FEATURES OF SUCCESSFUL CALCULUS PROGRAMS AT FIVE DOCTORAL DEGREE GRANTING INSTITUTIONS

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We present findings from case study analyses at five exemplary calculus programs at US institutions that offer a doctoral degree in mathematics. Understanding the features that characterize exemplary calculus programs at doctoral degree granting institutions is particularly important because the vast majority of STEM graduates come from such institutions. Analysis of over 95 hours of interviews with faculty, administrators and students reveals seven different programmatic and structural features that are common across the five institutions. A community of practice and a social-academic integrations perspective are used to illuminate why and how these seven features contribute to successful calculus programs.

INTRODUCTION

Calculus is typically the first mathematics course for science, technology, engineering, and mathematics (STEM) majors in the United States. Indeed, each fall approximately 300,000 college or university students, most of them in their first post-secondary year, take a course in differential calculus (Blair, Kirkman, & Maxwell, 2012). In the US, Calculus I is a university-level course that typically covers limits, rules and applications of the derivative, the definite integral, and the fundamental theorem of calculus. Typically, over half of Calculus I students also took a calculus course in secondary school, which usually focuses on techniques of differentiation and integration. In comparison, university-level Calculus I is usually more rigorous in its treatment of concepts (including limits, graphical interpretations, definitions, etc.) and applications. Proofs are typically not part of Calculus I at either the secondary or post-secondary levels.

Internationally, first year university mathematics courses are consistently credited with preventing large numbers of students from pursuing a career in a STEM area (Steen 1988; Wake 2011). In the United States, STEM intending students typically enroll in calculus (though not necessarily Calculus I). In many European countries, STEM intending students instead typically enroll in abstract algebra or proof-based calculus (Wake 2011), as calculus is covered in secondary school.

Recent studies show that in the US and elsewhere students show less interested in a STEM majors paired with an increased need for STEM professionals in the workforce (Carnevale, Smith, & Melton 2011; Hurtado, Eagan, & Chang 2010; van Langen & Dekkers, 2005). Thus for those students that do choose a STEM major, there is a pressing need for them to be successful in first year mathematics courses so that they can continue in their chosen STEM major and ultimately meet the growing demand of

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the workplace for STEM graduates (PCAST, 2012; Wake 2011). However, student retention in STEM majors and the role of first year mathematics in student persistence is a major problem (Hutcheson, Pampaka, and Williams 2011; Pampaka, Williams, Hutcheson, Davis, and Wake 2012; Rasmussen and Ellis, 2013; Seymour and Hewitt 1997).

In order to better understand the terrain of calculus teaching and learning in the US, we are near completion of a five-year, large empirical study funded by the National Science Foundation and run under the auspices of the Mathematical Association of America. The goals of this project include: to improve our understanding of the demographics of students who enrol in calculus, to measure the impact of the various characteristics of calculus classes that are believed to influence student success, and to conduct explanatory case study analyses of exemplary programs to identify why and how these programs succeed. In this report, we present findings from our case study analyses at five exemplary calculus programs at institutions that offer a doctoral degree in mathematics. Understanding the features that characterize exemplary calculus programs at doctoral degree granting institutions is particularly important because these institutions produce the majority of STEM graduates.

The overall five-year project was conducted in two phases. In Phase 1 surveys were sent to a stratified random sample of students and their instructors at the beginning and the end of Calculus I. The surveys were restricted to “mainstream” calculus, meaning the calculus course designed to prepare students for the study of engineering or the mathematical or physical sciences. Surveys were designed to gain an overview of the various mainstream calculus programs nationwide, and to determine which institutions had more successful calculus programs. Success was defined by a combination of student variables: persistence in calculus as marked by stated intention to take Calculus II; affective changes, including enjoyment of math, confidence in mathematical ability, interest to continue studying math; and passing rates. In Phase 2 of the project, we conducted explanatory case studies at 18 different post secondary institutions, where the type of institution was determined by the highest degree offered in mathematics. In this report, we present findings from analyses of the five case studies at doctoral degree granting institutions.

THEORETICAL BACKGROUND

Analysis of our case study data is grounded in two complementary perspectives, the first of which draws on the *community of practice* perspective put forth by Wenger and colleagues (Lave &Wenger; 1991; Wenger 1998). A community of practice is a collective construct in which the joint enterprise of achieving particular goals evolves and is sustained within the social connections of that particular group. In achieving a particular joint enterprise, such as the teaching and learning of calculus, a community of practice point of view highlights the role of brokers and boundary objects. A broker is someone who has membership status in more than one community and is in a position to infuse some element of one practice into another. The act of doing so is referred to as brokering (Wenger, 1998). Boundary objects are material things that allow people to cross between different communities and facilitate progress on their joint enterprise.

