DEMOCRATIZING MATHEMATICAL CREATIVITY THROUGH KOESTLER'S BISOCIATION THEORY

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The presentation challenges a frequently-expressed assertion: "There is no single, authoritative perspective or definition of creativity [in mathematics]" Kattou et al. (2011). It points to difficulties resulting from using accepted definitions in educational research (Wallas, 1926; Thorance, 1975). In this paper, the authors express concern about joining research on creativity with the research into giftedness and suggest the need for democratizing that approach. To that end, they introduce an alternative definition of creativity - bisociation, that is "a creative leap of insight" or an Aha moment (Koestler, 1964). Prabhu and Czarnocha argue for adopting Koestler's bisociation as "the authoritative perspective or definition of creativity."

THE STATE OF THE FIELD

Mathematical creativity may be the only gate through which to reactivate the interest and the value of mathematics among contemporary youth whose engagement in the field is hampered by disempowering habits expressed as "I can't do it," "I am not good in math," "thinking tires me" (Czarnocha et al., 2011). This teaching-research observation is in agreement with the research community: Lamon (2003) emphasizes the need for creative critical thinking and Mann (2005) asks for the explicit introduction of creativity as the component of learning in general. However, the conceptualization of creative learning varies due to the diversity of the proposed definitions of creativity. (Kattou et al., 2011) There is no single, authoritative perspective or definition of creativity (Mann, 2006; Sriraman, 2005; Leikin, 2011, Kattou et al., 2011) leaving practitioners without a clear and supportive viewpoint. However, a clear understanding of the cognitive and affective conditions for the creative act is important at present to be useful as the jumpstart for bridging the Achievement Gap in the US or start the numerical literacy campaign among the Tamilian Dalits of India (Prabhu, Czarnocha, 2008). There are two recently published excellent collections of papers, dealing with creativity in mathematics education, (Sriraman and Lee, 2011; Leikin et al., 2009). Both collections join the issue of creativity with the education of gifted students, indicating that the interest in creativity of all learners of mathematics *is not the central focus of the field*. There can be several reasons for so restrictive a focus on creativity: it could be due to the efforts of globalization so that "the winds are changing" (Sriraman and Lee, p. 2) or it could be that our understanding of the creative process is not sufficiently sharp to allow for the

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2014. In Nicol, C., Oesterle, S., Liljedahl, P., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 5, pp. 1-8. Vancouver, Canada: PME.

effective focus of research on the mathematical creativity by all students including, of course, the gifted. This observation raises the issue of democratization of creativity in mathematics research and teaching.

There are two definitions of the creative process on which many of the investigations are based. Wallas (1926) puts forth Gestalt-based definition of the process as consisting of preparation, incubation, illumination and verification. More behavioural in approach is Torrance's 1975 definition. It involves fluency, flexibility, novelty and elaboration. Leikin (2007) and Silver (1996) contracted it to fluency, flexibility and originality making the definition one of the bases for understanding creativity in mathematics education. Neither approach, however, addresses itself directly to the act of creativity nor to the structure of the "Aha moment" as the commonly recognized site of creativity itself (Sriraman, 2005). Fortunately, the theory developed by Arthur Koestler in his 1964 work, *Act of Creation*, does exactly that. It builds our understanding of creativity on the basis of a thorough inquiry into the Aha moment, which Koestler calls a bisociative leap of insight. The development of a comprehensive Theory of an Aha Moment is particularly urgent at present from the theoretical research viewpoint given the empirical work of Campbell *et al.*, (2012), who are investigating the Anatomy of an Aha Moment and the work our colleagues from computer creativity, a subdomain of Informatics, who are already employing bisociation for their data mining processes (Dubitsky et al., 2012).

KOESTLER'S PRINCIPLES OF CREATIVITY.

Arthur Koestler (1964) defines "bisociation" as "the spontaneous flash of insight, which …connects the previously unconnected frames of reference and makes us experience reality at several planes at once… " (p. 45) – an Aha moment. Koestler clarifies the meaning of "insight", by invoking Thorpe's 1956 definition of insight: "an immediate perception of relations". Koestler also refers to Koffka's 1935 understanding of insight as the "interconnection based on properties of these things in themselves." In the words of Koestler:

> The pattern… is, *the perceiving into situation or Idea, L, in two self-consistent but habitually incompatible frames of reference, M_1 and M_2*. The event L, in which the two intersect, is made to vibrate simultaneously on two different wavelengths, as it were. While this unusual situation lasts, L is not merely linked to one associative context, but *bisociated* with two. (p. 35)

Consequently, the creative leap or "an immediate perception of relations" can take place only if we are participating in at least two different frames, matrices of discourse. Examples of such simultaneous two frames of thought abound. One of them, present during the instruction of elementary algebra, is the theory of the number line based on (1) the framework of the number theory and (2) the framework of the geometrical line, memorialized through the creativity of the Dedekind axiom of one-to-one correspondence between real numbers and points on the line. Another one is the teaching-research methodology, which is the integration of the teaching framework

with the framework of research, a highly creative and effective method of teaching and doing research on teaching and learning at the same time (TR/NYCity in B.Czarnocha et al. 2014). Koestler offers examples of bisociation in the discovery of electromagnetism out of two separate investigations, that of electricity and that of magnetism; he mentions wave-particle duality, of course, as well, and many others.

