EXAMINING THE COHERENCE OF MATHEMATICS LESSONS FROM A NARRATIVE PLOT PERSPECTIVE

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This paper aims to clarify how coherence of 'structured problem solving' mathematics lessons can be produced by comparing the lessons of three teachers from a narrative perspective. Results of our analysis showed three main coherence characteristics of lesson teaching sequences: (1) sequence scenes are recursively developed based on the previous scenes, (2) there is a scene of setting a learning goal in terms of the conflict between what students know and what they do not know, and (3) coherent plots are grounded in certain mathematical content knowledge. We conclude by introducing a metaphor of living theatre to better understand the coherence of lesson structure.

INTRODUCTION

This paper aims to clarify the coherent qualities of mathematics lessons commonly referred to as "structured problem solving" (Stigler and Hiebert, 1999) as conducted by effective teachers. Since the TIMSS video study, Lesson Study has drawn global attention as a means for improving the quality of mathematics lessons and teachers' knowledge for teaching. Stigler et al. (1999) identified a pattern, or script, in effectively taught mathematics lessons in Japan: reviewing the previous lesson, presenting the problem for the day, students working individually or in groups, discussing solution methods, and highlighting and summarizing the main point. This script has been historically developed by Japanese teachers for cultivating students' mathematical thinking abilities and attitudes as well as their knowledge and skills.

However, we should not directly equate the above teaching pattern with an effective mathematics lesson, because there is a range of teacher efficacy from effective to ineffective, and a range of lesson success from successful to unsuccessful, even if the pattern is indeed adopted by most of the primary school teachers in Japan. Namely, for teacher development it is not effective to simply use this pattern. Rather, it is important to know how lesson coherence can be produced. We believe that there is a substantial difference in lesson quality depending on whether a lesson is developed like a narrative or in isolated steps. It has been reported that Japanese mathematics lessons can be characterized as coherent accounts of a sequence of events and activities that comprise the classes, as if they were a story or drama (Stigler and Perry, 1988; Shimizu, 2009).

We believe it is essential to explore how such coherent accounts are created for studies of teachers' knowledge. Ball et al. (2008) provide a framework for mathematical knowledge for teaching (MKT) that elaborates on subject matter knowledge and pedagogical content knowledge. They mention several research tasks in situating such knowledge in the context of its use, such as how different categories of knowledge

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come into play over the course of teaching. On this point, Silverman and Thompson's (2008) MKT framework based on 'key developmental understanding (KDU)' as "a conceptual advance that is important to the development of a concept" (Simon, 2006, p. 363) seems useful for planning lessons. However, it remains unclear what processes of a lesson a teacher can practically realize using such knowledge, particularly to produce coherence in teaching.

THEORETICAL BACKGROUND

Several researchers have noted that children's learning is narrative in nature. Dewey (1915, p. 141), for instance, stated, "(Children's) interest is of a personal rather than of an objective or intellectual sort. Its intellectual counterpart is the story-form…Their minds seek wholes, varied through episode, enlivened with action and defined in salient features—there must be go, movement, the sense of use and operation—inspection of things separated from the idea by which they are carried. Analysis of isolated detail of form and structure neither appeals nor satisfies." This suggests that even if we collect all of the parts that constitute a lesson structure, it will not attract the attention of children unless it is in a story-form. Mathematics education studies have also seen effective lessons as being in story-form. Krummheuer (2000) understood classroom situations as "*processes of interaction*: students and teachers contribute to according to their sense and purpose of these events" (p. 22) in terms of classroom culture; this view was influenced by Bruner's (1990) view of narrative as having the following characteristics: sequentiality, a factual indifference between the real and the imaginary, a unique way of managing departures from the canonical, and a dramatic quality. Zazkis and Lilijedahl (2009) tried to shape mathematics learning as storytelling to enhance students' interest in, and to engage them with, mathematical activities. They listed the following general elements of good stories: plot, beginning, conflict and resolution, imaginary elements, human meaning, wonder, and humour.