The second set of ideas that we employ to make sense of our case study data draws on research in Higher Education that has extensively studied factors related to student retention at the post-secondary level, with a focus on the effects of student engagement and integration on persistence (e.g., Kuh et al., 2008; Tinto, 1975, 2004). According to Tinto's integration framework (1975), persistence occurs when students are socially and academically integrated in the institution. This integration occurs through a negotiation between the students' incoming social and academic norms and the norms of the department and broader institution. From this perspective, student persistence (a measure of success in calculus) is viewed as a function of the dynamic relationship between the student and other actors within the institutional environment, including the classroom environment.

METHOD

The survey results from Phase 1 provided information on which institutions are enabling students to be more successful in Calculus I (as compared to other institutions of the same type) per our measures of success. From this information, we were able to determine 18 institutions across all institution types that were more successful than others. Success was defined as a combination of increased student interest, enjoyment, and confidence in mathematics, persistence onto Calculus II, pass rates in Calculus I, and previously identified success on national measures of student understanding of calculus. Table 1 provides a brief description of the five selected doctoral granting institutions and why each was selected.

Institution		**Why Selected**
D1	• Large • Public	• Increased confidence, interest in math, and intention to take Calc II; Higher than expected Calc I pass rate
D2	• Small • Private • Technical	• Increased confidence, enjoyment of math, interest in math, and intention to take Calc II; Higher than expected Calc I pass rate
D3	• Small • Public • Technical	• Increased confidence, enjoyment of math, interest in math, and intention to take Calc II; Higher than expected Calc I pass rate
D4	• Large • Public	• Prev. identified implementation of best practices; high scores on national assessment of conceptual understanding of Calc
D5	• Large • Private	• Increased confidence, enjoyment of math, interest in math, and intention to take Calc II

Table 1: Description of case study sites.

Survey results, however well crafted and implemented, are limited in their ability to shed light on essential contextual aspects related to *why* and *how* institutions are producing students who are successful in calculus. The case studies were therefore designed to address this shortcoming by identifying and contextualizing the teaching practices, training practices, and institutional support practices that contribute to student success in Calculus I. As argued by Stake (1995) and Yin (2003), explanatory case studies are an appropriate methodology to study events (such as current practices in Calculus I) in situations in which the goal is to explain why or how, and for which there is little or no ability to control or manipulate relevant behaviors.

Four different case study teams (one per each type of institution—community college, bachelor, masters, and doctoral) conducted three-day site visits at the selected institutions. During the site visit each team, which consisted of 2-4 project team members, interviewed students, instructors, and administrators; observed classes; and collected exams, course materials, and homework. Common interview protocols for all 18 case studies were developed, piloted, and refined in order to facilitate comparison of calculus programs within and across institution type.

At the completion of each site visit the case study teams developed a reflective summary that captured much of what was learned about the calculus program, including key facts and features that were identified by both the case study team and the people interviewed as contributing to the success of the institution's calculus program. A more formal 3-4 page summary report was then developed by reviewing the reflective summary and transcripts and sent to the respective department of each institution as part of the member checking process (Stake, 1995).

At the five doctoral degree granting institutions, we conducted 92 interviews with instructors, administrators, and students for a total of more than 95 hours of audiorecordings. All interviews were fully transcribed and checked by a second person for accuracy and completeness. In order to manage this vast amount of qualitative data, a tagging scheme was developed to facilitate the location of relevant interview excerpts related to one of more of 30 different areas of interest. These areas of interest include such things as placement, technology, assignments and assessments, instructor characteristics, etc. Each interview was first chunked in terms of what we refer to as a "codeable unit." A codeable unit consists, more or less, of an interviewer question followed by a response. If a follow up question resulted in a new topic being discussed by the interviewee, then a new codeable unit was marked. Each codeable unit was then tagged with one or more of the 30+ codes. This data organization strategy then enable us to systematically identify all instances in which any interviewee addressed a particular topic area. Once these instances were located, then a more fine-grained grounded analysis proceeded. We used the facts and features documents to conduct initial cross case analysis to identify common features across the five doctoral degree granting institutions.

The set of 30+ codes was developed by representatives from each of the four different case study teams and consists of both a priori codes from the literature and codes for themes that emerged from the reflective summaries. The final set of 30+ codes underwent an extensive cyclical process in which representatives from each case study team coded the same transcripts, vetted their respective coding, which then led to refining, deleting, and adding new codes and operational definitions. Two different team members coded each transcript and the two coders resolved any discrepancies.

DISCUSSION

Cross case analysis of the five doctoral degree granting institutions led to the identification of seven features that contribute to the success of their calculus program. We first highlight what these seven features are followed by a discussion of the seven features in light of the communities of practice perspective and Tinto's academic and social integration perspective.

