NUMBER'S SUBTLE TOUCH: EXPANDING FINGER GNOSIS IN THE ERA OF MULTI-TOUCH TECHNOLOGIES

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*In this paper, we explore a richer sense of finger gnosis with respect to three- and four-year-olds' interactions with a novel iPad application (*TouchCounts*), focusing on their responses to an "inverse subitising" task. The direct and tactile nature of their engagement with* TouchCounts *leads to a striking shift from index finger incrementation to deployment of several fingers all-at-once (in a cardinal touch gesture) to achieve a given target number that is then spoken by the iPad. This form of finger gnosis differs from the more ordinally based differentiation of fingers that is discussed in the psychology literature.*

INTRODUCTION

In nascent numeration with very young children, there is a telling ambiguity concerning the status and nature of fingers in relation to counting. This dual role is well captured by the English expressions 'using fingers to count *with*' and 'using fingers to count *on*'. Fingers can serve as both a physical extension of what Rotman (1987, p. 27) calls the 'one-who-counts' (counting *with* my fingers) as well as the thing-to-be-counted (counting *on* my fingers): fingers are thus simultaneously subject and object, both of the person and of the world. In inhabiting this dual status (being both me and not-me), fingers provide echoes of the analyst Donald Winnicott's notion of 'transitional object': "an intermediate area of experiencing to which inner reality and external life both contribute" (1971, p. 2). (See also Maher, 1994.) When a four-year-old asserts, "Don't do it! I'm just fingering it out!" (Phillips, 1996, p. 82), in that slippage from 'figure' to 'finger' there is a literal as well as metaphorical truth being expressed. In this paper, we explore aspects of fingers' transitional object status with regard to counting by means of three- and four-year-olds working on a novel application, *TouchCounts* (Sinclair & Jackiw, 2011), which makes central use of the iPad's ability to respond to multiple tactile inputs synchronously.

FINGER GNOSIS AND THE DEVELOPMENT OF NUMBER SENSE

Within the field of developmental psychology, subitising (which connects to the mathematical task we report on in this paper) refers to the ability to enumerate the items in a set quickly, without counting. This notion has been claimed to be a core component upon which all other mathematical abilities are built (see Butterworth, 1999; Penner-Wilger *et al*., 2007). For Butterworth, subitising provides initial access to cardinality, allowing children to "categorise the world in terms of numerosities – the number of things in a collection" (p. 6). Such access – and to number sense more generally – appears to be strongly dependent on 'finger gnosis' (literally "finger

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knowledge"), defined as the ability to differentiate one's own fingers without any visual clues when they are touched. Gracia-Bafalluy and Noël (2008) show that improving children's finger gnosis by training them on finger differentiation tasks increases their numerical performance.

Our research explores the nature of finger gnosis as it relates to children's interaction with *TouchCounts*, which involves the use of various finger gestures (tapping, swiping, pinching, flicking) to produce numbered objects and spoken words. Multi-touch enables direct mediation, allowing children to produce and transform objects with fingers and gestures, instead of acting through a keyboard or mouse. This added sensory input seems to play no role in developmental psychology studies, but may provide a powerful accompaniment to the visual and oral forms of communication that are currently privileged in that research. The word *gesture* has been used by touchscreen interface designers to describe specific configurations and actions of the finger(s) on the screen (swiping, tapping, etc.). These kinds of gesture are different from those typically discussed in the mathematics education literature in two ways: they involve contact with a screen and they *perform an action*. Similar to the performative speech act (Austin, 1962; Searle, 1969), which refers to language that performs on the world, we use the term "performative gesture act" to describe these tangible, input gestures.

DESIGN OF *TOUCHCOUNTS*

Currently, there are two sub-applications in *TouchCounts*, one for Counting (1, 2, 3, …) and the other for Adding (1+2+3+…). Here, we focus exclusively on the former (see Sinclair & Metzuyanim, 2014), for a more complete description). In this world, a user taps her fingers on the screen to summon numbered objects (yellow discs). The first tap produces a disc containing the numeral "1". Subsequent taps produce sequentially numbered discs. As each tap summons a new numbered disc, *TouchCounts* audibly speaks the English word for its number ("one", "two", …). Fingers can be placed on the screen one at a time or simultaneously. With five successive taps, for instance, five discs (numbered 1 to 5) appear sequentially on the screen, which are counted aloud one by one (see Figure 1a). However, if the user places two fingers on the screen simultaneously, two consecutively numbered discs appear at the same time (Figure 1b), but only the higher-numbered one is explicitly named ("two," if these are the first two taps). The entire 'world' can be reset, to clear all numbered discs and return the 'count' of the next summoned disc to one.

