generative actions they can transpose to other activities (Koelsch et al., 2014).

Finally, central to any task are its *goals:* what English learners will learn by completing that task. For example, the goal of a parabola-sorting task is to understand which features of parabolas, such as graphs, matter. To attain this goal, according to one of the three curricula we studied, students will need to create a *product* (e.g., a set of labels and classifying criteria) while engaging in a *process* (e.g., looking at and describing graphical features).

Communicative Gaps in Task-Based Learning Designs

Beyond Ellis's categories of split and shared conditions, the field of second language acquisition has also considered the notion of "gaps" in language-focused tasks, which are designed to be bridged through interaction in the target language. Two kinds of gaps are relevant here: opinion gaps and information gaps.

An *opinion gap* occurs when different speakers in a discussion share what they think. These opinion gaps occur naturally in such activities as choosing where to go for dinner or how to solve a mathematical problem. In these cases, there is no "right" answer per se. Within mathematics education, however, sharing different approaches may be considered an opinion gap if there are no clear criteria or a formal process in place for reaching consensus. In these cases, it is not sufficient to share approaches or opinions just for the sake of variety, but instead there should be a mathematical "point" such as efficiency (Sleep, 2012, p. 936).

Beyond these opinion gaps, second language acquisition instructional designs that use split conditions have employed *information gaps* to provide authentic opportunities to use language (Ellis, 2003). In a typical scenario, one party can see something that the other does not, and thus must use language to communicate information that the other party needs. The party who happens to hold the information is not necessarily more knowledgeable; rather, the task has created a condition in which that party must effectively communicate certain information to his or her partner. For instance, a student might have a card with a graph of a parabola that the partner does not see. The partner without the complete information must listen to the description, perhaps asking questions to refine and clarify meaning, all in order to sketch the parabola accurately as assessed by the partner with the complete information (Chu & Hamburger, 2019). This information-gap task therefore highlights the need to use language to communicate mathematical facts and ideas across an information gap.

Design of Mathematics Tasks for English Learners

Recent findings have emphasized two key actions in mathematical instruction: (a) the importance of fully drawing upon English learners' cultural and linguistic resources as they work on challenging mathematical tasks (de Araujo, Roberts, Willey, & Zahner, 2018), and (b) offering English learners explicitly supportive opportunities to engage in mathematical practices while having productive discourse with their peers (National Academies of Sciences, Engineering, and Medicine, 2018). To assist in implementing these actions, some mathematical language routines have been identified as ways to support English learners as they refine understanding, interpret story problems, or construct and critique arguments (Zwiers et al., 2017). These routines may be understood within the analytic framework to be primarily structures that support mathematical tasks with particular goals. Other taxonomies of tasks have been constructed based upon information flows and mathematical relationships, such as an expansion of an information gap task, as described above, to include a student pair having different information (e.g., graphs of parabolas) that they will then compare and contrast (Chu & Hamburger, 2019).

Research Question

The empirical portion of this study, which is based upon the six dimensions (schema) identified by Ellis (2003), is concerned with the classroom activities provided to students who are learning English as a second language as they learn mathematics. The descriptive research question for this study is: What is the nature of communicative tasks offered to English learners as they acquire key concepts of eighth-grade mathematics? Specifically:

- What is the goal, product, and process of tasks?
- How are information, conditions, and structure employed in tasks?

Methods Overview

Sample

We employed purposive sampling of eighth-grade curricula aligned with the Common Core State Standards in Mathematics (Patton, 2002). In this sampling, we aimed not to exhaustively select a representative sample of lessons, but rather to empirically test our conceptual framework in widely implemented instances of classroom mathematical tasks that would provide robust information about different types of student interaction. Such an empirical application allows for the identification of areas where existing curricula can be amplified for communicative opportunities or more radically redesigned to enhance English learners' access to and achievement of high-quality mathematical learning.

