VISUOSPATIAL WORKING MEMORY AND EARLY MATH SKILLS IN FIRST GRADE CHILDREN

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ABSTRACT

This study aimed to investigate the relationship between different components of active visuospatial working memory and math ability in young children. In a longitudinal study, we compared the contributions of active visual and spatial working memory (WM) tasks in early math performance at two times: the beginning of the first class of primary school (T1) and the end of the first class of primary school (T2). Two tests were conducted with 43 young participants to investigate active visual WM (Imaginative Puzzles) and active spatial WM (Corsi Backward). Measurements related to pre-math ability (BIN 4-6 test) at T1 and math skills (AC-MT 6-11 test) at T2 were accomplished. The relationship between visual and spatial WM and math ability was analyzed using a regression model in which the predictors were identified through a forward selection based on the use of the BIC index (Bayesian Information Criterion). Results show that at the beginning of primary school, basic knowledge of magnitude and numbers is strongly influenced by spatial WM. T1 pre-math performance is the sole predictor of mathematical performance at T2. These results suggest different implications of domain-general and domain-specific variable on early math performance, depending on the child's development period. This finding brings additional evidence to the debate on the relationship between visuospatial WM and math ability in young children.

KEYWORDS

Visual Working Memory, Spatial Working Memory, Early Math, Pre-Math, Longitudinal Study

1. INTRODUCTION

The literature of early math skills development emphasizes that, at the beginning of primary school, this type of learning involves both domain-specific skills related to numerical cognition, such as understanding of quantity, recognition of numbers and counting skills (Geary, Hamson, & Hoard, 2000; Krajewski & Schneider, 2009), and domain-general cognitive skills that predict not only mathematics achievement but also other general learning abilities (De Smedt et al., 2009; Fuchs et al., 2005; Passolunghi, Mammarella, & Altoè, 2008; Passolunghi, Vercelloni, & Schadee, 2007).

Among the domain-general cognitive abilities involved in math learning, the literature assigns a key role to working memory (Hitch, 1978; McLean & Hitch, 1999; Gathercole & Pickering, 2000; Rasmussen & Bisanz, 2005; Bull, Espy, & Wiebe, 2008; Passolunghi et al., 2008) and its visuospatial components (e.g., Krajewski & Schneider, 2009; Passolunghi & Mammarella, 2010; Alloway & Passolunghi, 2011; Passolunghi & Cornoldi, 2008).

Working memory (WM) is a mental space that controls, processes, and maintains relevant information to complete a complex cognitive task. According to the original Baddeley and Hitch model (Baddeley & Hitch, 1974; Baddeley, 1998) WM consists of a unitary central executive (CE), considered to be the heart of the model because it controls and manipulates relevant information, and two sub-components, the phonological loop and the visuospatial sketchpad, that maintain verbal and visuospatial information. Shah and Miyake (1996) questioned whether the CE should be considered a unitary construct, suggesting that, even at the level of higher cognitive processes, it is necessary to make a distinction between the verbal and visuospatial domains. At a lower level of processing, Logie and Pearson (1997) proposed separating the visual and spatial WM maintenance sub-components, with the visual component active in the recall of shapes and textures and the spatial component devoted to the recall of spatial locations and sequences.

The Cornoldi and Vecchi (2003) Continuity Model is a WM model that recognizes both the distinction between storage and processing and the presence of distinct verbal, visual, and spatial components at various levels of cognition. This model has a cone structure characterized by two main dimensions: the horizontal continuum and the vertical continuum. The horizontal continuum defines the type of information used in a given task and, at this level, it is possible to hypothesize the existence of semi-independent systems and, therefore, to identify possible dissociation between specific types of information (e.g., verbal, visual, and spatial codes), while the vertical continuum represents the level of active control associated with a particular cognitive task.