The depth of Koestler's approach to creativity doesn't rest here. Within his conceptual framework, "creativity is the defeat of habit by originality". That means that bisociation not only is the cognitive reorganization of the concept by "an immediate perception of relations", but also it can be an affective catalyzer of the transformation of habit into originality (Figure1).

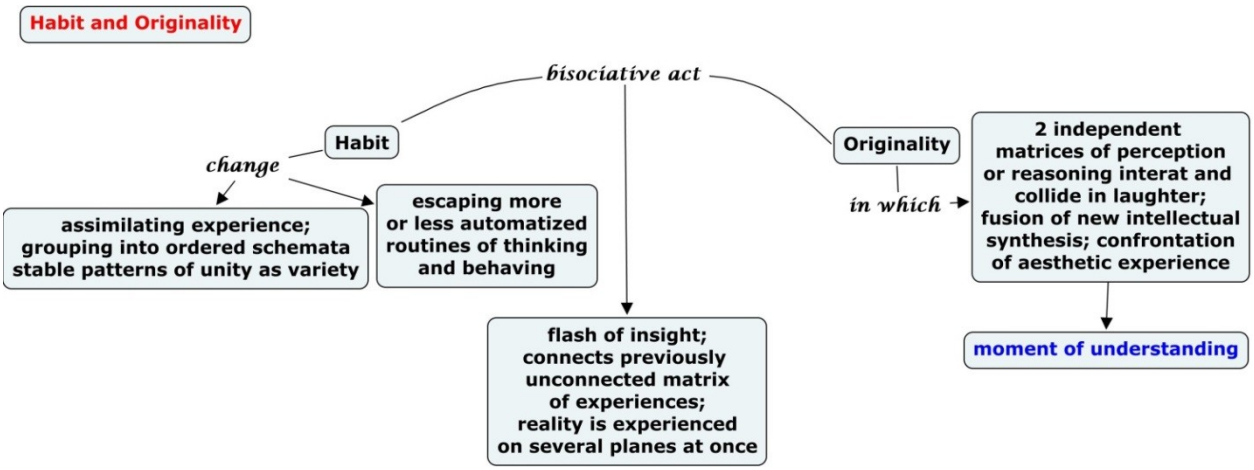

Figure 1: Habit to originality through the "flash of insight" (Prabhu, 2014)

The presence of this cognitive/affective duality of creativity, of the Aha moment, can provide the intrinsic motivation to bridge the Achievement Gap in US and in other centres of educational inequality, according to Prabhu, (2014). In fact, the first teaching experiments introducing the principles of the Act of Creation into classrooms were conducted "to address the emotional climate of learners" in the remedial/developmental classes mathematics classroom. The transformative relationship between habit and originality formulated at the very basis of the bisociation theory confirms Liljedahl's 2004 meta-findings that the "…Aha experience has a helpful and strongly transformative effect on a student's beliefs and attitudes towards mathematics…" (p. 213). However, in stressing that "the aha experience is primarily an affective experience", he is neglecting its equally significant cognitive component. (Liljedahl, 2009). Quoting Poincare, Koestler brings out explicitly the cognitive element of the Aha moment: "Ideas rose in crowds; I felt them collide until pairs interlocked, so to speak, making a stable combination" (Poincare qtd in Koestler,p. 115). Note the process of grasping stable relations of pairs of concepts in accordance with Koestler's definition of bisociation.

THE IMPACT OF BISOCIATION UPON UNDERSTANDING OF CREATIVITY

Koestler's 700+ page *Act of Creation* argues convincingly that bisociation is the common structure across the domains of Humor, Scientific Discovery and Art Sublimation making it the principle underlying any creative act of invention. supporting Hadamard's view that

> Between the work of the student who tries to solve a problem in geometry or algebra and a work of invention, one can say that there is only the difference of degree, the difference of a level, both works being of similar nature (1945, p. 104).

Thus the standard division of creativity into absolute and relative is misleading because it seems to suggest an essential difference between the two. Similarly, in each intellectual domain the tools and the language through which creativity is expressed vary, but the process of insight through bisociation is exactly the same. Hence, the conventional distinction between general creativity and domain specific creativity doesn't hold water.