Because we wanted to sample for lessons that explicitly focused on conceptual development rather than problem solving, we first located tasks that are part of the Mathematics Assessment Resource Service's (MARS) Mathematics Assessment Project (Mathematics Assessment Resource Service, 2011; Swan, 2007). These MARS "classroom challenges" are provided as 20 lessons in each of Grades 6, 7, and 8, and then in the two high school courses of algebra and geometry. Among these, we selected 12 eighth-grade lessons that we labeled as "conceptual development" because of the critique that students may lose sight of the key conceptual goal of the lesson when problem solving and sharing varied approaches (Sleep, 2012). Finally, we narrowed our choices to six lessons spanning three of the clusters in the eighth-grade curriculum: Number & Quantity, Algebra, and Geometry.

To compare this corpus to a curriculum that has made explicit pedagogical accommodations for English learners, we selected the eighth-grade Illustrative Mathematics curriculum (Zwiers et al., 2017). For a total of three, we included the EngageNY curriculum, because it is widely used across the country and was one of the first freely available curricula to address Common Core standards (Common Core, Inc., 2013; Eureka Math, 2013). We matched the topics from the six lessons from the MARS challenge lessons to the corresponding lessons from the other two curricula that had the closest topical and conceptual fit (Table 1).

Table 1.

Sample of Lessons Reviewed

Cluster	MARS	Illustrative Mathematics	EngageNY
Number & Quantity	Translating Between Repeating Decimals and Fractions	Infinite Decimal Expansions (8.15)	Converting Repeating Decimals to Fractions (7.10)
	Estimating Length Using Scientific Notation	Multiplying, Dividing, and Estimating with Scientific Notation (7.14)	Choice of Unit (1.12)
Algebra	Solving Linear Equations in One Variable	All, Some, or No Solutions (4.7)	Classification of Solutions (4.7)
	Defining Lines by Points, Slopes, and Equations	Equations of all Kinds of Lines (3.11)	The Graph of Linear Equation in Two Variables (4.13)
Geometry	Representing and Combining Transformations	Making the Moves (1.4)	Sequences of Rigid Motions (2.10)
	Identifying Similar Triangles	Similar Triangles (2.8)	More about Similar Triangles (3.11)

Analysis

Based upon the conceptual framework, we developed a set of analytic questions for each of the six dimensions of task design, which, as noted, we based on the schema we developed from those first identified by Ellis (2003) (Table 2). For the empirical part of this study, we applied this framework to existing mathematics lessons and tasks. To code the selections from the three curricula, we answered the questions, with at least one lesson from each curriculum coded by two raters, with discussion to resolve any coding discrepancies.

Table 2.

Task Analytic Framework

Dimension	Analytic Question
Goals	What are the mathematical conceptual and pedagogical goals of this task?
Product	What mathematical artifact will the group create as they complete the task?
Process	In what mathematical practices, requiring concepts and language, will students engage to complete the task?
Information	What mathematical objects, data, or situations will students receive?
Conditions	How will the mathematical information be distributed among the different participants in the group?
Structure	What are the steps and turns provided for students to take in completing the task?

Results

In this section, we present findings organized by the six different task dimensions within our analytic framework to compare the three curricula (see the appendix, pp. 28–29, for a complete coding matrix).

Broad Consensus in Goals

Across the six topics, the lessons from each of the three curricula had similar goals in terms of what students were supposed to achieve mathematically by completing the task. The only exceptions were that, in some cases, the goals for a particular task within a lesson were more narrowly defined in one curriculum than in another. For example, one Illustrative Mathematics lesson distinguished only between linear equations with no solutions or infinitely many solutions, compared to the corresponding lessons in MARS and EngageNY, which also allowed for a third possibility (i.e., linear equations that have exactly one solution).

Variation in Making Common Products

Lessons from MARS consistently asked students to create posters collaboratively, positioning and affixing cards with mathematical objects or operations and showing different relationships between them. Students were further instructed to write annotations and explanations that justified their work. In contrast, tasks within Illustrative Mathematics and EngageNY primarily had students work independently and then share their work orally, either with a partner or with the whole class; there was no public product required for students to create or co-create.