- *Coordination*. Calculus I (as well as PreCalculus and Calculus II) has a permanent course Coordinator. The Coordinator holds regular meetings where calculus instructors talk about course pacing and coverage, develop midterm and final exams, discuss teaching and student difficulties, etc. Exams and finals are common and in some cases the homework assignments are coordinated.
- *Attending to Local Data*. There was someone in the department who routinely collected and analyzed data in order to inform and assess program changes. Departments did this work themselves and did not rely on the university to do so. Data collected and analyzed included pass rates, grade distributions, persistence, placement accuracy, and success in Calculus II.
- *Graduate Teaching Assistant (GTA) Training*. The more successful calculus program had substantive and well thought out GTA training programs. These ranged from a weeklong training prior to the semester together with follow up work during the semester to a semester course taken prior to teaching. The course included a significant amount of mentoring, practice teaching, and observing classes. GTA's were mentored in the use of active learning strategies in their recitation sections. The standard model of GTA's solving homework problems at the board was not the norm. The more successful calculus programs were moving toward more interactive and student centered recitation sections.
- *Active Learning*. Calculus instructors were encouraged to use and experiment with active learning strategies. In some cases the department Chair sent out regular emails with links to articles or other information about teaching. One institution even had biweekly teaching seminars led by the math faculty or invited experts. Particular instructional approaches, however, were not prescribed or required for faculty at any of the institutions.
- *Rigorous Courses*. The more successful calculus programs tended to challenge students mathematically. They used textbooks and selected problems that required students to delve into concepts, work on

modeling-type problems, or even proof-type problems. Techniques and skills were still highly valued. In some cases these were assessed separately and a satisfactory score on this assessment was a requirement for passing the course.

- *Learning Centers*. Students were provided with out of class resources. Almost every institution had a well-run and well-utilized tutoring center. In some cases this was a calculus only tutoring center and in other cases the tutoring center served linear algebra and differential equations. Tutoring labs had a director and tutors received training.
- *Placement*. Programs tended to have more than one way to determine student readiness for calculus. This included: placement exams (which were monitored to see if they were doing the job intended), gateway tests two weeks into the semester and different calculus format (e.g., more time) for students with lower algebra skills.

The fact that all five of the more successful calculus programs at doctoral degree granting institutions had someone whose official job included coordinating the different calculus sections is noteworthy. This role of coordinator was not something that rotated among faculty, such as committee assignments do, but rather was a designated and valued permanent position. The existence of this position is, however, only part of the story. An equally important part of the story is the role that calculus coordinator, among others, played in creating and sustaining a community of practice around the joint enterprise of teaching and learning of calculus. In other words, calculus was not seen as being under the purview of one person, such as the coordinator, but rather calculus was viewed as community property.

Nonetheless, the calculus coordinator played a unique role within their community of practice. In particular, the calculus coordinator functioned as a broker between the more central members in the department that typically teach calculus and the many newcomers. At doctoral institutions, these newcomers to the calculus joint enterprise include visiting research or teaching faculty, post docs, lecturers, and graduate teaching assistants (GTAs). The regular meetings that the calculus coordinator convened provided occasions for newcomers to be enculturated into the norms and practices related to calculus. Long-term members of the community also used these meetings to reflect on their own and other's practices. This reflection contributed to the sense of calculus as community property, as well as to the negotiation of communal practices.

We identified a number of boundary objects that helped to facilitate this enculturation, including historical records of passing rates, current grade and persistence data, student evaluations, various training manuals (especially for GTAs and visiting faculty) and the development of common assignments and assessments. Other brokers in the joint enterprise of teaching and learning calculus included, for some of the five doctoral institutions, the graduate teaching assistant trainers and leaders, department chair and the person whose responsibility it was to collect and disseminate to the department local data concern student pass rates and persistence and/or the correlation between these measures of success and the placement process. We conjecture that their attention to local data and continual improvement efforts contributed to a climate in which those involved with calculus teaching were always striving for improvement. Indeed, it was striking to us that none of the five case study institutions considered themselves to be particularly successful in calculus. That is, none of the five institutions in our case studies felt that they had everything just right.

A community of practice perspective helps to illuminate the how and why particular calculus programs are successful from a point of view that highlights faculty and administration. In our view, Tinto's academic and social integration perspective sheds equally important insight into how and why calculus programs are successful from a student point of view. In particular, almost without exception the students we talked with at the five doctoral institutions noted that they felt their calculus course was academically engaging and challenging (despite the fact that the vast majority had taken calculus in high school) but that there were a number of resources available to them to help them be successful. These resources included well-developed math help centers where students felt they received the help they needed and availability of instructor's and GTAs office hours. Other factors that contributed to students' academic and social integration included student centered instruction, common space in the math department where students could gather to work on homework, dorms that provided them with opportunities to interact with like minded fellow students, and in some places a cohort system or strong student culture that provided cohesion between students.

In summary, our ongoing analysis of the five successful calculus programs at doctoral institutions is highlighting a number of structural and programmatic features that other institutions would likely be interested in adapting. The ongoing theoretical analysis points to the importance of how these structural and programmatic features come together for faculty so that calculus is seen as community property and for the academic and social integration so critical for students' continued interest, enjoyment, and persistence in calculus. Our analysis that combines a community of practice perspective with the seminal work of Tinto on academic and social integration also sets the stage for the development of a more comprehensive model of successful college calculus programs.

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