'VALUE CREATION' THROUGH MATHEMATICAL MODELING: STUDENTS' DISPOSITION AND IDENTITY DEVELOPED IN A LEARNING COMMUNITY

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This study examined how mathematical modeling activities within a collaborative group impact on students' perceived 'value' of mathematics. With a unified framework of Makiguchi's theory of 'value', mathematical disposition, and identity, the study identified the elements of the value-beauty, gains, and social good-with the observable evidences of mathematical disposition and identity. A total of 60 college students participated in 'Lifestyle' mathematical modeling project. Both qualitative and quantitative methods were used for data collection and analysis. The result from a paired-samples t-test showed the significant changes in students' mathematical disposition. The results from the analysis of students' written responses and interview data described how the context of the modeling tasks and the collaborative group interplayed with students' perceived value.

INTRODUCTION

Studies reported that when students see themselves as capable of doing well in mathematics, they tend to value mathematics more than students who do not see themselves as capable of doing well (Eccles, Wigfield, & Reuman, 1987; Midgley, Feldlaufer, & Eccles, 1989). To see the value in mathematics, it is essential for students to believe that mathematics is understandable, not arbitrary; that, with diligent effort, it can be learned and used; and they are capable of figuring out mathematical problems based on their experiences. Kilpatrick and his colleagues (2001) introduced "productive disposition" as one of key components of mathematical proficiency and defined as the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy"(NRC, 2001, p. 131). "Mathematics disposition" (NCTM, 1989, p233) was also included in the National Council of Teachers of Mathematics Evaluation Standards. Developing such a disposition toward mathematics requires frequent opportunities to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics. It becomes a question of what learning environment supports students to engage in meaningful learning of mathematics and to develop positive disposition as well as self-concept. A number of studies demonstrated that mathematical modeling, which plays a prominent role in the new Common Core State Standards for Mathematics (CCSSM), promotes socially situated learning environments with group collaboration, classroom discussion, initiative, and creativity and it has the potential to develops positive disposition toward mathematics and strengthen their mathematical identity (Ernest, 2002; Lesh & Doerr, 2003). The studies highlight that learning

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mathematics extends beyond individuals' learning concepts, procedures, and learners learn to be a part of a community of practice and become participants in the mathematics being practiced (Boaler, 2002). How a student learns mathematics involves the development of the student's identity as being a part of a mathematics classroom community (Anderson, 2007). A mathematical identity consists of a participative "mode of belonging" related to one's participation in a mathematical community of practice typically, the mathematics classroom (Wenger, 1998).

THEORETICAL FRAMEWORK

As a unified framework of Makiguchi's theory of value creation (1930), mathematical dispositions outlined by NCTM Evaluation Standards 10, and identity (see Table 1), this study identified the elements of the value with the observable evidences of mathematical disposition and identity.

Mathematical Disposition, Identity, Sense of belonging	Makiguchi's Elements of Value
• Interest, curiosity, and inventiveness in doing math	Beauty
• Confidence in using math to solve problems and communicate ideas • Willingness to persevere and become persistent in math tasks • Flexibility in exploring math ideas and trying alternative methods in solving problems • Appreciation of the role of mathematics in our culture and its value as a tool and as a language • Inclination to monitor and reflect on their own thinking and performance • Valuing of the application of mathematics to situations arising in other disciplines and everyday experiences	Gains
• See oneself as a learner, and doer of mathematics • Sense of belonging in a learning community, global citizenship	Identity Social value (Social Good)

Table 1: Theoretical framework (Makiguchi's theory of value, disposition, and identity)

The concept of value in the notion of Makiguchi (1930; Bethel, 1989) takes into account the subject and object relationship (students' relationship with mathematics in this study), which reflects human creativity. In the notion of Makiguchi (1930; Bethel, 1989)'s value creation, it is critical that students feel happiness, enjoyment, and pleasure in their own processes of investigating and understanding mathematics, as a result, students construct meaning, and value is created. In Makiguchi's concept of value, the three elements of the value are the following:

Beauty is perceived to be an emotional and temporary value. The value of *Gain* is an individual value and self-development, and beneficial aspect that is related to the whole of man's life. *Social good*, however, is a social value and is related to the life of the group. The value of good is the expression given to the evaluation of each individual's voluntary action, which contributes to the growth of a unified community composed of the individuals (Makiguchi, 1930; Bethel, 1989).

