IMPROVING PROBLEM POSING CAPACITIES THROUGH INSERVICE TEACHER TRAINING PROGRAMS: CHALLENGES AND LIMITS

Ildikó Pelczer[1], Florence Mihaela Singer[2], Cristian Voica[3]

[1]Concordia University, Canada, [2]University of Ploiesti, Romania, [3]University of Bucharest, Romania

The paper presents the results of a study based on a training program for in-service mathematics teachers, targeting to improve their skills of problem posing and qualitative appreciation of problems. During this training program, we found an improvement in participating teachers' availability to discuss and analyse math problems, but also resistance to adapt posed tasks to the students' thinking.

INTRODUCTION

The knowledge unique for an effective teaching of mathematics has been a research focus in mathematics education ever since Shulman (1986) introduced the concept of pedagogical content knowledge. Ball, Thames and Phelps (2008) further refined Shulman's initial framework and proposed a structuration of mathematical knowledge for teaching into components as common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT). From these, the SCK represents the mathematical knowledge needed for teaching and it is needed in tasks as "modifying tasks to be either easier or harder, finding an example to make a specific mathematical point" (Ball et al., 2008, p. 400).

Ball et al. were primarily interested in operationalizing the acquisition of SCK in pre-service teacher training. However, research shows that task adaptation - as a form of problem posing – is a challenging activity for in-service teachers, independently of their teaching experience (Silver, Mamona-Downs, Leung, & Kenney, 1996). A second factor adding on the complexity of the in-service teachers' case is the fact that many of the teachers obtained their initial training under a different educational paradigm. A third element worth mentioning is the teachers' in-depth experience with curriculum materials, textbooks and evaluation systems. This experience can lead to situations where teachers assign differentiated importance to elements of the content to be taught, whether this is expressed in problem types, concepts or strategies for problem solving and thus limit their adaptations. Consequently, professional development programs for in-service teachers aiming to create conditions for acquiring or refining their SCK might require a completely different approach from the one employed in the case for pre-service teachers.

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In this paper, we look at an implementation of a teacher-training program and analyse the impact (benefits and limitations) of its design on in-service teachers' ability to pose multiple-choice problems relevant for students' learning with understanding.

In particular, the questions we ask in this paper are: Which elements of an in-service teacher training program might prove to be useful in helping teachers improve their problem posing skills? Which aspects of problem posing (PP) might prove to be effective to achieve this goal? More specifically, our hypothesis was the following: if, based on structured strategies, we systematically expose teachers to PP contexts, then their willingness to discuss and analyse problems will increase, and their ability to pose and solve problems focused on students' understandings will improve.

BACKGROUND

In-service teacher training programs (ITTP), or professional development programs, are seen as part of the teachers' learning process throughout their career (Broad & Evans, 2006). Their purpose is to provide, between others, the context for acquiring deep and broad content knowledge and knowledge about teaching and learning. Traditional ITTP were organized as "formal, highly structured activities outside the context of teachers' actual work" (Schlager, Fusco, & Schank, 1998, p. 2). However, Broad and Evans, synthesize some characteristics of effective ITTP that contrast traditional ITTPs. Based on their literature review, the authors enumerate the features of ITTP that makes them useful: they link teacher and student learning; they must be personalized, and, the key element for its success is collaboration, shared inquiry and learning from and with peers. Under these conditions it is more likely that teachers will adopt the newly learned approaches in their classroom teaching.

A second aspect relevant to our paper is problem posing (PP), in particular posing multiple choice questions. Literature on PP in mathematics education has increased significantly in the last two decades. Here we adhere to the definition given by Silver et al. (1996) according to which PP is defined as the creation of new problems or modification of an existing one. As far multiple choice problems are concerned, the literature consists mainly of tips, hints and recommendations on how to build distracters with no indications for a more systematic approach. The task of creating multiple-choice questions is challenging since the distracters need to consist of answers to which the teacher can (clearly) associate an interpretation in terms of the student's knowledge and understanding. However, hint and feedback formulation is a challenging task for teachers (Singer & Voica, 2013).