Focus on Calculations and Procedures in Task Processes

Across all three curricula, mathematical processes predominantly involved carrying out calculations and procedures. For the EngageNY curriculum, frequently only one procedure would have been appropriate to the task, and indeed many of these tasks were labeled as “exercises,” indicating that they were meant as the application of a predetermined procedure.

There was one task in the Illustrative Mathematics curriculum that had a different process, one involving the sequencing of steps in an annotated algebraic calculation for solving equations that relates repeating decimals to fractions. This sorting task had little ambiguity, however, as there was only one sequence that could reasonably result.

In terms of linguistic processes, the MARS lessons asked students to construct explanations or rationales and then to evaluate the reasoning of others, either by agreeing with or challenging differing interpretations. While this process was part of the structure of the task, supports for engaging in this way were not explicit (e.g., in the form of formulaic expressions or other language useful for evaluating arguments).

Emphasis on Representations, Procedures, and Relations

Every MARS task focused on information, given on cards that contained different representational families (such as equations, fractions, and decimals); these cards frequently contained sets of cards that were labeled based upon these representational families. Several of these tasks also included blank cards on which students could supply their own objects, relationships, or information to complement what they were given.

For the Illustrative Mathematics and EngageNY curricula, information was primarily given in the form of story problems or exercises. One exception, mentioned above, provided students with steps and explanations of one student's algebraic solution presented in the curriculum; these steps and explanations were to be sorted. The other exceptions in these two curricula were in geometry lessons in which students were given different results or different scenarios they then had to describe or explain to their partners.

Primarily Shared Conditions

The MARS tasks reviewed had shared conditions, in which all information is visible to all parties. This information was frequently in the form of representations and relationships to be sorted. Most of the lessons from the other two curricula also consisted of shared conditions; an exception was three geometry lessons that presented split conditions. For the geometric transformations lesson in the EngageNY and Illustrative Mathematics curricula, pairs of students were given different figures that were the results of an unspecified transformation. Using coordinates, one member of the pair was then supposed to describe their shape to their partner, and then go on to explain a transformation that could result in that particular shape. In another Illustrative Mathematics lesson on similar triangles, students received different sets of angles, from which they individually tried to construct different triangles. One part of the task was to find another student with the same angles in order to compare the triangles they had constructed.

Largely Unspecified Structure

The EngageNY and Illustrative Mathematics curricula typically had a structure of individual work, followed by sharing with a partner or the whole class. No further guidance was given to assist students in communicating with peers and the teacher. These structures are compatible with the notion of an opinion gap, in which partners or group members share their individual approaches or solutions, but without a process to reach consensus.

In contrast, the MARS tasks each had specific steps that partners or small groups were to undertake as they completed the task. A sample is reproduced below:

Matching Card Set A

1. Take turns to match a card in scientific notation with a card in decimal notation.
2. Each time you match a pair of cards, explain your thinking carefully and clearly. Place your cards side by side on your desk, not on top of one another, so that everyone can see them.
3. Partners should either agree with the explanation, or challenge it if it is not clear or not complete. It is important that everyone in the group understands the matching of each card.
4. You should find that two cards do not have a match. Write the alternative notation for these measurements on the blank cards to produce a pair. (Mathematics Assessment Resource Service, 2015, p. P-4)

This clear structure for tasks also emphasized the need for consensus at each step, while providing guidance for how to make thinking and reasoning visible. Further instructions were provided for each stage of tasks, including instructions for creating posters and writing annotations.

Discussion and Application

While this study was intended as a practical application of a task-design framework from second language acquisition rather than an exhaustive or comprehensive study of curricula, several patterns emerge from this initial review. First, there is general alignment across curricula in terms of the conceptual goals and processes of tasks. Second, the MARS tasks often differ from Illustrative Mathematics and EngageNY in terms of product (i.e., a poster that publicly displays connections and categories), information (i.e., objects given as parts of representational families), and structure (i.e., specific steps that small groups are to undertake in embedding criteria and requirements, such as challenging ideas and reaching agreement). Finally, the split conditions, a regular feature of task-based pedagogy in second language acquisition, were seen just in the Illustrative Mathematics and EngageNY curricula for geometry lessons in which different students received different objects. Overall, however, outside of the MARS tasks' specific instructions, there appears to be little attention paid to the design of tasks that would enhance English learner opportunities to participate fully and equitably. Without clear structures to ensure their participation, English learners may be left out of many of these potentially rich mathematical interactions.

