# MAPPING CONCEPT INTERCONNECTIVITY IN MATHEMATICS USING NETWORK ANALYSIS

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*This paper reports from a broad investigation of mathematics knowledge as dependent on interconnected concepts. The paper focuses specifically on illustrating how network analysis may be used in examining spatiotemporal relationships between learned mathematics concepts, or curriculum outcomes, and concepts inherent in assessment items. Connections both within and between year levels are shown, based on primary years' multiple-choice assessment items related to measurement. Network analysis provides a potentially powerful tool that may offer educators greater specificity in approaches to the design of revision and intervention through a view of complex rather than linear conceptual connectivity in mathematics learning.*

## INTRODUCTION

This paper uses analysis based in network theory, a modern development of graph theory, to illustrate connections between measurement items as part of a larger project *MathsLinks: Spatiotemporal Links in Mathematics Learning in Classroom and Online Environments*. A major thrust of this project is an examination of the connections between learned concepts as curriculum outcomes (e.g., Woolcott, 2013) and concepts inherent in assessment items. Network representations of such connections provide a spatiotemporal view of conceptual development in mathematics, with illustration here of complex connectivity in assessment items within and across year levels.

This project is based in a growing awareness that knowledge is interconnected and it utilises the strong groundwork for quantitative and qualitative investigation laid down in approaches using complexity theory (e.g., Davis, Sumara & Luce-Kapler, 2008). Although such approaches have been applied only recently in educational studies, student knowledge of mathematics has been linked specifically to complex and non-linear concept connectivity using network theory, with Mowat & Davis (2010) viewing mathematics in terms of 'complex networks'. Successful learning, in this view, depends on the development of major network junctions, or hubs, that support non-linear conceptual development, as well as the development of weak connections that circumvent hub failures (Khattar, 2010).

## BACKGROUND

Network theory is a widely used and powerful tool for representing and examining relationships in terms of system connectivity, and follows a well-established analytical methodology that allows qualitative mapping and quantitative analysis of the

2014. In Nicol, C., Oesterle, S., Liljedahl, P., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 5, pp. 385-392. Vancouver, Canada: PME.

relationships between nodes connected in a network (Newman, Barabási, & Watts, 2006). Network analysis has been applied widely across differing disciplines, largely because the rules governing network relationships remain independent of the nature of the subjects being linked (Newman et al., 2006). The main focus of Mowat and Davis (2010) is an argument that mathematics can be integrated through an examination of the complex linkages between mathematics concepts based on the embodied metaphors of Lakoff and Núñez (2000). A sidebar to this argument, however, is that mathematics concepts so integrated must be linked as networks. This seems to have support from the notion of expertise gained through the development of schemas, themselves arguably a type of network (Sweller, van Merriënboer, & Paas, 1998). The idea of knowledge linked as networks implies not only that mathematics concepts are linked together, but also that they are linked to other concepts in what Khattar (2010) considers as bodily experiences that are experienced emotionally.

Contemporary mathematics curricula, however, can be seen as constructs that are, in effect, a sequence of disconnected 'learned concepts' (e.g., see Chapter 1 in Glatthorn, Boschee, Whitehead & Boschee, 2012). Devlin (2007) has argued that a mathematics learner may have a functional understanding of a taught concept, as a learned concept, if the learner shows, through assessment, some level of understanding of that concept. A mathematics curriculum concept, in this sense, is a concept being taught that is being defined in terms of what the learner can do with it. A primary school teacher, for example, may consider student knowledge of addition of one-digit numbers to be a concept, but later to consider knowledge of addition of any two-digit numbers to be also a concept. The view of a mathematics concept as determined by a curriculum and its assessment, however simplistic, is useful in that the links between learned concepts may be traceable, using assessment results, in terms of functional understanding (Woolcott, 2013). It may be possible, for example, using a sequence of assessments, to determine if a primary school student, who has answered successfully a question involving knowledge about circles, has knowledge of other mathematics concepts that have led either linearly, or through a network of supporting links, to that knowledge (e.g., Lamb, 1999, in Mowat & Davis, 2010).

## METHODOLOGY

Large-scale testing programs, such as the Australian National Assessment Program – Literacy and Numeracy (NAPLAN) (ACARA, 2012) and the Australasian Schools Mathematics Assessment (ASMA) (EAA, 2012) include multiple-choice test items for assessing mathematics curriculum outcomes. Feedback from such testing is limited to assessing student responses against the outcome-based items. Network analysis methodology illustrates here how a more complex view of mathematics learning, generated from item data, may assist educators in understanding how concepts are related and why students find it difficult to make key connections between concepts.