Situating the definition of creativity in the illumination stage of the Wallas definition itself provides a new perspective upon questions raised in recent discussions on the subject. In particular, Sriraman et al., (2011) assertion can be qualified:

> …when a person decides or thinks about reforming a network of concepts to improve it even for pedagogical reasons though new mathematics is not produced the person is engaged in a creative mathematical activity. (p. 121)

Whether the process described above is or is not a creative mathematical activity can be decided on the basis of Koestler's distinction between *progress of understanding* – the acquisition of new insights, and *exercise of understanding* – the explanation of particular events (p.619). If for example, I decide to design a developmental course of arithmetic/algebra based on my knowledge of the relationship between arithmetic and algebra (generalization and particularization), which involves the redesign of the curriculum, that is its "the network of concepts", I am engaged in *the exercise of understanding of mathematics*, distinctly different from creative *progress of understanding in mathematics*. It may however, depending on the initial knowledge of the teacher, be a creative activity in pedagogical meta-mathematics, that is understanding mathematics from the teaching point of view – the content of professional craft knowledge.

The bisociation theory, in which on the one hand creativity is "an immediate perception of relation(s)", and on the other it is the affective catalyzer of the transformation of habit into originality, interacts well with MST methodology. (Leikin, 2009). It predicts the absence of the difference between absolute and relative creativity observed by authors of the experiment. Moreover, the observed fall in the expression of originality reported by Leikin, (2009) as well as the correlation between creativity and originality is natural in the context of the relationship between habit, creativity and originality – a point made explicit in the often quoted Koestler's assertion "*Creativity*

is the Defeat of the Habit by Originality". The authors point correctly to the fluency and flexibility as the carriers of the habit which diminished the originality of student subjects: "…when students become more fluent they have less chance to be original". This apparently complementary relationship between fluency and creativity *dictates an utmost care* in conducting the research into creativity with the help of the definition which includes fluency, because it may result in undesired lowering of creativity. And that we don't want, especially in the "underserved communities". This observation brings in the old question to the fore: What is the optimal composition of fluency and creativity in the preparation of teachers of mathematics, as well as in classroom teaching?

CLASSROOM IMPLEMENTATION OF THE THEORY – V.PRABHU (2014)

Design of Triptych based Assignments

The Act of Creation defines bisociation that is "the creative leap of insight, which connects previously unconnected frames of reference and makes us experience reality at several planes at once." How to facilitate this process? Koestler offers a suggestion in the form of a triptych, which consists of three panels…indicating three domains of creativity which shade into each other without sharp boundaries: Humor, Discovery and Art.

Each triptych stands for a pattern of creative activity which is represented on them; for instance

Cosmic comparison ⟷ objective analogy ⟷ poetic analogy

The first is intended to make us laugh, the second to make us understand, the third – to marvel. The creative process to be initiated in our classes of developmental and introductory mathematics needs to address the emotional climate of learners, and here is where the first panel of the triptych comes into play, Humor.

Having found humour and the bearings of the concept in question, the connection within it have to be explored further to "discover" the concept in detail, and finally to take the discovery to a form of sublimation by Art.

Triptych assignments facilitate student awareness of connections between relevant concepts and thus they facilitate understanding. However, what maybe even more important, the accompanying discussions help to break the "cannot do" habit and transform it into original creativity. There was a significant improvement (measured by the instructor's intuitive assessment and tests results) in the experimental statistics class.