We consider the concept of plot as being crucial to analyzing a quality lesson. Krummheuer stated that "a plot characterizes the sequence of action in its totality: it describes something that is already fixed… But an unfolding plot connotes something fragile, not yet entirely executed, still changeable. Both aspects are essential and the tension between these two dimensions of this concept is crucial for its adaptation for classroom interaction and its function for learning" (p.25). It seems that there are two of these aspects that correspond to the planning and the practicing of a lesson, respectively. We consider the following script, identified as a Japanese lesson structure (Stigler and Hiebert, 1999), as a way of adding the role of plot to a narrative structure.

- Reviewing the previous lesson;
- presenting the problem for the day;
- students working individually or in groups;
- discussing solution methods; and
- highlighting and summarizing the main point.

However, as we stated above, even if teachers use this pattern in their teaching, there is a range of possible lesson evaluations from very good to very poor. Shimizu (2009) suggested that lessons conducted by effective teachers can be compared to stories or dramas. A coherent account of a lesson can be explained as a well-formed story which "consists of a protagonist, a set of goals, and a sequence of events that are causally related to each other and to the eventual realization of the protagonist's goals. An ill-formed story, by contrast, consists of a simple list of events strung together by phrases such as "and then…", but with no explicit reference to the relations among events" (Stigler and Perry, 1988, p. 215; cf. Shimizu, 2009). Thus, it is important to examine how a coherent plot in teaching can be produced during a lesson. In addition, it is important to remember that the protagonists are the students, and that thus their ideas and feelings are central components of the story, and that a teacher may assume that as many students as possible will play active roles. On the contrary, the lesson may not be effective if the only active persons are a teacher and just a few capable students.

METHODOLOGY

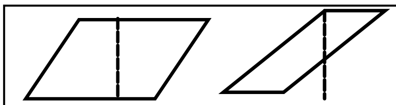


Figure 1

We asked six teachers to conduct lessons: A) 2 experienced teachers who specialize in mathematics teaching, B) 2 experienced teachers who do not specialize in mathematics teaching, and C) 2 teachers who have a few years' experience. We selected the content 'area of a parallelogram for which the height cannot be known from a straight line on its inside' from a fifth grade mathematics textbook (Fig.1, right). We assumed that children would have difficulty in the height, and that differences in teaching among the teachers would appear when dealing with this difficulty.

During a preliminary meeting with each teacher, we introduced multiple methods for finding the area of the parallelogram. Then, we asked him/her to conduct their lesson to help students find multiple solutions to the problem and to understand the concept of area beyond simply understanding how to solve the problem. We also interviewed each teacher to better understand what he/she valued most in designing and practicing his/her daily lessons.

The lessons were recorded with video cameras and field notes. We made transcripts of the video data. In our data analysis, we first extracted all meaningful interactions to examine whether a teacher's questioning or instruction evoked student responses, and how he/she subsequently responded to the students. Next, we conceptualized each interaction unit in terms of the teacher's intention and the interaction's practical effects, before trying to reconstruct the entire picture of the lesson structure, that is, the 'plot', by examining how the interactions were connected to each other. Finally, we compared the reconstructed lesson structures of the six teachers' lessons and tried to clarify the characteristics which comprised the creation of a coherent plot.

Below, we present the results of our analysis of the lessons conducted by three of the teachers: Mr F (the above type A, 35 years of experience), Ms Y (type B, 18 years of experience), and Mr S (type C, 3 years of experience).

RESULT 1: THE CASE OF MR F'S LESSON

First scene: Reviewing the formula for the area of a parallelogram

Mr F began by reviewing the formula for finding the area of two parallelograms (base 6, height 4; base 3, height 1) (Fig. 2). Here, the interactions between Mr F and the students showed a pattern. First, Mr F asked the value of the area and the students answered 24 by counting the unit squares or by using the area formula. Next, Mr F asked what formula they used, and they answered 6×4. Moreover, Mr F asked what 6 and 4 referred to in the figure, and one student indicated the base and height locations by tracing along the figure with her finger on the blackboard. In particular, Mr F made her check the vertical relationship between the base and height and trace the height of the shape in several places. The pattern of interaction here was: answer   formula   meaning of the values in the formula   arbitrary places of height of the shape. We found that this pattern of interaction was also used in the case of the 3×1 formula.

