IS ELIMINATING THE SIGN CONFUSION OF INTEGRAL POSSIBLE? THE CASE OF CAS SUPPORTED TEACHING

Eyup Sevimli, Ali Delice

Gaziosmanpasa University & Marmara University

This study explored how the challenges encountered during integral sign determination process change after various learning processes. In this comparative investigation which is based on qualitative data, the students in the CAS group were subjected to technology enhanced teaching whereas the students in the traditional group were subjected to the traditional centered teaching approaches. Sign determination challenges of the students according to the groups were determined by means of pre- and post-application tests, and semi-structured interviews were employed as supportive data. The findings show that the students in CAS group, in comparison to the students in traditional group, had less "negative area" misconception in definite integral after teaching processes. In this investigation, it has also been discussed how teaching technology influences eliminating misconception.

INTRODUCTION

In many studies in the literature, misconceptions and challenges encountered in calculus lessons are mentioned. It is reported in many studies that calculus lesson students whose operational abilities have developed particularly in traditional class environment have difficulty in understanding, associating and interpreting concepts at basic level (Orton, 1983, Cornu, 1991; Rasslan & Tall, 2002; Berry & Nyman, 2003, Sofronas, De Franco, Vinsonhaler et al., 2011). Uniform presentation of information within the learning content and accustoming of students to solve questions in same pattern with mechanical steps are considered as the primary cause of this case. The fact that although students are successful in routine calculation problems requiring operational information they are confused about conceptual level has required revising lesson content and learning approaches. In this context, one of the steps that have been taken for fertilizing learning process is Calculus Reform Movement. As a result of this movement, textbooks and learning programs have been revised and they were rearranged according to the reform approach (Murphy, 1999). Key elements of reform approach are multiple representations. Accordingly, conducting only algebraic operation steps is not adequate to understand calculus subjects; in addition, interpreting inter-conceptual relations and choosing and using representations suitable for specific cases are also necessary (Dreyfus, 1991; Berry & Nyman, 2003). Calculus Reform Movement supporters claiming that teaching content and method must be reorganized in a way to offer opportunity for multiple representations support the process of integration of technology into learning environment (Murphy, 1999; Vlachos & Kehagias, 2000). A number of previous studies suggest that technology support can be benefited for eliminating the challenges encountered during teaching and learning

2014. In Nicol, C., Oesterle, S., Liljedahl, P., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 5, pp. 153-160. Vancouver, Canada: PME.

process of calculus (Berry & Nyman, 2003). The definition made by Vlachos and Kehagias (2000) for CAS-supported learning pattern is presentation of teaching contents organized according to multiple representations by means of technology support and this definition is based on in this study. This research is a part of a wider project which is concerned with students' understanding of the first-year calculus and project's pre-findings which is related to "*The role of CAS for concept images of definite integral*" was presented in previous PME conference (Sevimli & Delice, 2013). By means of this study, how misconceptions and challenges about integral observed in traditional calculus classes and that have correspondence in the literature are affected by CAS-supported teaching process was evaluated.

THEORETICAL FRAMEWORK

Integral concept, which is included within the fundamental subjects of higher education and which is the primary subject that students have difficulty in making sense of, is analyzed under definite and indefinite integral topics. Since definite integral involves previous subjects such as limit, derivative and function knowledge and requires solving techniques with various rules, it is considered among the primary and difficult subjects of higher education (Orton, 1983; Rasslan & Tall, 2002). Challenges encountered about definite integral is either associated with the nature of the concept or it can originate from pedagogical reasons. Accordingly, while Cornu (1991, p. 158) mentions about three reasons of cognitive challenges in calculus subjects, he lists them as epistemological, psychological and didactic oriented challenges. Some studies reports that traditional class students that can successfully solve integration problems that are difficult to calculate even with pencil and paper have difficulty in explaining and interpreting concept definitions at basic level (Orton; 1983). In teaching content of traditional classes, more time is allocated for algebraic interpretation of integral subject and more stress is put on calculation sense of integral (Berry & Nyman, 2003; Sofronas et al., 2011). Some cognitive challenges encountered in the class environment in the studies on integral can be listed as follows: limited concept image, lack of awareness of multiple representations, misconception, difficulties in contextual problem, misusing of Fundamental Theorem of Calculus etc. (Orton, 1983; Oberg, 2000; Rasslan & Tall, 2002; Sevimli & Delice, 2013).

