

MATHEMATICAL ACTIVITY IN EARLY CHILDHOOD: IS IT SO SIMPLE?

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The aim of this paper is to build up an argument about the importance of a mathematical analysis of young children's activity in relevant for the age educational tasks. Most of current approaches (psychological, social, and pedagogical) are limited to the study of the development of children's thinking, paying less attention to the involved mathematical concepts. In the paper these approaches are briefly presented and an attempt is then made to analyse the mathematical activity within and beyond them. Finally, implications and some examples from a program of early mathematics aimed at developing authentic mathematical activity is provided.

INTRODUCTION

Research in early childhood mathematics education highlights its importance; young children, working in appropriate educational and pedagogical environments, show interest and have the potential to develop remarkable mathematical ideas (e.g., Mulligan, & Mitchelmore, 2013; van Oers, 2013; English, 2012; Gisburg et al., 2008; van den Heuvel-Panhuizen et al., 2008; Perry et al., 2008). Most countries provide considerable early mathematics education programs to support children in developing basic mathematical concepts, but also to encourage practice with processes (problem solving, reasoning, etc.), mental skills, routines of mind and creativity (Sarama & Clements, 2009).

There are many perspectives -psychological, social, cultural, pedagogical and recently neurophysiological, which attempt to contribute to the understanding of early mathematics development but there is less reflection and research examining the mathematical nature of this development (Newton & Alexander, 2013). It is documented that children, through a range of relevant experiences, challenges and activities, are enabled to develop interesting ideas, but it remains ambiguous whether these are mathematical ideas and if young pupils reach to a level of *thinking or acting in a mathematical way* (which is the goal of most current curricula). Moreover, it appears that, despite the considerable amount of studies and proposals related to early childhood, there is less progress in school, i.e. teachers' implementation of relevant approaches, tasks and materials.

One of the key factors could be the lack of understanding of the mathematical meaning shaped in the classroom and developed by children. All the aforementioned approaches deal with issues having to do with 'mathematics': mathematical development, mathematical thinking, mathematical activity and so on. But, how do we *define* and how do teachers understand and deal with the 'mathematical' part in these

expressions? How can a meaning, an activity or an outcome be characterized as 'mathematical' and how do young children apprehend it?

In the present paper, pursuing answers to above questions, we attempt to take a more substantial look at the mathematical aspect of several proposals related to early childhood mathematics education. This way we hope to contribute in building up an argument about how mathematics itself is related to both learning and teaching and provides essential answers to early mathematics education. We first present shortly different approaches (psychological, social, and pedagogical) related to this education and then we attempt to analyze the mathematical activity within and beyond them. Finally, we provide some examples of our proposal concerning a program and tasks aimed at developing authentic early mathematical activity.

THEORETICAL APPROACHES IN EARLY MATHEMATICS EDUCATION

After a long period during which early mathematics education was almost non-existent or was dealing with simplistic activities concerning numbers and shapes, widespread and extensive research gave rise to different scientific and educational approaches that contributed to changes in national curricula with special recommendations for this section of mathematics education.

Starting with Piaget and his psychological approaches, later researchers (Sarama & Clements, 2009) studied systematically young children's mathematical thinking and developed what they call "learning trajectories". According to the authors:

Learning trajectories are descriptions of children's thinking as they learn to achieve specific goals in a mathematical domain, and a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking. (p. 17)

This approach, based on a theoretical frame that the authors call 'hierarchical interactionism', is focused on children's thinking; thus there are activities and tasks related to the progression of this thinking and its relevant levels. The engagement of children with these tasks is supposed to lead them to some mathematical ideas, but the connection between children's thinking and relevant mathematical concepts (or aspects of them) don't appear so clear. For example, while a child recognizes a shape and discusses about it or uses it to compose a larger configuration, what part of the development of geometric knowledge does s/he access? How does s/he draw on the mathematical characteristics of the relevant concepts, objects, properties, relationships, definitions?

On this matter, Levenson, Tsamir and Tirosh (2011), in their work about early childhood geometry, add a 'mathematical view' to the development of geometry, proposing the formation of geometrical concepts with the use of the expression 'working definitions' that children can use for identifying and showing figures properties, relationships, comparing and communicating. The researchers, based on Fisbein's and Vinner's work about concept images and concepts in general, attempt to

develop an approach of geometric figures in line with mathematical concept definitions.

Important and systematic work on early mathematics was carried out by English (2012) and Mulligan and Mitchelmore (2013) who also worked on developmental aspects of children's thinking. Their work was not limited to specific mathematical content domains such as arithmetic or geometry, but dealt with the structural elements of mathematics, examining and connecting them with children's mathematical understanding. These studies constituted an important development that opened a new direction to early mathematics education, beyond numbers and shapes. However, they also raise some concerns regarding access to mathematical ideas: working with patterns and common structures isn't only a component of the mathematical activity that has to be combined with other actions to support children's conceptual formation?