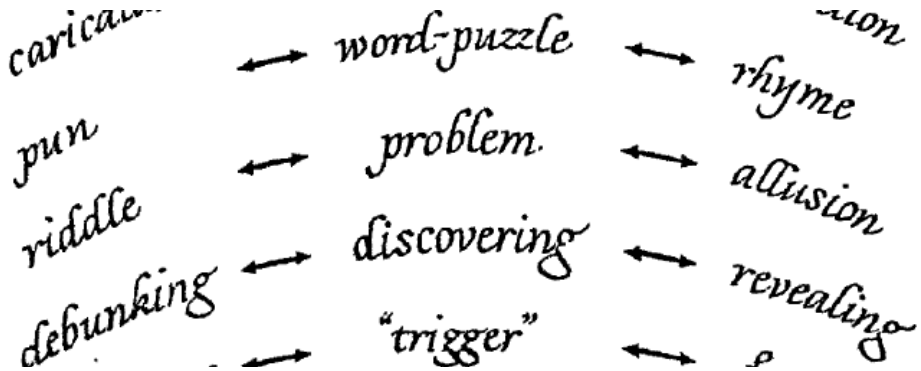

Figure: 2 Koestler's triptychs

Examples of triptych assignment used in the class of introductory statistics

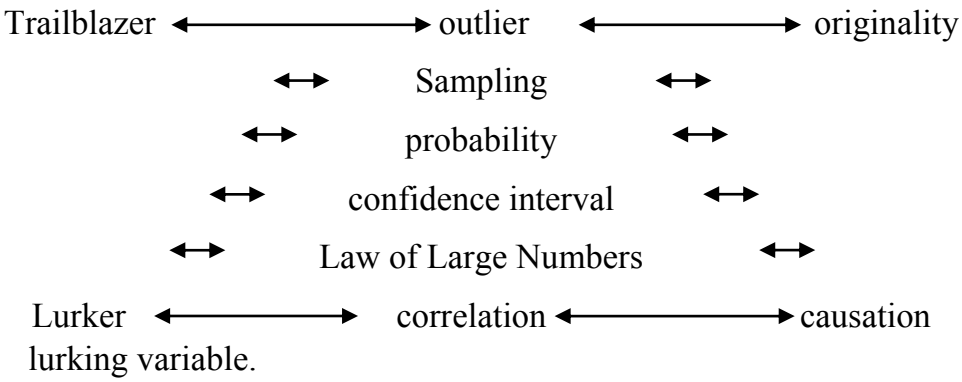

Figure 3: Triptych assignment

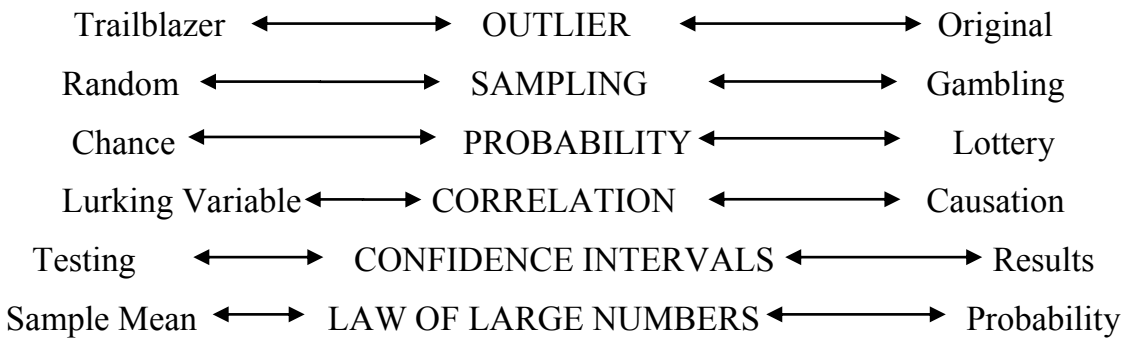

Figure 4: Triptych student response

Use of triptychs in the mathematics class brings back the puzzle inherent in mathematics.

What is the connection between stated concepts? What could be the concepts connected to the given concepts? – A forum for meaning making is created in connecting the prior knowledge, with synthesized, reasoned exploration. The question "how" is answered by the question "why" through the use of mathematical triptychs."

CONCLUSION: A PROPOSAL

This short review of our efforts to understand creativity indicates serious weaknesses in the field, which undermine the educational effectiveness of creativity.

In light of widely spread conviction that there is no single, authoritative perspective or definition of creativity as expressed by Mann, Sriraman, Leikin, and Kattout et al., we are *proposing bisociation as the authoritative definition of creativity in the field of mathematics*. Its relationship to two basic definitions (1) coming from Gestalt approach as well as (2) from a more behavioristic school depending on fluency, is clear. In the first case it focuses on the stage of illumination, the actual stage of creativity; in the second case, it suggest that fluency, which can correlate well with creativity, can undermine it at the same time. Clearly fluency does not measure nor defines creativity but instead some composition of creativity with a habit. Bisociation, on the other hand, is the "pure" act of creation in the making. Its disassociation from fluency is very important for the facilitation of mathematical creativity in the remedial and elementary mathematics classrooms of community colleges, where it is exactly fluency that's missing. It is the definition of creativity for everyone, because "everyone" knows Aha moment. Koestler flatly asserts that "minor subjective bisociation processes…are the vehicle of untutored learning" (p. 658). Taking bisociation as the definition of creativity ensures democratization in mathematics education. It's interesting to note that our colleagues in computational creativity have discovered recently Koestler's bisociation for the creative information exploration (Dubitsky et al., 2012). The simplicity of bisociative facilitation through the discovery & creative problem solving in the context of a triptych approaches provides us with ready pedagogical techniques of teaching and researching it. It would be very useful to understand better the process of scaffolding the bisociation; this understanding can come only if bisociation is observed en vivo, that is in the classroom, more in the context of qualitative research approach at present than quantitative.

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