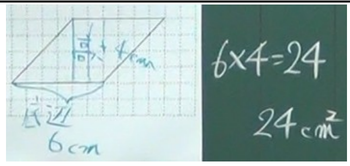


Figure 2

Second scene: Setting a problem through an experience of the conflict

Mr F presented a problem as follows.

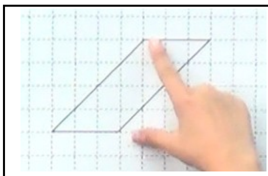


Figure 3

Mr F:	I have one issue with this. I am bothered by this parallelogram. Do you understand my trouble?
Student 1:	The previous parallelograms had this line. This time, we can't draw this (line) (Fig. 3).
Mr F:	I tried to find the height, but there's nothing there! Oh, there's no height!
Students:	But, but… (Several students raised their hands to respond.)
Mr F:	But, does the parallelogram have an area?
Students:	Yes, it has an area.
Mr F:	Yes, it does. This is a parallelogram. But we can't use the area formula because we don't know the height. Don't you feel like crying?

The problem setting was like the beginning of a narrative in which the students were involved in an issue troubling Mr F, where the two circumstances ('there is no height' and 'the area formula can't be used')were given as the problematic aspects of the issue. We note that the problem was set based on the preparation conducted in the first scene.

Third scene: Setting a goal by comparing between the known and the unknown

Mr F next proposed setting a learning goal for the students. One student said, "Let's find the height", but the task at hand was not simply to find the height of the shape. Mr F then tried to direct the students' interest to figuring out a formula to find the area of a parallelogram of unknown height based on the student's statement.

Mr F:	Oh, yes. The height of this is dubious. If you know the height, then…
Students:	We can know the formula!

Moreover, he clarified the task by aligning three parallelograms and confirming that the formula could now be used only for 6×4 and 3×1 parallelograms (Fig. 4). Then, the students were able to set a goal: to find the area of a parallelogram of unknown height using a formula. We note that this aligning of the three parallelograms implicitly prepared the students with insight for two ideas to solve the problem by seeing it as half of a 6×4 parallelogram and as four 3×1 ones.

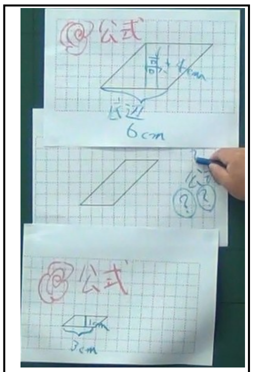


Figure 4

Fourth scene: Individual activities and redefining the goal

The students individually tried to solve the problem. However, Mr F found that some students just wrote the formula 3×4=12 procedurally (Fig. 5), which was different from the set goal of understanding the situation based on the known parallelograms using the area formula. Mr F then stopped these students and restated the task for all the students again.

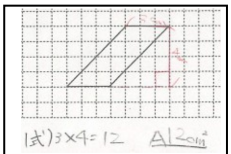


Figure 5

Mr F: Some of you may be thinking of this as the height. As it is now, we don't know whether this is the height or not, because it doesn't meet the base. So, you can't set this as the height (Fig. 6). Consider using the formulas you already know.

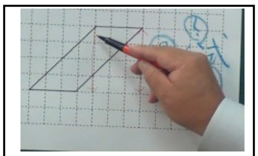


Figure 6

Mr F's redefining of the task in this way seemed to work successfully because all the students then started considering the problem using the known parallelograms. We found a total of 13 distinguishable solutions in the students' notebooks.

Fifth scene: Class discussion (1): Sharing the fundamental idea

We found that Mr F employed one particular type of interaction in which he tried to deepen one basic idea by using plural voices during the class discussion. First, Mr F invited the students who had come up with the idea of using four 3×1 parallelograms to present their idea to the class. Mr F's writing on the blackboard gradually became more detailed as he interacted with the different students. We characterize this series of interactions as multi-layered.