One of the first studies on integral concept in the mathematical education literature was conducted to determine the misconceptions of students by Orton (1983). Emphasizing sign determination of students in his study he conducted to determine comprehension levels of students at introduction level of calculus about definite integral, Orton (ibid) expressed that the notion of limit of sums causes confusion in terms of algebra and stated that the biggest problem encountered arose from misconceptions named as 'negative area'. Negative area misconception is caused by interpretation of students the area above x-axis as positive and below x-axis as negative in area calculation problems. However, within [a,b] interval, since heights of the rectangles below the curve will be $-f(x^*_k)$ if $f(x)\leq 0$, ($x^*_k \in [x_{k-1}, x_k]$) area formula will be $\int_a^b -f(x)dx$ (Hughes-Hallet et al., 2008). Oberg (2000) attribute the main problem encountered about sign confusion to lack of integral in geometrical sense. Accordingly, students that can interpret the behavior of a function over a graphic representation have less difficulty in area calculation problems. Rasslan and Tall (2002), embarking with a similar research question, reported that students did not calculate definite integral value by means of sum of positive and negative areas, actually this sign confusion repeated systematically. Although students had sign confusion about definite integral in many studies as it was mentioned in the previous paragraph, a study evaluating the role of teaching processes for encountered misconception and/or challenges was not found. In line with the suggestions of the previous studies, a perspective for the role of use of technology in eliminating misconception/concept challenge in sign determination process was presented in this study.

METHOD

Research Design and Study Group

This study was designed according to multiple case study since teaching processes are assessed with a holistic approach over misconception of integral. The study was carried out in Calculus II during the 2011-2012 spring term. The participants of this study consists of 84 undergraduate calculus students at a state university; out of these students two groups have randomly been assigned, one as traditional group (n=42) and the other as CAS group (n=42). When assessing whether traditional and CAS groups are comparable, their marks in Calculus I in the previous term have been taken as criteria. It has been established that both groups have same scores in Calculus I and that groups are equal to each other in terms of their academic achievement.

Settings

The treatments in traditional and CAS groups in Calculus II are carried out during six weeks. In this period the role of two teaching approaches on eliminating misconception of integral were tested. Both approaches have been followed by the researchers. In the control group, where the course has been delivered in the traditional approach, the course notes from previous students have been made use of, and a traditional calculus textbook which generally emphasizes symbolic representation and focuses primarily on definition, theorem and proof processes has been used. Differing from traditional approach, technology support was benefited to provide different representations for a concept in the CAS-supported teaching. *LiveMath* software embedded textbook which was adjusted as per calculus reform and emphasizes translations between/within representations were used in CAS group (Hughes-Hallet et al., 2008). Teaching activities prepared according to multiple representations for preventing from misconception.

Data Collection Tools

Data collection techniques were test and interviews. Concept Definition Questionnaire (CDQ) used for determining students' misconception of definite integral before and after teaching processes and semi-structured interviews conducted for understanding students' problem-solving process in terms of misconception.

Concept Definition Questionnaire (Pre& post test)

The questions took place in previous studies are used to determine the students' misconception of definite integrals (Orton, 1983; Rasslan & Tall, 2002; Robutti, 2003). CDQ includes misconception and difficulties met during the teaching of integral, particularly "*negative area*" misconception. The questions in CDQ have different characteristic from each other in terms of obstacles at determining sign which might be depending on context of the question and the multiple representations used in the question. While the questions might be relevant to calculation of the integration and area with respect to context, they also might be algebraic and graphical with respect to representations in terms of characteristic. In Figure 1, an example questions from CDQ is presented with area context and algebraic representation. CDQ had been used in prior research (Sevimli & Delice, 2013) and three experts in mathematics (education) evaluated CDQ in terms of face and content validity. CDQ was given to the CAS and traditional groups as pre and post tests.

