THE INTERPLAY BETWEEN LANGUAGE, GESTURES, DRAGGING AND DIAGRAMS IN BILINGUAL LEARNERS' MATHEMATICAL COMMUNCIATIONS

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This paper provides a detailed analysis of the mathematical communication involving a pair of high school calculus students who are English language learners. The paper focuses on the word-use, gestures and dragging actions in the student-pair communication about calculus concepts when paper-based static and then touchscreen dynamic diagrams. Findings suggest that the students relied on gestures and dragging as multimodal resources to communicate about dynamic aspects of calculus. Moreover, examining the interplay between language, gestures, dragging and diagrams made it possible to uncover English language learners' competencies in mathematical communications. This paper points to an expanded view of bilingual learners' communication that includes gestures, dragging and diagrams.

INTRODUCTION

The goal of my research has been to extend Moschkovich's (2007) sociocultural view of bilingual learners, to "uncover" bilingual learners' mathematical competencies when they communicate about significant calculus concepts. Although some research has shed light on bilingual learners' non-linguistic forms of communication such as gestures and diagrams (Gutierrez, Sengupta-Irving, & Dieckmann, 2007; Moschkovich, 2007, 2009), this work has not addressed the use of digital technologies, and dynamic geometry enviornments (DGEs) in particular—which have been shown to facilitate student communication by providing visual and dynamic modes of interaction (Ferrara, Pratt, & Robutti, 2006; Falcade, Laborde and Mariotti, 2007)—and the interplay between these multimodal resources for analysing bilingual learners' mathematical communications.

The current research questions concern the kinds of multimodal resources that bilingual learners use to communicate about certain calculus concepts using a touchscreen-based DGE. In particular, I investigate:

1. What characteristics of communications, and what kinds of *mathematical discourse practices* (Moschkovich, 2007) do bilingual learners engage in, when working with touchscreen-based DGE?
2. How may this analysis uncover bilingual learners' competencies and resources in mathematical communications?

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THEORETICAL FRAMEWORK

Moschkovich's (2007) sociocultural view of bilingual learners questions the efficacy of the *vocabulary* and *multiple meaning* perspectives for understanding bilingual mathematics learners because such perspectives focus on what learners don't know or can't do. The vocabulary perspective views the acquisition of vocabulary as a central component of learning mathematics for bilingual learners. The multiple meaning perspective focusses on learning to use different meanings appropriately in different situations. In contrast to these deficit perspectives, the *sociocultura*l view focuses on describing the resources that bilingual learners use to communicate mathematically.

The sociocultural view draws on a situated perspective of learning mathematics. From this perspective, learning mathematics is a discursive activity "that involves participating in a community of practice, developing classroom socio-mathematical norms, and using multiple material, linguistic, and social resources" (p. 25). In the sociocultural lens, bilingual learners are seen as participating in *mathematical discourse practices*—practices that are shared by members who belong in the mathematics or classroom community. In general, "abstracting, generalising, searching for certainty, and being precise, explicit, brief, and logical are highly valued activities across different mathematical communities" (p. 10). Moschkovich argues that analysing the extent and type of mathematical discourse practices can highlight the competencies of bilingual learners: "even a student who is missing vocabulary may be proficient in describing patterns, using mathematical constructions, or presenting mathematically sound arguments" (p. 20).

Complementary to Moschkovich's sociocultural view of bilingual leaners, I adopt Sfard's (2008) communicational theory, which conceptualises learning as a change in one's mathematical discourse. Sfard's approach highlights the way in which thinking and communicating (for Sfard, this includes talking and gesturing) stop being but 'expressions' of thinking and become the process of thinking in itself. In terms of bilingual learners' use of multiple resources in their mathematical discourse, Sfard (2009) suggests that utterances and gestures are two modalities that serve different functions in the thinking-communicating process. Namely, gestural communications ensure all interlocutors "speak about the same mathematical object" (p. 197). Moreover, gestures and diagrams are forms of visual mediators that learners may utilise as resources in mathematical discourse. Although Sfard has not adequately addressed the distinction between dynamic and static gestures, diagrams and visual mediators in general, the distinction is important for this paper because of the potential for the dynamic visual mediators in DGEs to evoke temporal and mathematical relations in calculus concepts. In addition, bilingual learners who are still grasping the English language may draw on dynamic visual mediators such as gestures and DGEs as multimodal resources to communicate.

In summary, I use Sfard's communicational theory to analyse bilingual learners' thinking as they communicate about calculus concepts given two types of visual mediators, static and dynamic. I focus on their word use and gestures as features of their mathematical discourse and their mathematical discourse practices within the activities. This enables me to analyse the interplay of resources situated in their use of static and dynamic diagrams and to uncover their competencies in mathematical communications.

METHODOLOGY OF RESEARCH

The participants of the study were three pairs of 12th grade students (aged 17 to 18) enrolled in two sections of the AP Calculus class in a culturally diverse high school in Western Canada. The participants were selected for their relatively low English ability–all of them have only been studying in Canada in an English-speaking schooling environment for two to three years. The detailed data analysis that follows focusses on one pair of bilingual learners, Ana and Tammy, whose native language is Mandarin and who had the lowest English language ability amongst the three pairs.

The study took place at the end of the school year in the participants' regular calculus classroom, outside of school hours. At the time, the participants had just finished enrolling in a year-long AP Calculus course where key concepts in calculus were taught using an iPad-based DGE called *Sketchpad Explorer* (Jackiw, 2011). Therefore, the students have experienced with exploring and discussing, in pairs, concepts such as the definition of a derivative, derivative functions, related rates, and the Fundamental Theorem of Calculus through geometrical, dynamic sketches.