The literature on the relationship between active WM and math skills is mainly related to the Baddeley and Hitch model and highlights how the CE is the WM component that best predicts mathematics performance. The CE involvement results come principally from studies that used verbal, active WM tasks, such as backward digit span, counting recall, and listening span tests (see Raghubar, Barnes, & Hecht, 2010 and Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013 for reviews).

Even though the CE is considered to be the best predictor of mathematics performance, there are few studies on the relationship between the CE (the active WM) and math performance conducted with separation of the active visuospatial domain and the active verbal domain. Instead, most of the works that have investigated the involvement of the CE in math learning have considered verbal and visuospatial active memory task performances together with a unique score (e.g. Passolunghi and Lanfranchi, 2012; Simmons, Willis and Adams, 2012). The results of these studies confirm the involvement of active WM skills in early mathematical performance and learning but do not allow us to identify the presence of a unique contribution of visuospatial active WM abilities in math abilities development.

A clear suggestion about the utility of separating the verbal and visuospatial components of active WM in relation to learning abilities is found in St Clair-Thompson & Gathercole (2006), who explored the influence of active visuospatial WM and active verbal WM on school performance in a sample of adolescents. Their results showed a strong relationship between visuospatial WM and math, while verbal WM was involved only in the English score. Further evidence that active visuospatial WM is individually implied in math learning comes from Cornoldi, Della Vecchia, and Tressoldi (1995) who studied sixth-grade children with low visuospatial ability and observed that these children had greater difficulty in math than in other school disciplines; the children had low scores in both active and passive visuospatial tasks, but the effects were more apparent in active visuospatial tasks. Bull et al. (2008), in a similar vein, observed in a longitudinal study that visuospatial WM, evaluated in first grade through the Corsi Backward test, predicted mathematical performance in the third class of primary school.

These results suggest that active visuospatial abilities are involved in math learning and that the involvement of active visuospatial processing may be different at different ages during math learning processes.

As for the distinction between spatial and visual active WM, to our knowledge, there are no studies that have directly investigated the different contributions of visual and spatial active WM to early numeracy.

In the current study, we investigated the implications of different sub-components of active visuospatial WM (according to the Cornoldi and Vecchi Continuity Model) on the developmental trend of pre- and early math skills. We performed a longitudinal design structured with two pivot times: Time 1 (T1), the beginning of the first class of the primary school; Time 2 (T2), the end of the first class of primary school.

In our work, we have maintained a distinction between visual and spatial active tasks, considering the involvement of the different components of the CE in the very first stage of math learning. We also take into account the relationship between pre-math abilities and later math performance, since recent studies show that early numeracy abilities are important domain-specific predictors for math learning (e.g., Passolunghi et al., 2007).

Our first goal was to observe whether there is a different contribution from the active visual and spatial WM abilities in children's math performance, measured at two different moments. The literature describes research on passive STM visual and spatial processes linked to math learning, but there are no studies that have kept the spatial and visual domain distinct in the active WM. If active processing follows the same path of passive processes, we should expect different contributions from the visual and spatial WM tasks at different development stages; the spatial areas are most involved in the youngest children's performance, while the visual areas are most involved in the subsequent years of schooling (McKenzie, Bull, Gray, 2003; Holmes et al. 2008). Finally, we expect continuity between pre-math competence and early math performance during the first year of primary school.

2. METHOD

2.1 Participants

Forty-three Italian children (23 males and 20 females) were recruited from a public school in Cagliari (Italy). Children were tested at the beginning of the first primary school year (October, time 1; mean age = 77.7 months; SD = 3.32; range: 71-84 months) and at the end of the first primary school year (May, time 2; mean age = 82.7; SD = 3.73). All participants showed typical development as identified by local educational services, and all were native Italian speakers. Both the school and the children's parents agreed to let the students take part in the research study, and all signed informed consent forms. Ethical approval was granted by the ethics committee of the Department of Education, Psychology, Philosophy of the University of Cagliari.