Semi-structured interviews

After administering post-CDQ, semi-structured interviews were conducted with four participants to get additional knowledge about integration processes and to understand the role of CAS-supported teaching in terms of elimination of misconception. These four participants in the interviews were selected using the purposeful sampling technique. Main selection criteria were that each participant taught by different teaching approach (CAS or traditional) and that they had different integral misconception. The participants were asked to explain their answers to the questions in CDQ.

Data Analysis

Pre & Post CDQ's data was first assessed in terms of students' misconception. To define the difficulties students have in determination of sign at before and after treatments, "*negative area*" confusion and "*positive value*" generalization which are frequently seen in the literature are utilised as categorization (Orton, 1983; Rasslan & Tall, 2002). According to these categorizations the change in determining sign confusion is compared with respect to characteristic of the questions over the study groups. Interview data was tagged for analysis using an open coding method. Participants' arguments when determining integral sign are exemplified as it is.

FINDINGS

Pre & Post CDQ findings

Evaluations were performed over tests that were conducted before (Pre-CDQ) and after (Post-CDQ) teaching application to determine the role of CAS support on eliminating sign confusion. No sign confusions were encountered pre-CDQ and post-CDQ in the problems (Integration/Algebraic) that were delivered by means of algebraic representation and that require operational integral calculation. While operation result was negative in 40% of the answers in the CAS group and 33% of the answers in the traditional group for the problems delivered by means of graphic representation requiring integral calculation, positive results were reached (Table 1). It was observed that, students found positive results by taking the negative values within the integral calculation into absolute value. The challenge encountered in such type of solution is that students interpreting every graphic problem as area problem in integral consider the interval as positive even where the integral function is negative, and generalize operation sign as positive.

Characteristic of Question	Type of Difficulties	Pre-CDQ (%)		Post-CDQ (%)	
		CAS	*Tra*	*CAS*	*Tra*
Integration/Algebraic	-	-	-	-	-
Integration/Graphic	Positive value	40	33	14	31
Area/Algebraic	Negative area	55	50	16	43
Area/Graphic	Negative area	36	29	9	21

Table 1: Distribution of pre-CDQ and post-CDQ sign confusions of the groups according to question type.

Post-CDQ findings showed that the sign that needed to be negative was determined as positive in 14% of the answers in the CAS group and 31% of the answers in the traditional group for integration/graphic characteristic question. When compared to the pre-CDQ findings, it can be suggested that CAS-supported teaching process considerably reduce "*positive value*" confusion encountered in integral problem delivered by means of graphic representation. It was observed that percentage of the students having "*positive value*" confusion was similar in the traditional group.

The questions delivered by means of algebraic or graphic representation in Pre & Post CDQ were applied to both groups to determine the reflections of sign confusion encountered in definite integral onto area calculation problems. Pre-CDQ findings revealed that "*negative area*" confusion is encountered more in area/algebraic characteristic questions when compared to area/graphics characteristic questions. Pre-CDQ findings show that at least one of every two students in both groups had negative area confusion for the questions delivered by means if algebraic representation requiring area calculation in integral. It was observed that post-CDQ negative area confusion encountered in the questions with area/algebraic characteristic decreased for both groups, however CAS support was more determinant in eliminating this challenge. Comparisons between the groups showed that "*negative area*" confusion encountered in area/algebraic characteristic questions was eliminated in CAS group in great extent when compared to traditional group.