This paper shows examples of representations (maps) based on network analysis of measurement items, about 6-8 items per assessment, from a larger analysis of

2009-2012 ASMA across primary school Years 3-6 in NSW, Australia. The Year 6 network results represent a single class sample of 62 students. The power of this analysis in comparing concepts longitudinally is illustrated using a map generated from results of one student who had completed ASMA in each of the years 2009-2012.

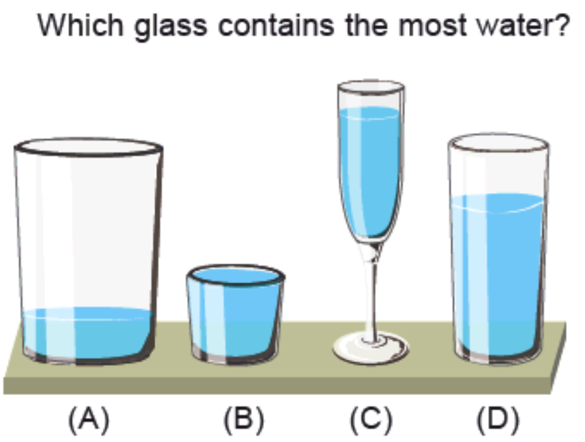
## Concept survey and matrix coding

A matrix of coded data generated from a concept survey of all measurement items, was analysed and maps generated using NetDraw (Borgatti, 2002). Each of the items was assigned one or more outcomes from the NSW K-6 Syllabus (BOS, 2012). Adapting Newman's Error Analysis (NEA, see White, 2010), additional inherent concepts were generated as 'access concepts' (Do I understand the question?) and 'answer concepts' (Can I now answer the question?). A limitation in using NEA for multiple-choice items is that analysis of student strategies cannot be used. Words as concepts, however, were included (e.g., Radford, 2003), as well as overarching concepts that allowed interpretation of diagrams (e.g., Lowrie, Diezman & Logan, 2012). An example of the concepts surveyed is shown in Figure 1 for a Year 5 ASMA practice question.

**The Measurement Outcomes**

MES1.3 Compares the capacities of containers and the volumes of objects or substances using direct comparison

MS1.1 Estimates, measures, compares and records volumes and capacities using informal units




**Inherent item concepts**

**Access**

Recognise 3D representation in 2D image

Concept of volume

Object contains a liquid

Objects may contain different volumes

**Answer**

Select informal unit to describe volumes

Estimate volumes

Compare volumes

*Words*: Which, contains, most

Figure 1: Concepts determined for a Year 5 multiple-choice measurement item. Practice item used with the permission of EAA.

For each of the ASMA Years 3-6 measurement items, responses and survey results were coded as follows: correct items and associated outcome/concepts as 1; incorrect items and associated outcome/concepts as 0. In network maps constructed using the matrix, nodes are either outcomes/concepts or items. Table 1 shows a sampling of the coded matrix for a Year 6 student with Item 2 correct and Item 5 incorrect.

| Outcomes/Concepts | Item 2 | Item 5 |
|---|---|---|
| MES1.5 (NSW K-6 outcome) | 1 | 0 |
| Recognise graph (access concept) | 1 | 0 |
| Read columns in graph (answer concept) | 1 | 0 |
| MS2.4 (NSW K-6 outcome) | 0 | 0 |
| *Word*: bought (access concept) | 0 | 0 |
| Adds mass in grams (answer concept) | 0 | 0 |

Table 1: Matrix coding for items and outcomes/concepts, for a Year 6 student with Item 2 correct and Item 5 incorrect. (Sample only - not all concepts shown.)

### Direct and inferred network connections

As well as direct relationships between items and outcomes/concepts (Figure 2), analysis utilised two types of inferred relationships: connections between the concepts/outcomes associated with an item; and connections between all concepts/outcomes of two or more items that shared a concept/outcome. The inferred network maps in Figures 3 and 4 are designed to provide additional structural overviews of any concept connectivity. Two types of weightings have been calculated for these network connections: simple weighting based on total numbers of students with correct/incorrect item responses (Figure 2), and; weightings based on class averages of these item responses (Figure 3). Network maps provide a diagnostic tool that can act as a guide to assessed mathematics knowledge and potential interventions and, although associated metrics can also provide additional insights, such as patterns of conceptual linkage, these are included elsewhere in this project.