A set of questions to guide educators as they reconsider different aspects of task design is given in Table 3. We focus on the last five dimensions, given the overall agreement across the three curricula on goals that are sufficiently challenging in alignment with the ambitious demands of new standards.

Table 3.

Design Considerations for Amplifying Tasks

Dimension	Questions to Consider
Product	● How clearly are components of the final product modeled? ● To what extent will the product require small groups to negotiate and make decisions about content?
Process	● How clearly are the underlying and essential mathematical practices modeled for students? ● What examples of language for engaging in mathematical practices are offered to students?
Information	● To what categories do the mathematical objects belong and how are these categories signaled to or solicited from students? ● What are the underlying mathematical relationships between the objects that students are asked to explore?
Conditions	Could alternative conditions enhance opportunities for students to interact? ● Split ● Split to shared ● Jigsaw formats
Structure	● How do the steps of the task ensure that all students *must* participate for the group as a whole to succeed? ● How clearly do steps signal their purpose in facilitating the completion of the task?

In the rest of this article, we consider ways in which mathematics teachers and curriculum designers may amplify the design of existing tasks, by providing clear structures and conditions under which information is offered to students.

Requiring Negotiation in Creating Products

The MARS tasks ask students to develop clearly defined products, often in the format of pasting cards down and providing their reasoning turn by turn. While this clarity makes student reasoning public in a way that the other curricula do not, there is a danger that the required product, a poster, serves as the exclusive record of what the students have done rather than as an opportunity for students to synthesize their understandings and negotiate the joint creation of a poster to display their more fully emerged understandings. The running-record nature of such posters may also tend to reflect the contributions of more assertive students. Two questions are therefore important for educators to consider in deciding on what products to ask from students:

- How clearly are components of the final product modeled?

- To what extent will the product require small groups to negotiate and make decisions about content?

In the context of creating posters, instructions ought to clearly demonstrate the different components, as well as criteria for quality. A rubric can provide students with critical guidance as they create posters. Sometimes, however, less may be more—students may engage in an "information dump" as they create posters, indiscriminately providing everything that they have done (Chu, 2013). By requiring small groups to select a smaller number of examples, representations, or connections collaboratively, the poster that is ultimately produced can facilitate strategic discussions that engage English learners in reflecting on higher order features of mathematics.

Modeling Clear Mathematical Processes and Practices

While there is general agreement and adequately ambitious processes across the three curricula, tasks could provide greater clarity in terms of the mathematical practices that students are engaged in and the more generative language that will assist English learners to engage in those practices. Two questions are salient for educators as they consider the processes at which tasks aim to achieve:

- How clearly are the underlying and essential mathematical practices modeled for students?
- What examples of language for engaging in mathematical practices are offered to students?

To provide English learners with explicit models, it is helpful to identify the finer grained practices that are constituent of a broader mathematical practice (Chu & Hamburger, 2019; Koelsch et al., 2014). For instance, as students are sorting cards to identify patterns, they may find it useful within the mathematical practice of "look for and make use of structure" to "put into groups and take groups apart." Specific language that will assist includes "If I take apart . . . I have . . ." and "If I put together . . . I get . . ." (Chu & Hamburger, 2019, p. 222).

Eliciting Categories and Focusing on Relationships

With regard to the information offered to students, the MARS tasks clearly signal the different categories to which information provided to students belong (e.g., measurements, objects, and scalar comparisons). It may also be productive to solicit these categories from students so that they are more centrally involved in connecting different schema and representational families. Further consideration should also be given to the diverse underlying mathematical relationships that the selected mathematical objects have, including relationships of order, equivalence, correspondence, and other specific attributes.