Pre-CDQ findings showed that approximately one third of the students in both groups had "*negative area*" confusion in the area/graphic characteristic questions before the application. It was observed that, similar to the area/algebraic characteristic, "*negative area*" confusion encountered in area/graphic characteristic problems was reduced more in the CAS group when compared to the traditional group. When the general situation is considered, it can be stated that one third of the students had difficulty in determining integral sign after traditional teaching process.

Interview findings

Interviews were made with two each participant (CAS-P_1, CAS-P_2, Tra-P_1, Tra-P_2) from each group having "*negative area*" and/or "*positive value*" confusion to determine whether the challenge encountered in the process of determination of integral sign is a kind of misconception. Participants were confronted with their solutions for the question with area/algebraic characteristic and they were asked why they reached negative area when the function was negative. More than half of the participants having difficulty could not visualize the data presented algebraically and could not notice the intervals where the function switches sign. It is remarkable in the solution in Figure 1 that although the Tra-P_2 draw the graph and shaded the area of the region to be calculated, she did not count in negation of the sign in the area of the region below *x*-axis. It is wonder for what reason the Tra-P_2 used the graph and why she did not benefited from its content and the related analyses were supported by the interview findings.

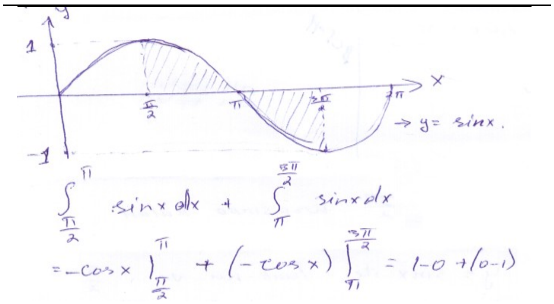
Question	***Solution sample***
Find the areas of the regions enclosed by the function *f(x)=sinx* and *x*-axis for $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$	

Figure 1: Questions and solution example for area/algebraic characteristic.

Since Tra-P_1 and CAS-P_2 did not draw graph, they noticed that they incorrectly wrote the equation corresponding the related area. CAS-P_1 determined the integral sign correctly for the problem with area/algebraic characteristic and stated that the area cannot be negative just like speed. Tra-P_2 benefited from the argument that, when the function is negative, it will be negative in alteration when she was explaining her solution.

> ...This graphic shows increase when it is above *x*-axis and decrease when it is below *x*-axis. Total change will be the sum of positive and negative changes. Therefore, the areas above and below the axis will cancel each other … (Tra-P_2)

DISCUSSION

Limiting definite integral with only area image may cause sign confusion in other algebraic calculations. The participants of the traditional group who stated in the interviews that they considered integrals of positive valued functions as area had sign confusion when the function sign was negative. In this study, this confusion named as "*positive value generalization*" is caused by explication of geometrical interpretation of definite integral as the area only below the curve. Students of CAS group seeing the algebraic and graphical approaches within teaching frequently and as a whole could easily differentiate geometrical sense of integral from algebraic calculation sense. Sevimli and Delice (2013) demonstrated that multiple representation opportunity supports richer and more variable image formation for integral. In this context, it can be remarked that CAS support emphasizes area sense of integral as well as calculation sense and thus provide support for making sense of calculation process.