METHODOLOGY

Participants

The sample used in this experiment consisted of 51 in-service mathematics teachers at junior and high level, participants in a training program. The ITTP was organized by the authors of this paper and consisted of 5 days training within a summer institute. The purpose of the training was to enhance teachers' assessment related competences. More specifically, the training targeted at improving teacher's competency to pose multiple-choice type problems and analyse them based on a set of criteria.

Tasks and design of the study

This study is based on detailed observation of the participants' behaviour during discussions and their written work submitted during the training period. During the program, overall organized as a sequence of interactive workshops, participants had to solve a series of tasks such as: Discussion and comparison of problems' formulation from the point of view of mathematical coherence and consistency, degree of difficulty and their usefulness in teaching particular concepts; Formulating distracters for a given problem; Modifying problem elements such as data, the problem question or the distracters; Formulating hints and feedback for choosing certain distracter. By the end, participants were required to pose a multiple-choice problem along with: feedback to students on each distracter, a hint for solving the problem and two modifications of the initial problem (one easier and one more difficult). The problems were presented and their qualitative aspects discussed in-group, during a final session.

For each task performed during the training program, participants completed work sheets where they noted the solution of the tasks, further proposals originating from their colleagues and comments/observations raised during the group discussion. All these files were scanned and posted on an e-learning platform associated to the training program. The final session, during which each participant presented his/her problem in front of the group, was audio and videotaped.

We analysed, holistically, the problems generated during this workshop and the whole group discussions related to them. In this analysis, we looked at various aspects, as: the didactic potential of the proposed problems – comparisons with easier and harder version included; the quality of the feedback given on distracters; quality of hints for solving the problem; the focus of the collectively conducted analysis. Here we present a qualitative study of those data.

RESULTS AND DISCUSSIONS

The ITTP presented few challenges. We see these challenges originating from two sources. On one hand, in Romania there are no specialized programs on PP and, consequently, this approach is a novel offer in ITTPs. On the other hand, many of the participants in our study were supervisors involved in the organization of mathematics Olympiads. The specific goal of these contests is to select highly trained students by facing them with difficult/advanced problems. In contrast, a teacher is more interested in advancing their students' understanding through the problems they offer them. From this point of view, Olympiad problems are more performance oriented, while the problems that teachers need in their classroom teaching are learning oriented. The need to change participants' vision about the finality of problems proved to be a challenging task for the organizers at the beginning of ITTP.

The training program: challenges and failures

We started the program with the optimistic expectation that teachers' competency in posing and solving problems (by having as reference point students' understanding) will improve. However, the conclusions must be nuanced.

The ITTP aimed, among other goals, to develop a self-reflexive attitude of the participants in what concerns the quality of the proposed/chosen problems for class work. In order to create the circumstances for the development of a reflexive attitude, the workshops were designed to be interactive, with systematic feedback from peers and instructors. Our impression, as instructors, from the workshops was one of real progress in participants' ability to pose and analyse problems from the point of view of its affordances to promote student learning with understanding. However, when the participants performed their final products, in an individual manner, their old conceptions proved stronger in influencing the PP process than group discussions.

In the following, we present the analysis of some components of trainees' posed problems; the selection highlights key-points of recurrent situations and each example is representative for the general case.

Example 1. The following problem was proposed by M:

Consider the following sequence of numbers: $a_1 = \frac{7}{3}$, $a_2 = \frac{11+2\sqrt{7}}{\sqrt{7}+2}$, $a_3 = \frac{17+\sqrt{70}}{\sqrt{10}+\sqrt{7}}$. *Then*, $a_4 = \ldots$

In solving the posed problem, M starts from the following sequence of equalities: $a_2 = \frac{11+2\sqrt{7}}{\sqrt{7}+2} = \frac{7+4+\sqrt{7}\cdot\sqrt{4}}{\sqrt{7}+\sqrt{4}} = \frac{\sqrt{7}^3 - \sqrt{4}^3}{7-4}$. M considers as natural the idea that, after processing as above the term a_2 and observing the other two given terms, the solver could "guess" the following rule: $a_n = \frac{\sqrt{3n+1}^3 - \sqrt{3n-2}^3}{3}$. Therefore, he can identify the "correct" answer ($a_4 = \frac{13\sqrt{13} - 10\sqrt{10}}{3}$).