RESEARCH QUESTIONS

The purpose of this study is to develop and evaluate a model for students to *create value* in learning mathematics. With the unified framework of Makiguchi's theory of value, mathematics disposition, and identity, this study examines how 'socially-situated' mathematical modeling activity within a collaborative learning community can contribute to students' development of their mathematical disposition, identity, and sense of community as well as students' creating mathematical meaning. The guiding questions for this study are as follows:

1. What changes (if any) are observed in students' mathematical disposition that results from learning mathematics through mathematical modeling within a learning community?

   Specifically, How do students perceive value of beauty and gains, in Makiguchi's notion, of learning mathematics before and after experiencing mathematical modeling activities within a collaborative group?

2. How are students' mathematical identities transformed from their involvement in mathematical modeling activities within a collaborative group?

3. How are students' perceived social values, in Makiguchi's notion, of learning mathematics observed during mathematical modeling activities within a collaborative group?

   1) How does the collaborative group create a sense of belonging to the group that can be realized through engaging in mathematical modeling activities with group members?

   2) How do students interpret mathematical results within the socially situated context of modeling activities?

METHODOLOGY

Both quantitative and qualitative methodologies were used in data collection and analysis, investigation, and interpretation. Multiple data sources including surveys, interview data, students' written tasks and journals were collected (see Figure 1).

These data sources provided participants with multiple opportunities of their reflecting and sharing thoughts about how these experiences impacted their disposition and identity. The participants were a total 60 students who enrolled in college algebra courses taught by the researcher. The curricular task for the study is a modified version of the mathematical modeling project developed by the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) at Rutgers University. The project introduces the ecology of humans as a topic, and ecological foot printing is developed as a tool for assessing human impact and as a decision-making tool. These topics are relevant to social and environmental issues in which students engage in everyday lives. The investigator attempted to provide students with the tasks that require everyday knowledge, critical thinking, and a collaborative work. The mathematical modeling project was conducted within groups of four or five for four consecutive weeks. After completing the first week of conducting the project, students were asked to collect their own data. The Mathematical Disposition Survey (MDS) was conducted at the beginning of the study and the end of the study, and the results were compared. The mathematical disposition survey instrument is a modification of the one developed by Kisunzu (2008). Students' written tasks and journals were collected after each class. A total of eighteen focal students were selected for interview based the results from the analysis of Mathematical Disposition Survey and students' journals. Semi-structured interview offered students the opportunity of giving detailed statements on their written tasks, questionnaires, and journals. The researcher took field notes and audio-taped all the activities in classroom and interviews.

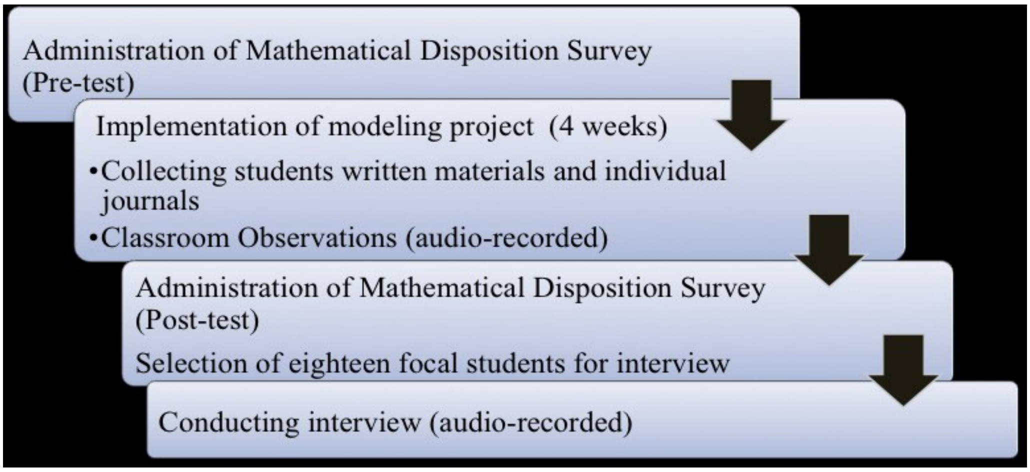

Figure 1: Procedures (Multiple methods)

RESULTS

The result from a paired samples t-test showed the significant changes in students' mathematical disposition between pre and post survey. There was significant difference in the mean scores for Mathematics Disposition Pre-test (Mean =132.57, SD= 23.65) and for Post-test (Mean=138.97 and SD= 24.52 with condition of t=-3.25 with $p < 0.01$) (See Table 2.)