Another challenged emphasized in this study is negative sign confusion encountered in area calculation problems. Test and interview findings revealed that some of the students in the traditional group did not take the sign of the function into consideration in area calculation problems presented by means of algebra representation and negative sign confusion was actually confusion for some part of the students. It was determined in the interview findings that some students in the traditional group interpreted the area below x-axis as negative and above x-axis as positive. Orton (1983) remarks that the challenge in such solutions is a misconception while he bases the cause of this misconception on the rote teaching that have no conceptual basis. As a matter of fact, answer of a student from the traditional group "*even if the area is negative, I multiply it with minus*" supports the reasoning of Orton (ibid). These misconceptions may be originated from student, information or teaching process (Cornu, 1991). Differently from pedagogical challenges, Orton (1983) reports that integral concept has challenges arising from its own nature, while Dreyfus (1991) integral concept require advance mathematical thinking processes, and they altogether confirm presence of epistemology-originated challenges. The findings of this study are similar to the results of other studies on sign determination process (Oberg, 2000; Rasslan & Tall, 2002), and authentically show that CAS-supported environments create awareness in the process of sign determination in definite integral. The students of CAS group trying to interpret graphic data within the context of algebraic calculation and area senses of integral used analytic and visual judgments together and by means of association, and they were more successful in terms of this respect when compared to the students in the traditional group creating solutions basing on analytic judgment. Area calculation problems in the contents presented by means of *LiveMath* software in CAS group were associated with rectangles sum in Riemann's definition. In teaching applications visualized by means of technology support, the fact that heights of the rectangles below x-axis were $-f(x)$ was stressed and the contents that would provide making sense of sign change by students were employed. These arguments used by the students of CAS group when determining signs can be interpreted as technology being as scaffolding. Namely it may be claimed that technology helps students to construct or reshaped the knowledge and procedures during the integral problem-solving processes.

CONCLUSION

Study results showed that the students in the traditional group could not interpret the graphic data before and after the teaching application, and therefore had "*negative area*" misconception and "*positive value*" confusion. Many students in the traditional group tried to make the transitions between graphic and algebra representation through the rule-based approaches that do not have conceptual basis. After CAS-supported teaching process, the students more frequently benefited from graphic representation in area calculation problems in integral and could correctly interpret graphic data in the problems orientated at integration calculation. Therefore, previous sign confusions of the students in the CAS group were eliminated to a large scale. In the light of the results mentioned above, it is concluded that CAS-supported teaching pattern is more effective in eliminating some misconceptions and challenges encountered before the application or in the literature when compared to the traditional teaching approach.

References

Berry, J., & Nyman, M. (2003). Promoting students' graphical understanding of the Calculus. *Journal of Mathematical Behavior*, *22*,481-497.

Cornu, B. (1991). Limits. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 153-166). Dordrecht, The Netherlands: Kluwer Academic.

Dreyfus, T. (1991). Advanced mathematical thinking processes. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 25-41). Dordrecht, The Netherlands: Kluwer Academic.

Hughes-Hallett, D., Gleason, A. M., McCallum, W. G., Lomen, D. O., Lovelock, D., Tecosky-Feldman, J., … Frazer Lock, P. (2008). *Calculus: Single variable* (5^{th} ed.). New York: Wiley.

Murphy, L. D. (1999). *Computer Algebra Systems in calculus reform*. Retrieved from http://www.mste.uiuc.edu/murphy/Papers/ CalcReformPaper.html.

Oberg, R. (2000). *An investigation of undergraduate calculus students' understanding of the definite integral* (Unpublished dissertation). Missoula, MT: The University of Montana.

Orton, A. (1983). Student's understanding of Integration. *Educational Studies in Mathematics*, *14*(1), 1-18.

Rasslan, S. & Tall, D. (2002). Definitions and images for the definite integral concept. In A. Cockburn & E. Nardi (Eds.), *Proc. of the 26^{th} Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 4, pp. 89-96). Norwich, UK: PME.

Sevimli, E., & Delice, A. (2013). An investigation of students' concept images and integration approaches to definite integral: Computer Algebra System versus traditional environments. In A. M. Lindmeier & A. Heinze (Eds.), *Proc. 37^{th} Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 4, pp. 201-208). Kiel, Germany: PME.

Sofronas, K. S., De Franco, T. C., Vinsonhaler, C., Gorgievski, N., Schroeder, L., & Hamelin, C. (2011). What does it mean for a student to understand the first-year calculus? Perspectives of 24 experts. *The Journal of Mathematical Behavior*, *30*,131-148.

Vlachos, P., & Kehagias, A. (2000). A Computer Algebra System and a new approach for teaching. *Journal of Computer Algebra in Mathematics Education*, *7*, 87-104.