Therefore, educators should consider two questions as they design the information they provide students with for them to connect:

- To what categories do the mathematical objects belong and how are these categories signaled to or solicited from students?
- What are the underlying mathematical relationships between the objects that students are asked to explore?

Emerging work has identified how these underlying relationships may be related to different conditions and task designs, such as task structures that support students as they put different objects in order or identify the links that connect different mathematical objects (Chu & Hamburger, 2019).

Offering More Split Conditions

Educators should consider whether the following conditions could enhance opportunities for English learners to interact with each other:

- Split
- Split to shared
- Jigsaw formats

Split structures were implemented by some of the EngageNY and Illustrative Mathematics lessons, in which students received different geometric figures and had to describe those figures and identify transformations from a common reference figure. These classic information gap tasks can provide more equitable opportunities for all students to contribute (Ellis, 2003).

Sorting tasks may benefit from a split-to-shared structure, in which students sort the cards one by one in groups of two or four, with each student taking out a card, describing what is on it to the group, and then offering to that group an idea about how to sort that particular card (Chu, 2013). This hybrid structure integrates the split condition for information by not immediately revealing what an individual student has, requiring that student to orally describe its salient features and advance a tentative suggestion for sorting. Over time, however, the information is shared by the group as more and more objects are added and further categories and connections emerge or unfold over time.

Jigsaw structures—which enact elaborately split conditions—may also afford greater opportunities for students to interact. One instance is the "jigsaw project," in which students begin in a base group and then are assigned to an expert group in which they themselves become expert in a particular problem or case (Walqui & van Lier, 2010). In their expert group, students answer key focus questions that structure how they will share their problem or case with their base group. When they present their findings to their base group, the narrower focus provided by the selected questions facilitates the identification of similarities and differences and key ideas that cut across the different cases or problems. Other puzzle-like structures include providing groups of students with structured sets of clues that serve to model the different categories of attributes that mathematical objects have (Chu & Hamburger, 2019).

Specifying and Signaling Purposes of Structure

Finally, with regard to the structure of tasks, educators should consider two questions as they seek to enhance communicative opportunities for English learners:

- How do the steps of the task ensure that all students must participate for the group as a whole to succeed?
- How clearly do steps signal their purpose in facilitating the completion of the task?

An example of an equitable structure that requires the participation of all students is the split-to-shared implementation of sorting tasks, in which all students in a small group take turns orally describing new cards, which they then place within the shared space of examples that have been sorted so far. When a structure requires all students to take meaningful turns, individual participation enables group success; in addition, the different steps that students engage in during a task should clearly signal the broader importance of these steps for completing the task. MARS tasks frequently signal these purposes explicitly, but for English learners it may be beneficial to also offer models of language for engaging in these steps—e.g., in the form of generative, established expressions.

Conclusion

In sum, the analytic framework with six dimensions derived from second language acquisition and the aligned design considerations provide practical tools for educators to apply as they critically examine existing tasks and consider how to expand communicative opportunities in mathematics for English learners. It is encouraging that there is already general agreement about two of the dimensions, the ambitious goals and process of many mathematical tasks. Building on this agreement, educators will need to empirically test and refine the other elements—with a particular focus on conditions and structure—to design opportunities that are more inviting for English learners.

While there is much work to be done, a shared framework grounded in task design can over time serve to develop shared instructional practices and designs that improve outcomes for English learners. Further inquiry is necessary to understand how these task designs would fit into broader lessons and units. Curriculum developers will need to work closely with classroom teachers to understand what other supports may be necessary. With a common framing, new categories of conditions and structure may be developed and empirically tested to accelerate the achievement of English learners in mathematics.

References

Candlin, C. N. (1987). Towards task-based language learning. In C. Candlin & D. Murphy (Eds.), *Language learning tasks* (pp. 5–22). *Lancaster practical papers in English.* Lancaster, UK: Lancaster University.