	Pre-Test	Post-Test
Mean	132.57	138.97
Variance	567.12	607.65
Observations (N)	47	47
Pearson Correlation	0.85	
df	46	
t Stat	-3.248	
P(T<=t) two-tail	0.002	
t Critical two-tail	2.013	

Table 2: Descriptive Statistics and Paired Samples T-test (Mathematics Disposition)

For further investigation, the pre and post survey mean scores in each aspect of mathematics disposition (confidence, flexibility, perseverance, interest and inventiveness, meta-cognition, usefulness and appreciation) were analyzed by a paired samples t-test. Students' gain score from pre to post test was statistically significant for the aspect of flexibility (Pre: Mean=10.53, SD=3.69; post: Mean=12.09, SD=2.51 with t=-3.28, $p<0.01$), for the aspect of appreciation (Pre: Mean=16.49, SD=5.34; Post: Mean=18.06, SD= 4.61 with t=-2.62, $p<0.01$). Interview data were analyzed searching for evidence of 'changes' in disposition, identity, and students' perceived value resulting from engaging in modeling activities in a collaborative group.

Students' changes in disposition and identity

As the main parts of the modeling project involves collecting their own data, mathematizing the data, and interpreting the data, the aspect of modeling with students' own data promoted autonomy among students and alternative ways of solving problems (the aspect of flexibility) by reflecting their own thinking through collaborative group work:

Excerpt 1. Change in the aspect of flexibility and meta-cognition (Value of gains)

Chloe: Group members shared different ways of doing it, when they found different solutions from others, they thought about it and came back to talk about it. If you work with three other people, you can get different perspectives. For instance, there was a case that we had same answers but we found everyone solved them in different ways. Something like that, it was cool. Also no one depends on anyone since I myself had to live my days and collect my own data.

The aspect of modeling that was relevant to students' everyday lives seemed to have contributed to their development of positive disposition and personal identity as doers of mathematics:

Excerpt 2. Change in the aspect of interest and confidence, and identity transformation (Value of beauty, gain, and identity)

Ella: I had difficult in doing math entire my life until to this day. I feel like this project would be beneficial for the students like me. I think it is important to think analytically and think outside of box through this kind of project. I changed my view of math in the sense that it became enjoyable since this project gave me some excitement. The project used math but it was interesting.

Sense of belonging and social value: what it means to understand mathematics

There was the evidences indicating socialization in a group through emotional connections by asking for help and sharing stories of events with particular topics:

Lisa: We talked about our data, also our personal lives, why we had these numbers, what electric devices have used. I had a big number and she had a smaller number than mine. Then we talked about why and talked about the details in our personal lives. Especially with Alexis, cause we both live with family and others lived on campus so our numbers were pretty close but others' were very different from ours. We talked about it at the personal level. Generally, as for a group work, some people do not do their parts but in our case, everyone contributed their parts.

Interpreting mathematical results and social value: what counts as mathematical argumentations in modeling tasks

'What constitutes mathematics argument' was related to the affiliation with the group, and the real life context of modeling tasks helped them to establish socio-mathematical norms.

Interviewer: While working on this project in a group, how did you guys decide the solution is correct?

Deana: we took a look at them to see if they make sense, like realistic number not too high or too low, one girl's number was so low everybody else was high, so we told her "you did something wrong". Then we found that she forgot to add something.

With regards to the norms of what counted as an acceptable or valid mathematical explanation, students justified it based on the real life contexts by comparing data and examining the process of measuring footprints with group members.

CONCLUSION

In Makiguchi's theory of value, benefit or gain as advancement of the life of the individual in a holistic manner and that is beneficial aspect of the interactions with an object (mathematics in this study), for example, one did develop the level of confidence in doing mathematics and was able to express his or her idea within a group or developed one's willingness to navigate alternative ways of solving problems and monitor own thinking. The individual creates value through contributing to the well-being of the larger human community and society (Ikeda, 2001). Social value was created through students' interactions with the external context to mathematics and also with other members while working in a group. A mathematical identity and norms were related to students' sense of belonging related to students' participation in the mathematical community of practice (Wenger, 1998). Students deeply engaged with mathematics through modeling activities by sharing mistakes, listening to and offering suggestions about other's work, and thinking about rationales behind why particular decisions were meaningful. The development of dispositions can be understood as being shaped by the interrelation between the context of mathematics tasks and interactions with others. For further study, By examining students' modeling activities and interactions with peers in the classroom, one can understand better, how these elements interplay with students' construction of disposition and identity.

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