The socioeconomic status of the sample, measured by the Family Affluence Scale (Boyce, Torsheim, Currie, & Zambon, 2006), was middle class.

2.2 Procedure

An experienced psychologist tested children individually in a quiet room of the school in a single session according to the procedure defined for the tests, from Monday to Friday and from 8:30 a.m. to 11:45 a.m. daily. Each session lasted about 30 minutes.

The psychometric tools used to collect data included instruments designed to measure the active visuospatial WM, instruments designed to assess early numerical competence, and instruments to evaluate mathematics achievement. The same tests were presented at T1 (beginning of the first year of primary school) and T2 (end of the first year of primary school).

During the time elapsed between the first and second test sessions the children attended to the learning/teaching activities following the Italian Education Ministry guidelines, which do not include specific visuo-spatial skills training, and which lead to the acquisition of the basic concepts of mathematics through verbal activities integrated with visual ones such as completion games, puzzles and card games.

Active visuospatial working memory tasks. The visuospatial WM skills were tested through the Italian BVS-Corsi test (Battery for Visuo-Spatial Memory; Mammarella, Toso, Pazzaglia, & Cornoldi, 2008), by using two tasks: the Imaginative Puzzles test (identified in the next sections with the term VisWM) to evaluate the active visual WM, and the Corsi Backward test (identified with the term SpWM) to evaluate the active spatial WM.

Early numerical competence assessment. Children's early numeracy skills at T1 were evaluated through the "Battery for the evaluation of Numerical Intelligence from 4 to 6 years of age" (BIN 4–6) (Molin, Poli, Lucangeli, 2007), a standardized test providing norms for pre-school Italian children. The battery is made up of 12 subscales divided into four areas: lexical, which requires knowledge of numerical symbols (numbers recognition task, Arabic digits reading task, and Arabic digits writing task); semantic (comparing two Arabic digits to determine which represents the larger quantity and comparing two arrays of dots to determine which contains more dots); counting (counting forward and backward; ordering Arabic numerals; completing numerical series); and pre-syntactic (matching Arabic numerals with corresponding dots; the one-many task; ordering different objects cards from bigger to smaller and vice versa). The final score of Early Numerical Competence (ENC) is the sum of all the subscales of the battery.

Mathematics achievement. To assess the mathematical performance at T2, the Italian test "AC-MT 6-11 - Test for the evaluation of calculating and problem-solving abilities" (Cornoldi, Lucangeli, & Bellina, 2012) was used. The test was administered collectively to the entire class under the supervision of an experienced psychologist. AC-MT 6-11 is a paper and pencil test comprising five subtests: Operations (child must solve additions and subtractions); Number judgment (evaluates the semantic comprehension of number quantities); Tens and Ones task (evaluates the ability to process the syntactic structure of numbers); Larger to Smaller task and Small to Large task (evaluates the semantic representation of numbers). The final score of Numerical Intelligence (NI) is the sum of the scores of the five subtests.

2.3 Statistical Analyses

Descriptive statistics of variables and measures were calculated (Table 1).

Table 1. Descriptive Statistics of the Variables Considered in the Study for Each Time (n = 43). VisWM = visual WM, measured by the Imaginative Puzzles test; spWM = spatial WM measured by the Corsi Backward test; ENC = Early Numerical Competence measured by BIN 4–6 test; NI = numerical intelligence measured by AC-MT 6-11 test

	Time 1			Time 2		
	Mean	sd	Range	Mean	sd	Range
Age	77.7	3.73	71-84	82.7	3.73	76-89
VisWM	11.19	4.57	2-20	12.12	4.05	2-20
SpWM	2.60	0.66	2-5	2.93	0.67	2-5
ENC	100.05	4.27	88-106	-	-	-
NI	-	-	-	20.28	5.25	4-26

Exploratory correlation analyses were performed to examine the relations between math measures and active VSWM measures. The Pearson's index was used. The analysis showed significant correlations between ENC (T1) and: NI (T2), r (43) = .50, p < .01, VisWM (T1), r (43) = .36, p < .05, SpWM (T1), r (43) = .46, p < .01. Further, significant correlations were found also between NI (T2) and: VisWM (T1), r (43) = .40, p < .01, SpWM (T1), r (43) = .35, p < .05, VisWM (T2), r (43) = .31, p < .05. There was not a significant correlation between SpWM (T2) and NI (T2), r (43) = .23, p > .05.