During the analysis of the problem's elements (formulation and distractors; feedback to solver; hint for solving), we have not witnessed even a minimal concern for the solver. In designing the problem, its author has not thought of how the problem might be understood or seen by others. It seems that the poser started from a pre-existing idea he wanted to put forward, and the concern for the solver was hindered by an attitude such as: "If I thought about it, others will also do."

Such behavior was not a singular one; several participants displayed a proudness stemming from the fact that they proposed competition problems which could be solved by very few students. In quite a few problems proposed at the end of the ITTP we could identify the same attitudinal pattern.

Yet, teachers' attitude has to be considered in perspective: in a culture dominated by school competitions aiming at strict mathematical performance, problem solving often is reduced to a formal game with mathematical concepts.

Example 2. The following problem was proposed by D:

Mircea and Cristi have together 28 years. Victor and Mircea have together 26 years. Andrew is half of Cristi's age. Victor, Mircea and Cristi were together 42 years. Which of the four boys is the second, if they are considered in increasing order of the ages? a) Mircea b) Victor c) Cristi d) Andrei e) It is impossible to know.

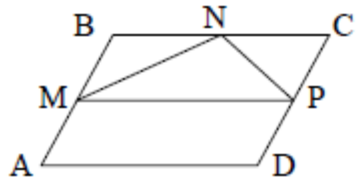
The background topic of this problem is an artificial one, being only used as a kind of "cover" for a system of linear equations. However, as such, it is just a reflection of the kinds of problems often encountered in the textbooks.

We focus another aspect now. The feedback to student on answer e) was formulated, as "The answer is absurd." If we evaluate the feedback from the point of view of the usefulness for the student, we have to conclude that this is not informative at all. The feedback doesn't help students to realize what was wrong in their solving. For example, if the student fails to translate the problem into a system of equations and solve the system, then for that student it is really impossible to know the answer! A better choice for this distracter might be: "all the four persons have the same age": a minimal understanding of the meaning of the given data could lead immediately a solver to decide that this answer is (really) absurd.

In fact, the feedback should be informative to the student; and teachers' activities should start from imagining difficulties students might encounter in contrast with mistakes they could commit. In this respect, comments like "your solution is not right" or "you are wrong" (seen relatively often in proposals of the participants) are useless for the solver and might have a negative impact on their confidence. Comments as such made by D (and others in our sample) might originate from a traditional view of teaching: where the teacher "delivers" methods and content and, thus, the sole responsibility for failure is on the student who "didn't try enough".

Example 3. V proposed the following problem:

The parallelogram from the next represents a garden, whose area is 24 dam^2. M, N, P are the mid-points of the sides AB, BC and CD. The area of MNP (cultivated with flowers) is equal to:



a) 6 b)3 c)9 d) 12 e)18.

V's intention was to propose a "realistic" problem. In his conception, the problem should get this characteristic just from using words that are from everyday use. However, the problem is purely mathematical: he even uses a geometric drawing and geometrical terms (midpoint).

From a mathematical point of view, the problem is correctly formulated and might be a useful problem for students studying area in context of special quadrilaterals. However, the quality of problem formulation, due to the mixture of language, is poor. What might explain the participants' inability to "see" this aspect of the problem? We hypothesize, again, that their experience as teachers is marked by strong focus on content (and the reasons for this are so varied that we shall not dwell on them at this point). Therefore, they perceive the problem already beyond its formulation: as to what it is once in pure mathematical terms. Our hypothesis is sustained by the fact that, once the problem is subject of discussion, they realize the shortcomings – however, when they pose the problem they seem completely "absorbed" by the mathematical aim of it.

The above examples reveal some important shortcomings in teachers' PP, such as: certain "blindness"/"short-sightedness" about problem formulation; un-informative feedbacks; artificial problem contexts and failed attempts to connect to everyday situations. These aspects were recurrent in most of the participants' problems.