Chu, H. (2013). Scaffolding tasks for the professional development of mathematics teachers of English language learners. In C. Margolinas (Ed.), *Task design in mathematics education* (pp. 559–567). Oxford, UK: International Commission on Mathematics Instruction.
Chu, H., & Hamburger, L. (2019). Designing mathematical interactions for English learners. *Mathematics Teaching in the Middle School, 24*(4), 218–225.
Common Core, Inc. (2013). *New York State Common Core math curriculum.* https://www.engageny.org/resource/grade-8-mathematics
De Araujo, Z., Roberts, S., Willey, C., & Zahner, W. (2018). English learners in K–12 mathematics education: A review of the literature. *Review of Educational Research, 88*(6), 879–919.
Ellis, R. (2003). *Task-based language learning and teaching.* Oxford, UK: Oxford University Press.
Eureka Math (2013). New York State Common Core math curriculum. https://www.engageny.org/resource/grade-8-mathematics
Hamburger, L., & Chu, H. (2019). Making slope a less slippery concept for English learners: Redesigning mathematics instruction with rich interactions. In A. Walqui & G. Bunch (Eds.), *Amplifying the curriculum: Designing quality learning opportunities for English learners* (pp. 115–137). New York, NY: Teachers College Press.
Heritage, M., Walqui, A., & Linquanti, R. (2015). *English language learners and the new standards: Developing language, content knowledge, and analytical practices in the classroom*. Cambridge, MA: Harvard Education Press.
Koelsch, N., Chu, H., & Bañuelos, G. (2014). Language for learning: Supporting English language learners to meet the challenges of new standards. *TESOL Quarterly, 48*, 642–650.
Martiniello, M. (2008). Language and the performance of English language learners in math word problems. *Harvard Educational Review, 78*, 333–368.
Mathematics Assessment Resource Service (MARS). (2011). *Performance assessment task: Sorting functions.* Retrieved from http://map.mathshell.org/tasks.php?unit=HA16&collection=9&redir=1
Mathematics Assessment Resource Service (MARS). (2015). *Classroom challenges: Estimating length using scientific notation.* Retrieved from https://www.map.mathshell.org/download.php?fileid=1664
National Academies of Sciences, Engineering, and Medicine (2018). *English learners in STEM subjects: Transforming classrooms, schools, and lives*. Washington, DC: The National Academies Press. https://doi.org/10.17226/25182
National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common Core State Standards*. Washington, DC: Authors.
Patton, M. (2002). *Qualitative research and evaluation methods* (3rd ed.). Thousand Oaks, CA: Sage.
Prabhu, N. (1987). *Task-based pedagogy.* Oxford, UK: Oxford University Press.
Sleep, L. (2012). The work of steering instruction toward the mathematical point: A decomposition of teaching practice*. American Educational Research Journal, 49*, 935–970.
Stein, M., Engle, R., Smith, M., & Hughes, E. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10,* 313–340.
Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2009). *Implementing standards-based mathematics instruction: A casebook for professional development* (2nd ed.). New York, NY: Teachers College Press.
Swan, M. (2007). The impact of task-based professional development on teachers' practices and beliefs: A design research study. *Journal of Mathematics Teacher Education, 10,* 217–237.
Valdés, G., Kibler, A., & Walqui, A. (2014). *Changes in the expertise of ESL professionals: Knowledge and action in an era of new standards.* Alexandria, VA: TESOL International Association.
Walqui, A., & Heritage, M. (2018). Meaningful classroom talk for English learners. *American Educator, 42*(3), 18–23, 39.
Walqui, A., & van Lier, L. (2010). Scaffolding the academic success of adolescent English language learners: A pedagogy of promise. San Francisco, CA: WestEd.

Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Principles for the design of mathematics curricula: Promoting language and content development.* Retrieved from http://ell.stanford.edu/content/mathematics-resources-additional-resources

Notes

*Unless otherwise identified with a page number from the accompanying citation reference, quotes used for certain terms and phrases common in the educational field are the authors' choice.
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APPENDIX: SUMMARY OF CODING OF LESSONS

Source	Lesson	Goals	Product	Process	Information	Conditions	Structure
MARS	Translating Between Repeating Decimals and Fractions	Match equations to decimals and fractions	Triplets of decimals, fractions, and equations	Generating new equations and fractions	Cards in three families	Shared	Find triplets (possibly generating missing info), and state reasoning to group
	Estimating Length Using Scientific Notation	Sort measurements with objects and scalar comparison between objects	Poster with measurements, objects, and comparisons	Sorting, connecting, and computing	Three sets of cards given serially	Shared	Partners: take turns to make pairs and reach agreements in each step
	Solving Linear Equations in One Variable	Sort linear equations by nature of solution set	Poster with explanations	Solving equations	Cards with linear equations	Shared	Place cards, give explanation; challenge possible
	Defining Lines by Points, Slopes, and Equations	Matching lines given by specific information	Pairs of cards with two points or point and slope	Computing equations, substituting values, calculating slope	Cards	Shared	Pair and provide explanations, matching slopes
	Representing and Combining Transformations	Connect shapes with transformations	Poster with explanations	Transforming shapes with rigid motions	Card sets with shapes and transformations	Shared	Pair and provide explanations
	Identifying Similar Triangles	Sorting triangles into similar, not similar, or cannot be determined.	Poster with explanations	Coordinating parts of triangles, mentally applying dilations	Cards with pairs of triangles and marked relationships	Shared	Pair and provide explanations to reach agreement
Illustrative Mathematics	Infinite Decimal Expansions (8.15)	Connect equations and calculations to fractions and decimals	Sorted order	Sequencing given steps in order	Strips with algebraic equation and explanation or work	Shared	None
	Multiplying, Dividing, and Estimating with Scientific Notation (7.14)	Comparisons of solar system distance, diameter, and mass	Answers to specific questions in terms of scalars	Using scientific notation with multipliers or division	Data table and questions	Shared	Pairs do "Notice and Wonder" with 10–12 minutes' work time,

							then group discussion
	All, Some, or No Solutions (4.7)	Distinguish algebraic identities from equations with no solutions	Classification into all or no solutions	Checking of values or algebraic manipulation	Set of equations	Shared	Individual thinking, then partner talk, then whole class
	Equations of all Kinds of Lines (3.11)	Plot line representing rectangles with fixed perimeter	Plot of line with ordered pairs	Checking of values	Problem	Shared	Individual think time to whole-class discussion
	Making the Moves (1.4)	Describe transformations	Oral description to create image	Description and clarifications in ordinary language	Card with image polygon	Split	Working in pairs, with a grid
	Similar Triangles (2.8)	Create and compare triangles with given angles	Pasta models of triangles with given angles	Trial and error and following directions	Three angles cut out from master; breakable pasta	Split	Individual work, followed by matching the comparison
EngageNY	Converting Repeating Decimals to Fractions (7.10)	Convert repeating decimals to fractions by solving equation	Conversion	Scaling by powers of 10 and comparing	Problems	Shared	Work in pairs
	Choice of Unit (1.12)	Express planetary masses relative to Earth	Ratios of masses	Division using scientific notation	Data set of planetary masses	Shared	Independent or small groups, then whole class
	Classification of Solutions (4.7)	Solve groups of equations and make comparisons	Categories and observations of solution types	Solving equations algebraically, grouping	Sets of equations grouped in families	Shared	Small-group work
	The Graph of Linear Equation in Two Variables (4.13)	Evaluate and graph linear equations	Table and graph for standard form	Substitution and solving	Set of equations and graph paper	Shared	Individual work followed by partner share
	Sequences of Rigid Motions (2.10)	Identify transformations relating pairs of shapes	Sequences of transformations	Transformation and sketching of intermediate stages	Shared problems	Split	Option: each student works a different scenario,

							then compares
	More about Similar Triangles (3.11)	Similar triangle problems	Computing lengths of missing sides	Ratios, scale factors, and solving proportions	Set of exercises	Shared	Independently or in pairs