From a socio-pedagogical perspective, the 'Learning Mathematics in Play' gave rise to important and interesting suggestions for early mathematics education. Typically, children play joyfully in game situations with mathematical features (Wager, 2013) or mathematical objects (like numbers or shapes), but these applications often end up with the need of the teacher's involvement in order to 'mathematize unintentional mathematical engagement in play' (Van oers, 2013). The later focuses his work on the use of language and communication within the Cultural-Historical Activity Theory perspective. While his approach has a clear orientation to mathematical thinking development, communication is again only a part of the process of mathematization and would also need (undefined) teachers' guidance for the appropriation of the relevant mathematical ideas.

In general, there are still many questions concerning early mathematics education: it is true that important aspects of mathematics can be found all around, in everyday situations and be used to develop children's mathematical learning; children are dealing with mathematical objects and situations and come to school with many mathematical ideas; they are acting in some mathematical content (counting, shape recognizing, measuring etc.) and are involved in actions and tasks that demand serious possesses, like problem solving, testing, explaining, reflecting, etc, using material and technology, with special mathematical features. However, are all these oriented to the development of mathematical thinking, knowing or acting? Do all these 'teach' them mathematics? Which part of what children do or we encourage them to do could be described as a well defined '*genuine mathematical activity*'?

MATHEMATICAL ACTIVITY

Teaching and learning of mathematics is not restricted to the development of mathematical concepts and procedures, but it mainly encourages the development of a human activity within situations and environments, institutionally formed by the educational system in schools. If we are interesting in developing this special human activity we need to define it: What is a *mathematical activity*? Which are its specific characteristics? What *criteria* can be used to evaluate whether an activity developed by

the students is or is not mathematical? Which *problems*, tasks or situations guide the development of this activity?

We find many similar or complementary approaches to the issue of what constitutes *mathematical activity* (in early childhood or generally). Most researchers consider as mathematical all the activities that involve specific type of working – processing including problem posing and solving, creative and flexible reasoning, communicating with arguments and documentation, reflecting and generalizing. Freudenthal (1983) understands the mathematical activity as a way of *modelling* to address and deal with real situations, while Brousseau (1997) as finding *appropriate solutions* for situation-problems. However, some researchers point out that learning mathematics overpasses problem solving, modelling and doing mathematics and concerns mainly *obtaining forms of reflection* about the world in a specific historical and cultural way, different from other forms of thinking. For them, acting of solving a problem without further explanation or transfer to a more general framework is only an aspect of the mathematical development (Radford, 2006).

Noss, Healy and Hoyles (1997) argue that mathematical meanings derive from *mathematical connections* that they consider as the important part of a mathematical activity (something that students usually do not learn to do). From another point of view, Ernest (2006) considers Mathematics as that area of human endeavour and knowledge that, more than any other uses a wide and unique range of signs and symbols; thus, he understands the process of *symbolization* as a basic part of mathematical activity and learning. In a different way, Steinbring (2005) addresses it as a dynamic link amongst situations – signs and concepts in his *epistemological triangle*.

In general, different views about mathematical development converge to the view that students need to reach a way of thinking that involves habits and mental routines and forms a high-level processing. Hence, combining different approaches we could argue that *mathematical activity* constitutes a set of (what we can call) *mathematical actions* that, based on the previous references, are summarized in the following (incomplete) list: search for properties and relationships, recognition of patterns and common structures, analysis and synthesis in parts and unit parts, connections, links to language, representations, signs and symbols, explanations / justifications, reflections and generalizations,... All these actions start with genuine questions, problems, unknown situations, games and involve conjecturing, solving, modelling, use of resources or tools, justification, metacognitive processes and formulations (e.g. Freudenthal, 1983; Brousseau, 1997; Radford, 2006; Perry & Dockett, 2008).

From the previous presentation it becomes clear that the simple engagement of children with mathematical objects does not always evoke relevant mathematical activity; moreover the activation of children alone is not sufficient for the development of a mathematical action. Thus, the study of forms of engagement with actions and tasks that are related to mathematical activity and supports children's mathematical development needs further exploration.

MATHEMATICAL ACTIVITY IN EARLY CHILDHOOD

The idea that simple practice in a concrete and local level does not mean generalizing of mathematical ideas or concepts is an old one (e.g. Nunes & Bryant, 1996). This position becomes more complicated and incoherent for early childhood as at this age children need to work with concrete material in everyday situations. Van oers (2013) analytically highlights:

Children evidently demonstrate behavior (like counting) that looks mathematical from the outside (as it is fairly in conformity with adult mathematical operations). These children, however, are often unable to apply this ‘knowledge’ in new situations, or answer questions about numbers...(p. 185)

Young children dispose an impressive amount of intuitive knowledge about space, quantities, patterns, measures, etc. evidenced by research (Sarama & Clements, 2009). This evidence gives an argument about the nature of this knowledge: is it ‘mathematical’, couldn’t it be just general, common or everyday knowledge, perceptual, kinesthetic, social, related to experiences, to needs, etc.? Certainly, this intuitive knowledge as well as the potential of young children to develop ideas and strategies, to find solutions or to communicate and explain could be seen as a base for the development of mathematical ideas. But at this age, if you don’t want to reduce mathematical knowledge to other conceptual development, we need to minutely study and analyze children’s activity in terms of mathematical work and outcome.