Table 2. Pearson's *r* correlation between active VSWM measures and early numerical competence tasks (ENC) at T1 and numerical intelligence (NI) at T2. VisWM: active visual WM; SpWM: active spatial WM

	VisWM (T1)	VisWM (T2)	SpWM (T1)	SpWM (T2)	ENC (T1)	NI (T2)
VisWM (T1)	-					
VisWM (T2)	.40**	-				
SpWM (T1)	.28	.47**	-			
SpWM (T2)	.28	.24	.26	-		
ENC (T1)	.36*	.52***	.46**	.35*	-	
NI (T2)	.40**	.31*	.35*	.23	.50***	-

*** $p < .001$; ** $p < .01$; * $p < .05$.

As second analysis step, we fitted two regression models to study the influence of active VSWM variables both on early numeracy abilities ENC at T1 and on math ability NI at T2. Relevant predictors were identified by a forward selection procedure based on the Bayesian Information Criterion (BIC) index (Schwarz, 1978), which selected the best model from a Bayesian perspective.

Results at T1 showed that ENC was predicted by only the active spatial WM task (β = .46, p = .001), which explained 21% of the variance in early numeracy ability. Results at T2 showed that NI performance was predicted only by the ENC measured at T1 (β = .50, p = .0004), which explained 25% of the variance in Numerical Intelligence.

3. DISCUSSION AND CONCLUSIONS

In the present study, we longitudinally studied the involvement of active VSWM on early numeracy abilities during the first year of primary school, distinguishing between spatial and visual active WM sub-components. In our work, children at the beginning of the first (T1), and at the end (T2) of the first primary school year were given two tasks that investigated active visual WM (Imaginative Puzzle task) and active spatial WM (Corsi Backward task). Moreover, early numeracy competence was measured with a pre-math normed test, (BIN 4-6 - Molin et al., 2007) at the beginning of the first year of primary school, and mathematical skills were measured with a normed math test (AC-MT 6-11 - Cornoldi et al., 2012) at the end of the first year of primary school.

We explore through a correlation analysis the relationship between early numeracy competence (BIN test score at T1) and later math skills (total AC-MT 6-11 test score at T2). Early numeracy abilities significantly correlated over time with later math skills. Children who, before starting school, had best learned how to handle numerical magnitude and numerical symbols, show—at the end of first grade—a better comprehension of the semantic and syntactic structure of numbers and are more able in written calculations. This result was expected, indicating that the first knowledge of magnitude and numbers provides an important foundation that influences the child along the first stage of formal mathematics teaching. It is a well-known phenomenon, in fact, that one of the best predictors of school success in mathematics is the level of early numeracy abilities (Gersten, Jordan, & Flojo, 2005; Jordan, Kaplan, Olàh, & Locuniak, 2006; Passolunghi et al., 2007; Passolunghi et al., 2008; Lyons, Price, Vaessen, Blomert, & Ansari, 2014).

Concerning the involvement of active WM components (visual and spatial) with numerical ability, the correlation analysis shows at T1 a good correlation between WM abilities, both visual and spatial, and pre-math abilities. At T2, instead, the correlation between math abilities and WM abilities is weaker: only the visual WM is significantly correlated with NI, while the spatial WM abilities are not significantly correlated with Numerical Intelligence score. It should be noted that the correlation between NI and earlier T1 WM competences is quite strong. Correlation analysis, so, suggests that, while before starting the formal path of math learning, both visual and spatial active WM abilities are correlated with pre-math knowledge, at the end of first grade the visual WM may be more influential than the spatial one.