In comparison with a relative isolation at the beginning, progressively during the workshops, participants expressed their willingness to engage in discussion of the problems proposed by peers and by themselves.

Their availability for involvement in the problem analysis process could be assessed through the dynamics of interventions during the workshops, but also through records from the final assessment, where the participants pointed out the usefulness and relevance of ITTP for the work of the teacher in a class.

We insert below some comments of participants from this last category.

> *"Even after 30 years of teaching, I learned a lot."*
>
> *"Most interesting parts were the discussions about changing distracters."*
>
> *"The part of maximal interest was about distracters and feedback to student."*

If at the beginning, most teachers did not want to expose their products to group discussion (mainly of fear of value judgments), towards the end we witnessed participants' openness in this regard and even a desire to get feedback on personal creations. Furthermore, we found that by the end of the seminar, the quality of problem analysis has improved. If at first discussions often slipped to collateral subjects, towards the end the themes addressed were converging towards key aspects of PP. Moreover, even the supervisors - initially reluctant to the idea of a training program with focus on PP - became actively involved in the process; not as much eager to share their own experience, but to learn more about the proposed methodology. We selected one more point that further illustrate the nature of discussions generated during the analysis of the posed problems.

A participant, M, suggested the problem below, which generated a discussion about the possibility of applying them to different classes:

Find out how many numbers in the sequence: 1, 4, 7, 10, ..., 301, are divisible by 5.

In presenting his solution, M used an algebraic method, which involves writing the numbers from the sequence as $3k + 1$ and identifying the numbers of the given form that are multiples of 5. Then, M indicated a second method of solving that goes more towards exploration: writing more terms of the sequence, one can observe that in each group of five consecutive terms, exactly one is divisible by 5.

Based on these solutions, participants discussed the possibility of proposing this problem for different classes, how distracters can be adjusted, or how the problem's difficulty level can be controlled. For example, the discussion revealed that a more difficult problem might be: "*Find out how many numbers in the given sequence are divisible by 5 or by 2.*" On this proposal, discussions continued on the possibility to adapt the initial solutions to the new problem. More specifically, it was concluded that it can be solved by the principle of inclusion and exclusion, or by an exploratory approach (in any sequence of 10 consecutive terms of the series, there is the same number of terms divisible by 2 or 5). Therefore, in the context of the ITTP sessions, even a "classical" problem led to relevant discussions on its exploitation in different class contexts.

Other presentations have also generated extensive discussions, touching aspects of the nature of the context of a problem, its veracity, correctness in a strict mathematical sense, and the possibility to correlate mathematical correctness and the need to create attractive contextual problems.

CONCLUSION

In this paper, we presented a study on the benefits and limitations of an ITTP as teachers' PP skills of multiple-choice problems are concerned. Two major aspects were identified. On the one hand, we observed a certain resistance from the behalf of teachers to shift in their problem posing process towards interpretations of students' thinking. Next, we synthesized the different manifestations of this resistance. During problem posing, teachers gave a superficial attention in formulating the problem: often, the problem formulation is elliptic or full of ambiguity, while background topic is irrelevant for students' motivation. Some formulations reflect a cognitive behaviour of the type: "if I had thought of this, surely the student will have the same idea". We interpreted such case as one of "blindness/short-sightedness" of the problem poser since it prevents him/her from seeing the problem objectively, from the readers' point of view. This attitude caused strong resistance to any suggestion of a need for change.

On the other hand, we document a change of participants' behaviour that happened on two dimensions, both observable in group interactions: an openness to discuss and analyse the quality of own posed problems, as well as a capacity to focus on some key-aspects of PP. The training program, thus, contributed to the development of a reflexive attitude as far as problem appreciation is concerned. These results illustrate that our starting hypothesis was only partially confirmed and that it needs further refinement.

We found a positive impact on group interaction, while individual products still exhibited thinking patterns linked to traditional views of teaching and learning. We interpret this situation as indication for changes to be brought to the training program. It seems that a follow up that systematically combines group interactions with individual tasks exploited during further interactions could significantly influence a PP approach focused on students learning with understanding. An ITTP built on such design principle might be a topic for future research.

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