The number of taps (made sequentially or simultaneously) is also the number of discs on the screen, which can reinforce the cardinality principle, since the last number "counted" (spoken aloud by *TouchCounts*) is exactly "how many" numbered discs there are. Even after children have counted a set of discs (up to five, say), when they are asked "how many" objects are in a given set, will often count the objects again (Baroody & Wilkins, 1999). The "how many" question seems to provoke a routine of sequential counting. In *TouchCounts*, the child is engaged in a somewhat different

routine – rather than counting a *given* set, she is actively *producing* that set with her finger(s) (perhaps to an instructor-given total) and elements of that set count themselves (both aurally and symbolically) as they are summoned into existence.

The Counting world directly supports two of the five aspects of counting identified by Gelman and Meck (1983): (1) when counting, every object gets counted once and only once (one-to-one correspondence principle); (2) the number words should be provided in a constant order. Also, the last number said by *TouchCounts* is always the number of items on the screen, it reflects a third of Gelman and Meck's 'aspects'.

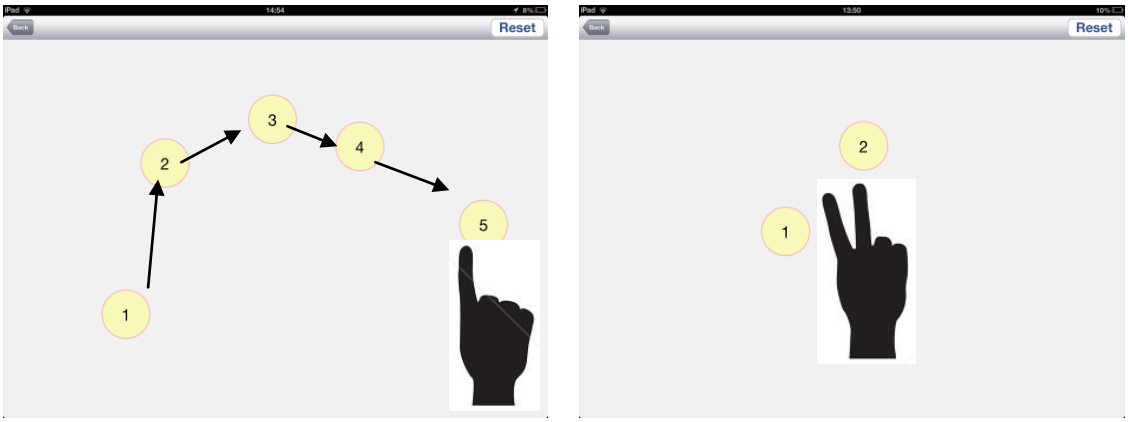

Figure 1(a): Five sequential taps – "one, two, three, four, five" is said;
1(b): A simultaneous two-finger tap – "two" is said.

THEORETICAL FRAMING

Broadly speaking, we take a non-dualistic perspective on thinking and learning. More specifically, we adopt an inclusive materialist approach in which the tool (in this case, *TouchCounts*) is seen as participating in an agential relationship with the user so that the tool and the user mutually constitute each other through interaction (de Freitas & Sinclair, 2013). In so far as the tool 'speaks' (and on occasion moves things) in interaction with the user, it takes on an animate role in the interaction, enabling but also preventing activity. We attend especially to the broad and varied ways of intervening involved in mathematical activity – including, bodily movements gestures and tone of voice. This is in accord with principles of embodied cognition, which posit that cognitive functions are "directly and indirectly related to a large range of sensorimotor functions expressed through the organism's movement, tactility, sound reception and production, perception, etc." (Radford, 2012, p. 4537). However, inclusive materialism insists on dissolving the rigid boundary that usually defines the human body and its sense organs.

An inclusive materialist approach also extends to mathematical concepts, not just to the concrete tools and bodies in the environment. We therefore focus on how the assemblage of finger/tool/number changes over time; how new materialities become part of the activity and affect its progression. The notion of finger gnosis thus strikes us as very interesting since it relates directly to embodiment, while also suggesting a

distribution of senses foreshadowed in the introduction, in which the fingers comprise a core presence in the assemblage of counting.

METHOD OF RESEARCH

The study took place over three months in a day care located close to a North American University, which provides a play-based environment in which children are free to choose from a range of activities, each of which offers different sets of materials. In order to fit into the environment, the iPad was placed on the carpet in the corner of the room and children were free to come or leave as they wished. At the beginning of the session we analyse here, many children crowded around the iPad, jostling to get a chance to play, but after about twenty minutes, a small group of four children formed and stayed for the remaining twenty minutes. The analysis begins at the point the group formed, when it was possible to record the interactions and actions of the children. The four children in the group were all three or four years old.