## RESULTS AND DISCUSSION

### Individual and class connectivity – Year 6 measurement items

The network map in Figure 2 shows direct connections (lines) between nodes representing items (squares) and their outcomes/concepts (circles) for incorrect items. Each item node is a hub, since all paths from one of its concepts/outcomes to another must pass through the item node. Connection weights were calculated from totals of incorrect item responses across the Year 6 class, with heavier lines representing larger numbers of incorrect responses. The circled node indicates an outcome/concept shared across 3 items, one of several that form the basis for the inferred connectivity maps. Figure 3 shows such an inferred connectivity map for the Year 6 class, focusing again on incorrect responses (concepts excluding words). Network maps such as Figure 3 allow an educator to identify key outcomes/concepts that were, on average, incorrect and that may need to be reinforced for successful future learning, in case of hub failure (see e.g., Khattar, 2010; Mowat & Davis, 2010). The type of inferred analysis in Figure 3 may be particularly useful in its representation of connections between concepts or outcomes which were being used correctly in one context and incorrectly in another, on average (e.g., the connection between the dotted squares). The dotted squares, for example, show concepts that were shared across items that, although incorrect in one item, were correct in other items, more than 50% of the time in this simple illustration.

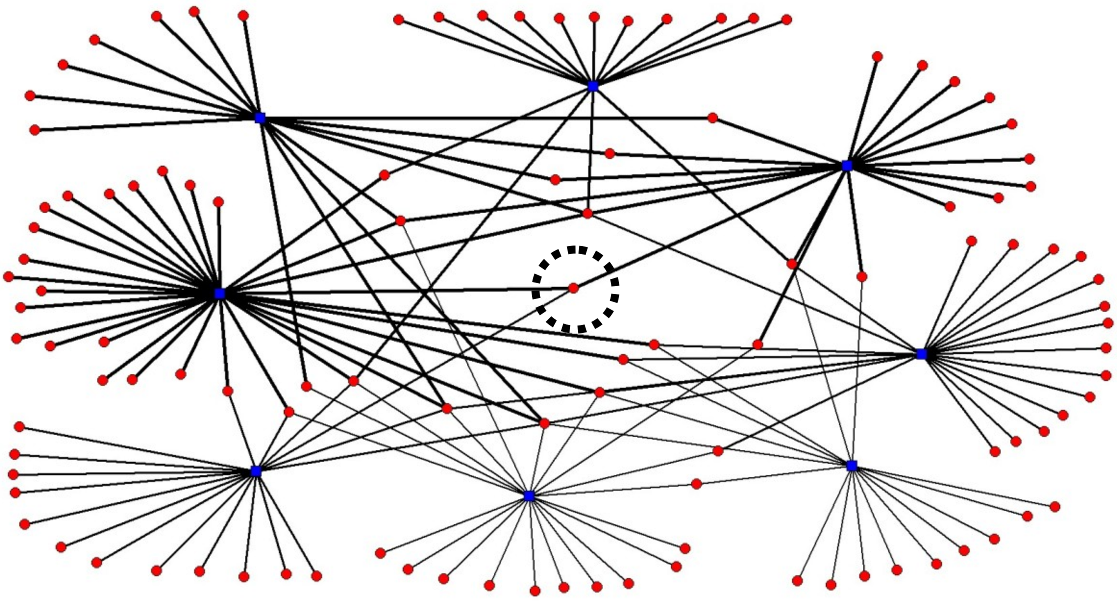


Figure 2: Direct connectivity map for Year 6 students with incorrect item responses

The heavier the line, the larger the number of incorrect responses. Items are indicated by filled squares and outcomes/concepts by filled circles. The dotted circle shows a shared concept, in this case 'a numeral written as a word'.