In early mathematics education, one could often wonder about the mathematical nature of tasks or actions carried out by children. A situation, a material, a story or another activity (such as cooking) are frequently presented in the classroom and the teachers ask questions to see if the children know how to count, or to compare bigger or smaller, or to give some location, or find a pattern or compose – decompose figures, accepting all these as mathematical actions and results (e.g. Doverborg, et al., 2011; van den Heuvel-Panhuizen, 2008; Sarama & Clements, 2009). But, these cases could raise questions about the development of authentic mathematical activity.

The special abstract nature of mathematics demands a long term development of each piece of knowledge, sometimes continuous but sometimes discontinuous, during which this knowledge in children’s minds is enriched, gets broader and is stabilized in a certain level (Confrey & Kazak, 2006). Thus, their teaching presupposes systematic experiences and activities from early age, during which the research or the teacher needs to follow not only the progress of children’s thinking but also the *progress of the knowledge itself* at this level of children’s thinking. The example of the use of ‘working definition’ in approaching geometric figures is very close to this position.

Concerning educational tasks, the suggestions in early childhood mathematics education usually take into account the previous experiences and knowledge of the children, their environment, their interests, their needs and so on. But their design needs also to be orientated by a framework that can connect the mathematical content

with the tasks and children's activity. Table 1 presents an example initiated by Keitel (2006) and adapted to early mathematical activity.

Content	Mathematical knowledge / meaning/ idea	What connection with the mathematical knowledge / meaning/ idea that aims to be developed by the task? Does it concern new knowledge, method, approach, reconstruction or widening of an older one? What connections to preexisting knowledge?
Task	Kind of task	Problem, realistic situation, project, research, testing, construction, model, data processing, representation, game, dramatization, implementation?
Tools	Representations/ material/ tools	What kind of language or representation is used for the task? Symbolic, synthetic (common elements), authentic related to the task? What kind of tools can be used? What recourses? What connections or aids?
Actions	Mathematical actions	What actions are proposed? Are there mathematical: search for properties/ relationships, pattern/ structure recognition, analysis and synthesis, connections, links to representations, explanation / justification, reflection and generalization. Do the children look for general solutions, methods, rules, general ideas?
Process	Mathematical processes	What processes are encouraged? Memorization/ application or imitation? Problem solving, dealing with situations, modeling, justification, metacognitive process, formulation, evaluation, creation?

Table 1: Questions for the design of tasks related to early mathematical activity.

Attempting to implement this approach, we organized a complete mathematical program with relevant content and tasks for ages 5-6 and 6-7 (the whole program is uploaded in www.nured.auth.gr/dp7nured/?q=el/userprofile/42). Following are some examples related to this program.

A PROGRAM DEVELOPING MATHEMATICAL ACTIVITY

The design of the program is based on the study of a coherent progressive development of *mathematical concepts and procedures*, analysed in their structural components and related to children's way of thinking. It aims at putting foundation in the basic concepts of the common mathematics curriculum through relevant tasks that encourage a high level mathematical activity for the target age group. Due to space limitation, we only present an example about Reflection Symmetry from the axis 'Space and Geometry', showing the focus on the mathematical aspects of the concept and the mathematical actions of children.

Preschool children identify quite easily and rather intuitively reflection symmetry in geometric shapes and other situations. Thus, the interest in working with this concept, even at this age, is not its holistic recognition in figures but its 'mathematical' approach

through (informal) understanding of its properties in symmetric shapes or symmetrical parts of a shape (same shape and size, equal distance from the axis and reverse orientation), with no formal presentation or teachers' guidance. To achieve this, we suggest tasks in which a transparent paper with a symmetrical part of a drawing is provided and the children have to complete it with the other symmetrical part. The paper is transparent so, after finishing their work, the children can fold the paper and control if their construction is right.

Depending on drawing and paper, the folding activity helps children realize one or more properties of symmetrical parts. For example, Figure 1 makes children



Figure 1

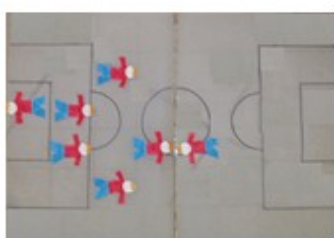


Figure 2

understand that they have to draw figures in equal distances from the axis: figures are already drawn, in same size, shape and orientation. If, after folding, there is a mismatch, the children need to reconsider distances. Similarly, Figure 2 helps children

understand both equal distances from the axis and change of orientation: figures are given (same size and shape) but they have reverse orientations. Mismatch after folding makes this change apparent.

Although the overall teaching approach is far from being completed, systematic implementation and observations of young children have produced important evidence about the development of mathematical activity in them (e.g. Tzekaki & Ikonou, 2009; Tzekaki & Kaplani, 2013). In the case of symmetry, a set of relevant tasks enabled children to approach the properties of reflection symmetry and 'formulate' them in a way. An ongoing research examines the development of this generalization, as part of mathematical activity, both in symmetry and other contents.

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