We focus on a five-minute interval because it was the beginning of the group's work together, and there was a clear change in the way they use *TouchCounts* to summon numbers. We offered an 'inverse subitising' task where children were asked to produce a target number by using two or more fingers all-at-once (rather than sequentially). In subitising tasks, students must determine quickly the number of objects in an array, which they then either say or type onto a keyboard. Here, instead of making an spoken or alphanumeric action based on a visual prompt, the children are to make an action based on an oral prompt, a gesture act. Unlike traditional finger counting, which is both ordinal and fixed, such an all-at-once gesture act is neither.

INVERSE SUBITISING

The interviewer (first author, henceforth "I") asked a pair of children (Owen and Ramona) to try to "get four together". They each tapped the screen with one finger once, making *TouchCounts* say "one", "two", then again almost at the same time, thus producing "four" (see Figure 2a). When I asked Katherine and Christine to make four together, they each tapped with one finger, stopping after *TouchCounts* said "eight". They tried again, this time stopping after *TouchCounts* said "sixteen". Thinking that perhaps the girls were having difficulty coordinating their work, I asked Katherine to "get to four by yourself". She placed all five fingers on the screen, which said "five" (see Figure 2b). Prompted by her use of more than one finger, I asked her then to "use lots of fingers to get to four". She placed her whole palm on the screen. When it was her turn, Christine did the same thing.

I then moved the iPad in front of Owen.

I: You try to use lots of fingers to get to four.

Owen: *Initially he stretches out his whole right hand, then curls and ripples from pinkie to index finger, then tucks his thumb under, and then straightens the remaining and touches the screen all-at-once (see Figure 2c).*

iPad: Four

Ramona: Ah. (*Very high pitch*) He did it.

I: He did it.

Figure 2: (a) Ramona and Owen working together to get 4; (b) Katherine placing her five fingers on the screen; (c) Owen placing four fingers on the screen.

I offered the iPad to Ramona. She raised her hand in the air and lifted her fingers one by one, then placed four of them on the screen. *TouchCounts* said "six". Thinking that she had inadvertently tapped other parts of her hand on the screen, I rolled up her sleeve and let her try again. This time she tapped sequentially four times on the screen and *TouchCounts* said "one, two, three, four". When asked to do it with lots of fingers, Ramona placed her whole palm on the screen, producing "twelve". She screamed, rolled over and, when asked if that was what she wanted, she exclaimed "no!" Christine was next slapped the iPad with her whole hand, also producing more than four. Christine tried again, as did Katherine, who imitated Christine's gesture.

I then gave the iPad to Owen and asked him to "use lots of fingers to get to two". He immediately put out his hand with his index and middle fingers outstretched and placed them on the screen. When it was Christine's turn, she also extended two fingers, but when she touched the screen, *TouchCounts* said "three" (she had inadvertently touched the screen with another part of her hand). She tried three more times, always holding out her two fingers, but each time *TouchCounts* said a number greater than two. Katherine decided to press Reset and to tap sequentially twice. Then Christine placed two fingers on the screen and *TouchCounts* said "two". I moved the iPad to Ramona, who lifted her left hand deliberately, extending one finger at a time and placed two fingers on the screen to get "two". I then asked Owen to "do three with lots of fingers", which he did successfully, as did Christine. Katherine then successfully placed three fingers on the iPad, as did Ramona. I congratulated the children for all managing to do "three with lots of fingers" and asked them to "do four". Owen succeeded quickly, as did Christine and Katherine. Ramona stretched out four fingers, but placed her palm on the screen so that *TouchCounts* said "five". This happened twice, and then she decided to tap successively four times.

EXPANDING THE SENSE(S) OF FINGER GNOSIS

At the very beginning, Ramona and Owen used their fingers to summon numbered discs and hear the count up to four while Christine and Katherine used them simply to

summon numbered discs, apparently without attending to the number of discs on the screen or the number words spoken aloud by *TouchCounts*. Ramona and Owen managed to get to four by using the oral/visual feedback from *TouchCounts* to stop tapping once they heard/saw "four/4", but Katherine and Christine tapped, without listening/looking for four/4. Perhaps the excitement of tapping outweighed the interest in performing the task. However, even on her own, Katherine did not use her fingers to get to four/4. Despite having tapped with one finger previously, both Katherine and Christine tried to get to four/4 by slapping the iPad with their hands, all the while giggling. For them, the request to "make four" seems to have been interpreted as a request to make some big number. While all the children were using their fingers to conjure numbered objects and number words, only Ramona and Owen's fingers were being used to produce particular ones. For Katherine and Christine, fingers were not yet counting tools (either counting *with* or counting *on*).