S2: Here is 1, 2, 3 and 4. Mr F: What is here? (He circled the bottom one.)	⇒ S3: The small parallelogram is 3×1=3. As there are 4, the answer is 12. Mr F: Can anybody else explain in the same way? (He wrote the formula.)	⇒ S4: The bottom one is 3. There are 4 parallelograms, 3 times 4 is 12. (He wrote the numbers.)	⇒ Mr F: What is the case of one step? Ss: Three. Mr F: What about for two steps? Ss: Six. …

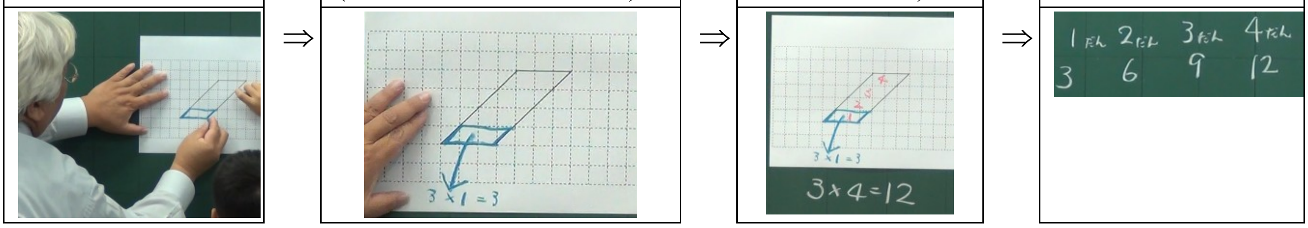
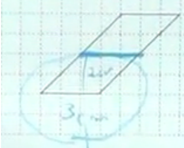


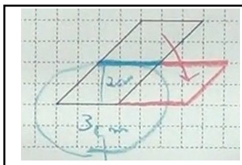
Figure 7

Sixth scene: Class discussion (2): Sharing various ideas

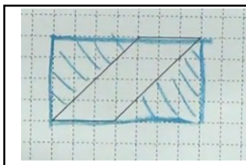
Mr F then invited the students to share their other ideas, and the following four ideas (Fig. 8.1-8.4) were presented.



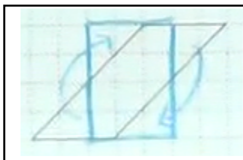
3×2×2
Figure 8.1



6×2
Figure 8.2



4×7-(4×4÷2×2)
Figure 8.3



4×3
Figure 8.4

Here, how to transform the parallelogram into known figures and the relevant formulas were confirmed. Then, Mr F classified these ideas into two categories, 'parallelogram based' (Figures 8.1, 8.2) and 'rectangle based' (Figures 8.3, 8.4).

Seventh scene: Class discussion (3): Rethinking the goal

Mr F then proposed a rethinking of the main goal, and asked the students again what the height of the shape was. The students answered that it was 4, but they were not confident about their answer. Here, Mr F told them to reflect on the idea shown in Figure 7, saying together with the students, "The height of the smallest one is 1 cm, the height of the parallelogram one step higher is 2 cm…" while circling each parallelogram as they spoke (Fig. 9). Moreover, he modified the table by changing the word 'step' to 'cm' and newly adding cm2, indicating the area of each smaller shape (Fig. 10). As a consequence, the students could reinterpret one 'step' as 1 cm of height and then understand that the area formula that they already knew was actually applicable to all parallelograms.

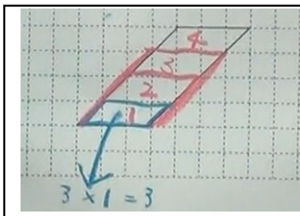


Figure 9

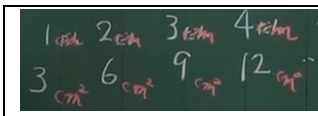


Figure 10

The lesson ended by applying the formula to other figures and summarizing the main learning points of the lesson. This was the final, eighth scene of the lesson.

RESULT 2: THE CASE OF MS Y'S LESSON

Ms Y's lesson followed the 5 steps of the typical Japanese teaching 'pattern' as identified by Stigler et al. (1999). However, a crucial difference from the class conducted by Mr F was that the students did not experience any conflict and did not share a common, explicitly stated learning goal. Ms Y simply presented the problem of 'finding the area of a parallelogram with base BC' (Fig. 11). She tried to prevent the students from considering side CD as the base, but this resulted in the following interactions.