To better explore these results, two regression analysis were computed. The first regression model aimed to study the influence of active VSWM variables on early numeracy abilities measured at T1 and showed that spatial working memory ability is the best predictor of mathematical performance; it explains the 21% of variance. However, a second regression analysis, in which all variables measured at the two times have been considered, suggests that at different time steps it is possible to find a different involvement of domain-specific and domain-general variables. In this second regression analysis the math performance measured at T2 is predicted only by the early numerical competence measured at T1, that explains 25% of the variance.

Therefore, at the very beginning of primary school, when there has not yet been a formal math teaching, the construction of basic knowledge about magnitudes and numbers and the first knowledge about the lexical and semantic aspects of numbers are strongly influenced by spatial active WM. Moving to the end of first grade (T2), our results show that at this stage the math performance is mainly based on the already possessed knowledges about magnitude and numbers.

Our exploratory study aimed to cast light on the involvement of visual and spatial active WM in the first phase of math learning, at a developmental stage for which no data are available: the first year of primary school. Even considered the limitation of a relatively scarce sample size, our results show that active VSWM influences early math performance in the very first phases of math learning and that there is a different individual contribution of visual and of spatial components.

At a theoretical level, this preliminary result suggests the importance to distinguish between visual and spatial WM components also at the processing level, following the suggestion coming from the passive WM literature (e.g. McKenzie et al., 2003).

As previously mentioned, the influence of active VSWM and the specific role of its constituent parts (visual and spatial) on math development in very young children has not been studied in the literature; therefore, our expectations for this study were mainly based on passive WM literature.

Our results seem to follow the same trend observed in researches that have studied the influence of the spatial and visual passive memory components on early numerical cognition: both our work and passive memory studies show a greater involvement of the spatial component than the visual one on pre- and early math abilities. For example, McKenzie, Bull & Gray (2003) found that 6-year-olds mainly use a spatial rather than a visual-type strategy to solve arithmetic operations. These results suggest that the link between numbers and space is particularly important for the development of numerical cognition and suggest the idea that the child, prior to math formal teaching, develops a spatial code for numbers probably thanks to a mental number line, as suggested by the Dehaene studies (1992). Accordingly with the number line idea, our data seem to suggest that in very early stage of math learning the spatial WM contributes to the formation of the first, mostly spatial, numerical representations.

In summary, our results show a clear influence of spatial skills on younger children's number and magnitude knowledge, while later, at the end of the first year of primary school, the association between spatial memory and mathematical performance is not significant either in the correlation analysis nor in the Bayesian model. This trend confirms passive WM literature findings and suggests, as for example hypothesize by Holmes and Adams (2006), that initially the child relies mainly on spatial strategies (while in a later stage a verbal strategy emerges) and that spatial WM functions as a workspace to support the transition from early concrete informal knowledge to the nascent formal mathematical knowledge.

Our exploratory longitudinal study showed that both domain-specific knowledge and domain-general variables are implied in the first math learning process, but with different weights along time. Prerequisites confirm themselves as the ground of the first math learning at the end of first grade, while before (beginning first grade) domain-general ability such as spatial active WM is an important variable in the math learning processes.

Our findings, at a theoretical level, suggest that the involvement of general domain abilities in early-math performance can be better understood within the framework of active WM models in which the spatial and the visual components are kept separate, as for instance the WM Continuity Model (Cornoldi & Vecchi, 2003).

The relationships between active VSWM variables and first math learning highlighted in our study are important for a better understanding of the components involved in the first phases of math abilities development and a better identification of strengths and weaknesses in children with early math difficulties. These relationships are worth of further investigation in future longitudinal research that will involve a larger number of subjects and need to be extended over a longer time span to better explore the involvement of active visuospatial working memory abilities along the math learning process.

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