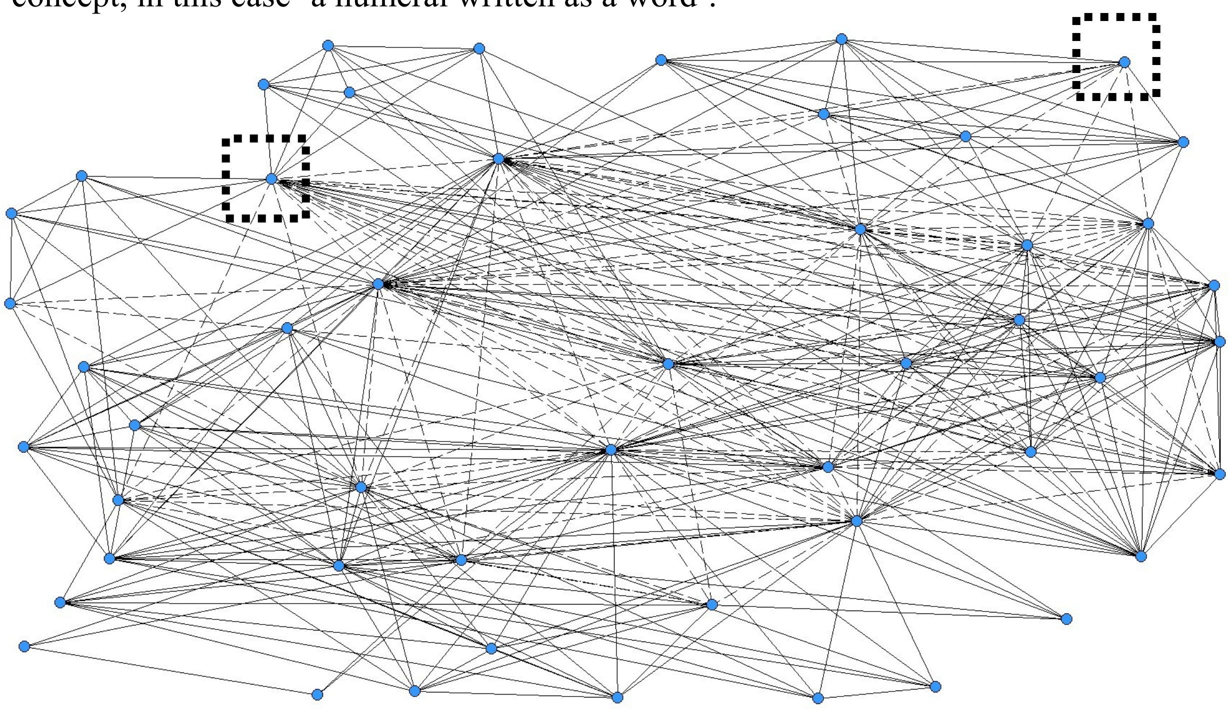


Figure 3: Inferred connectivity map, average weighted, for Year 6 students with incorrect item responses.

Solid lines are based on incorrect to incorrect connections and dashed lines on incorrect to correct connections between inherent item concepts (with word concepts not included for clarity). Concepts are indicated by filled circles. The dotted squares show two of the nodes that, on average, connect these concepts in incorrect and correct items.

The network analysis represented in Figures 2 and 3 (and other analysis not shown here) indicates to the teacher that a number of students in this class have not grasped particular measurement outcomes/concepts in this Year 6 assessment. These outcomes/concepts, therefore, may be a useful target for revision or intervention, even if it is only the centrally located outcomes/concepts that are targeted. The teacher could use such analysis to assist in design of revision or intervention around outcomes/concepts connected to the incorrect item responses for either the entire class or for individuals. Since this type of representation can also show nodes weighted by degree (number of connections), it offers further specificity for the classroom teacher as to relationships between items, outcomes and inherent concepts.

## Longitudinal connectivity – Year 3-6 measurement items

Figure 4 shows one of a number of possible inferred relationship maps that can represent longitudinal connectivity. In this case the map shows inferred connections between the incorrect Item 10 in Year 6 and items in Years 3-5. The item connections were inferred from shared outcomes/concepts, effectively reversing the inference process utilised to construct Figure 3. For a focus on curriculum, this analysis could also feature inferred connections between outcomes instead of items, or connections between outcomes and inherent item concepts.