Owen's deliberate gesture introduced a new element to the assemblage; all the children saw his hand, which became joined up to the vocalised four of *TouchCounts*. The children heard that the gesture produced four all-at-once, without passing through other numbers. When it was her turn, Ramona stretched her four fingers out one by one, instead of simultaneously as Owen had done. But the fingers touched the screen simultaneously, perhaps mimicking Owen's gesture. She had difficulty getting *TouchCounts* to say "four" though, and decided to revert to sequential tapping. Now the verbal sequence "one, two, three, four" had joined the assemblage. It is important to notice though, that each of Ramona's four fingers touched the screen in order to produce the ordinal sequence 1, 2, 3, 4 so that she was not just counting up to four *on* her fingers, but producing one, two, three, four *with* her fingers – each finger feeling the screen and producing a distinct number word and numbered circle. While it may be argued that Ramona's lack of manual dexterity got in the way of an Owen-like gesture act, we hypothesise that she may not be feeling the numerosity of her touch. She can present four on her hand, by extending her four fingers, so that her fingers can show four, but not yet use it to make four/4.

Despite seeing/hearing the Ramona-fingers-screen intra-action, both Katherine and Christine stretched out their hands, but slapped the screens. It was as if they were mimicking Owen's gesture, but without paying attention to the number of outstretched fingers or to the way in which those fingers touched the screen. Ramona and Owen responded by screaming and resetting, respectively, obviously aware that the action was incorrect and that the girls needed to try again.

When a new round of tasks was initiated, one in which the children were asked to use many fingers to make two/2, three/3 and then four/4, Christine and Katherine began to use their fingers very differently. Whereas a few minutes ago, when asked to "do four", they had slapped the screen almost haphazardly, by the end they both held up and placed four fingers on the screen. The speed at which Christine first held out her two fingers suggested some kind of subitising. She was confident enough about using these two fingers that she was willing to try several times to get *TouchCounts* to do as she

wished. Katherine's impatience, and decision to proceed with sequential taps, may have stemmed from a strong ordinal finger sense – with the index finger as the main tool for presenting number (counting *on*). However, the speed and dexterity with which each child made three/3 shows the momentum of the gesture act, with the fingers now used both to present and produce a given quantity. When it came to making four/4 all-at-once, only Ramona extended her fingers one at a time. This reverting from using all three fingers at once to using them sequentially suggests that she was not mimicking the other children's gesture; she knew, however, that counting up to four on her fingers would produce four discs, as well as the sound "four".

DISCUSSION

By the end of this five-minute time span, all the children could use their four fingers all-at-once on the screen, to make *TouchCounts* say "four". Owen was able to do this early on, but not the other three. Significantly, they did this by extending their fingers all at once as well, as a kind of gesture, instead of lifting them up one at a time (as occurred several times earlier in the episode). In this sense, there was a developing finger gnosis about fourness, in that four fingers were being touched to/by the screen. This form of finger gnosis differs from the more ordinally based differentiation of fingers that is discussed in the psychology literature, but seems mathematically significant as a form of 'knowing about and through one's fingers'. Unlike conventional subitising tasks, which rarely extend beyond five, 'inverse subitising' with *TouchCounts* has no upper limit, in the sense that a child may use all her fingers to make ten/10, but can also work collaboratively with other children to make even larger numbers. Our data (not presented here) show that, for numbers between five and ten, children quickly shift from counting on their fingers until they reach the target to a subitised gesture act producing the desired number of fingers all-at-once.

Returning to the notion of fingers as both subject and object for the one-who-counts, each child showed a slightly different relationship between them. Owen was the first to create a fourfold gesture by means first of a brief counting *on* his fingers before counting *with* them, as if a single touch. His subsequent gesture acts reflect this plural resource. Christine and Katherine's work with all-at-once gesture acts is quite distinct, with no independent finger movement (unfurling one by one). They seem to only count *with*. In contrast, Ramona moves back and forth between the newer all-at-once gesture acts and the more familiar single fingering. She seems aware they are different means to reach the same end.

This short episode shows learning occurring in that three of the children were able to do something they could not do at the beginning. We claim that this learning cannot be separated from the materialities and interactions of the situation. *TouchCounts* was centrally involved in the learning. However, what particular role did it play in supporting this learning? Based on the above analysis, three features seem relevant: (1) the children could summon numbers one by one or all-at-once, without having to be previously familiar with the numbers they were creating; (2) the spoken number words

could be connected to the tapping, providing feedback that encouraged self-correction, without external prompting; (3) the emotional engagement of the children – the screams, giggles, smiles, as well as the concentration, confusion and cooperation – cannot be overlooked. Further analysis of the affective flow in this episode would provide even greater insight into the assemblages' dynamic nature.

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