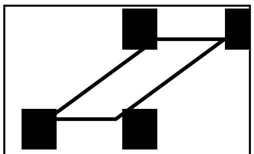


Figure 11

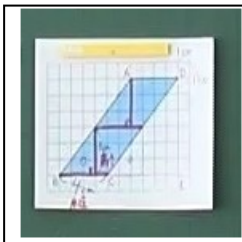


Figure 12

Student 5: I cut it horizontally and made it into two parallelograms. The formula is thus 2 of 4×3. It is 12×2, so the answer is 24 (Fig. 12).

Student 6: But, in the upper parallelogram the middle line changes to become the base.

Student 7: Yes, student 5 is wrong, because we must set BC as the base.

In fact, a similar series of interactions occurred twice. Ms Y did not try to redefine the goal, as was seen in Mr F's lesson. The lesson thus progressed in a disconnected way with respect to Ms Y's original intention and the students' actual thinking processes.

RESULT 3: THE CASE OF MR S'S LESSON

Mr S's lesson also followed the previously mentioned five teaching steps. However, we found two main differences in comparison with Mr F's lesson. First, the units of interaction often never exceeded one return consisting of the teacher's questioning, a student's response, and the teacher's approval. Additionally, one interaction unit was often not connected meaningfully with another. Indeed, Mr S often used the expression "and then" when shifting between scenes.

The second difference was related to subject matter knowledge regarding height. In Mr F's case, the height was reconstructed by reflecting on how many parallelograms of 1 cm height were stacked up together. On the other hand, in Mr S's lesson, the height was summarized as the length of the segment which lies at a right angle to the base, similar to the length of the pillar of a house. We believe that these differences had substantial effects on the students' ability to understand the height of the parallelogram; indeed, some of the students in Mr S's class asked him "So, in the end, what is the height in this case?" at the last scene of the lesson when the main points were summarized.

DISCUSSION

To discuss how lesson coherence can be produced, here we take Mr F's lesson as an exemplary case and compare it with those of Ms Y and Mr S. While all three lessons went through the five steps identified previously as the Japanese pattern, we observed that the eight scenes comprising Mr F's lesson formed a coherent plot: 1) Reviewing the formula for the area of a parallelogram; 2) Setting a problem through an experience of the conflict; 3) Setting a goal by comparing what is known and what is unknown; 4) individual activities and redefining the goal; 5) Sharing the fundamental idea (Class discussion); 6) Sharing various ideas (Class discussion); 7) Rethinking a solution for the goal (Class discussion); and 8) Applying the formula to other problems and summarizing the main point(s).

One characteristic of Mr F's class was that one scene was recursively developed based on the previous scenes. For example, setting up a problem in the second scene was based on the preparations performed in the first scene; similarly, setting a goal in the third scene was conducted by comparing the problem (unknown) in the second scene with the known parallelograms and the formulas discussed in the first scene. Thus, we believe that this recursive characteristic represents a crucial aspect of coherence.

A second characteristic consisted of the students' experiences of the conflict between what was known and what was unknown, including the goal-setting activity for coping with the conflict and the final attainment of the goal. These all combined to make the lesson into a coherent story. Without such goal-setting, Mr Y's lesson would not have been a well-formed, coherent story, and as a result the students may have tried to refute the correct method of finding the area of the parallelogram.

A third characteristic is that MKT based on KDU (Silverman and Thompson, 2008) increased coherence because the lesson was developed around the idea of how many parallelograms of 1 cm in height would need to be stacked. It seems that Mr F had understood beforehand that the idea would help lead the students to understand the formula for finding the area of parallelograms. This contrasts with Mr S's teaching, in which the height was summarized as just the length of a segment in his class.

Lastly, to focus on developing a sense of lesson coherence, we propose the term "living theatre" as a more appropriate metaphor. From this perspective, we can interpret the actions of Mr F to get as many students involved in the lesson as possible as his way of constructing a living theatre, with the students as the main actors (or role-players) on the classroom 'stage'. In addition, the teacher is also one of the main characters in this theatre; in this case Mr F began the lesson with a story describing his problem in trying to find the height of the parallelogram. We believe that such a spirit is the very nature of successful efforts to construct coherence in mathematics lessons.

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