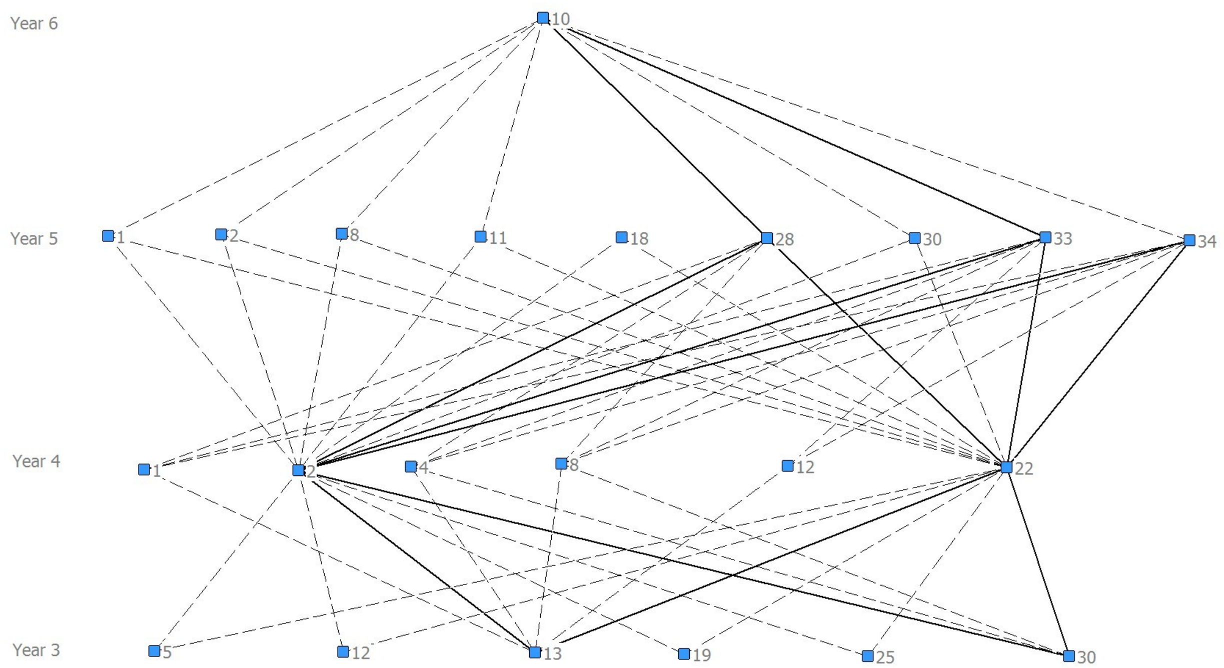



Figure 4: Inferred connectivity of the incorrect Year 6 Item 10 with items in Years 3-5.

Solid lines are based on incorrect to incorrect connections and dashed lines on incorrect to correct items. Items are indicated by filled squares with an item number.

Figure 4 shows how Year 6 items can be connected to items in previous years, effectively a 'concept trail' through past items, indicating which items and associated outcomes/concepts were/were not learned successfully. Analysis exemplified in

Figure 4, used in conjunction with that in Figures 2 and 3, may be useful, therefore, in designing revision or intervention that includes prior knowledge over time, as far back as Year 3 in this case, but over differing periods in such testing systems as NAPLAN. Two of the authors (Woolcott and Chamberlain) are using this longitudinal connectivity to trial an interactive 'App' designed to link curriculum outcomes and inherent concepts to intervention strategies for both multiple-choice and other styles of assessment items. The broader project aims to test the success of such strategies.

## IMPLICATIONS AND FUTURE RESEARCH DIRECTIONS

This paper provides an example of the network analysis we have been developing in order to examine the spatiotemporal interconnectivity of mathematical concepts. Although the application of network theory outlined here draws on extensive theoretical research on complex connectivity in mathematics (e.g., Lakoff & Núñez, 2000; Mowat & Davis, 2010), the illustrations aim specifically at an initial examination of whether network analysis is functional in the context of a school mathematics curriculum. This functionality is shown in the exemplar representations here as both direct and inferred connections between inherent concepts and outcomes derived from assessment items. The representation of longitudinal connectivity, in particular, gives a functional picture of conceptual development in mathematics over time. This paper shows examples of novel conceptual connections between outcomes and inherent item concepts, in this case for primary years measurement items, that are not currently utilised in the analysis of such large-scale testing programs as ASMA and NAPLAN.

The analysis here provides support for the view that new mathematics knowledge, even when described in terms of outcomes, requires prior knowledge (Sweller et al., 1998). Longitudinal representations may allow a more extensive analysis of prior knowledge than that undertaken in large-scale testing programs. The analysis here supports also broader analyses we are undertaking at differing conceptual levels, including analyses using embodied conceptualisations (Roth & Thom, 2009) and conceptualisations based on graphic elements in mathematics tasks (Lowrie et al., 2012) and pattern and structure (Mulligan, English, & Mitchelmore, 2013).

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