Each pair of participants was asked to discuss ten different diagrams—five static diagrams shown in PDF form and then five dynamic diagrams presented in *Sketchpad Explorer*. The five static diagrams (see Figure 1a) were taken from students' regular calculus textbook (Stewart, 2008), and the five dynamic sketches (see Figure 1b) were minimally adapted from the ones that the students had used in class during the school year. For the purpose of comparing patterns of communications, each of the five static diagrams had a corresponding dynamic sketch that involved the same target concept. After giving the instructions, the researcher turned on the camera located in front of and facing the student-pairs, and then left the room, until the students finished talking about all the diagrams. Each student-pair took around 25 minutes to complete the task.

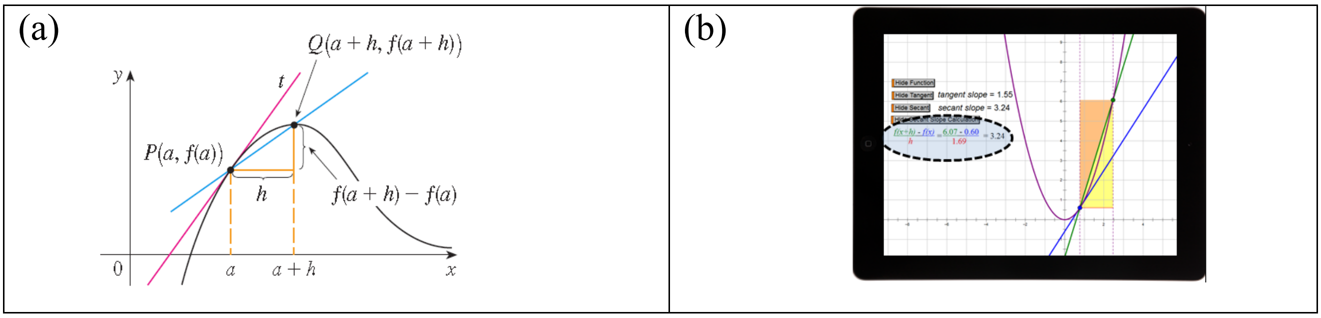


Figure 1(a): A static and (b): dynamic diagram conveying the definition of a derivative.

Figure 1b shows the screenshot of the dynamic sketch related to the definition of a derivative (with *Hide/Show* buttons "show function", "show tangent", "show secant" and "show secant calculation" all activated). As either point on the secant line is dragged, the corresponding numerical values of the tangent slope, secant slope, and secant slope calculation, *[f(x+h)–f(x)]/h*, are displayed with each value colour-coded.

ANALYSIS OF DATA

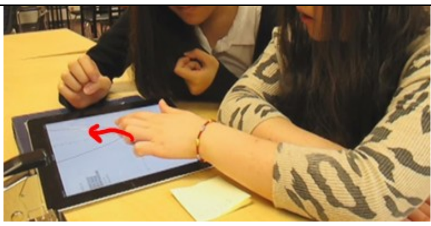
Below, I provide a detailed analysis of Ana and Tammy's discussion around the dynamic sketch described above, relating to the definition of a derivative. Prior to the episode, the students have already talked about the corresponding static diagram; some key analysis of that episode is discussed alongside. I divided the episode into two parts, each beginning with a transcript, for the purpose of identifying themes in each part.

Episode Part 1: Interplay between language, gestures, dragging and diagram

1	T:	From zero to positive <*Dragging/gesture 1s start* (Figure 2a)>, the slope is…
2	A:	The tangent line is increasing.
3	T:	Tangent line is increasing <*Dragging/gesture 1s start end*>. And from here to zero, it's decreasing.
4	A:	<*Dragging/gesture 2s start (Figure 2b)*> And at zero, the tangent line is zero. <*Dragging/gesture 2 end*>.

(a)

Dragging/ gesture 1s:



(b)

Dragging/ gesture 2s:

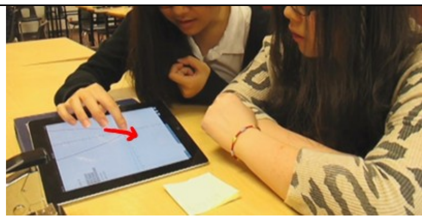


Figure 2 (a) and (b): T and A's dragging and gesturing actions in Episode Part 1.

When the students opened the sketch, two buttons were already in the "show" position; therefore, the graph of a parabola, $y = x^2$ and its tangent line at a given point appeared on the sketch. Ana and Tammy explored the dynamic sketch using the dragging modality. In the first exchange, Tammy's utterances, "tangent line is increasing" (line 1) was accompanied by dragging the point of tangency from left to right (dragging 1s), although technically it was the tangent *slope* that was increasing and not the tangent line. Following that, Ana seemed to mimic Tammy's utterance/dragging combination with "the tangent [slope of the] line is zero" (line 4) while she performed a similar dragging action to move the point of tangency towards the vertex (dragging 2s). These are two of the five series of dragging actions spanning between 2 to 5 seconds observed in the episode.

A further analysis suggests that these dragging actions were not merely dragging but also gestural communications—to communicate the dynamic features and properties in the sketch as obtained by dragging. To illustrate why the dragging actions are also considered gestures, it would be possible to imagine a static environment where the dragging modality is not available. If a speaker moves his/her finger along a graph while referring to the tangent slope as "increasing" or "decreasing", this action can be considered a kind of dynamic gesture for communicating the idea, "*as x varies along this graph*". In the current episode, the dynamic environment allows the dragging with one finger on the touchscreen and the gesturing with the index finger to blend together as one action. Hence, I refer to this action as *dragsturing*. The importance here is that *dragsturing* is one action subsuming both dragging and gesturing characteristics, in that it allowed the point to be moved on the screen (dragging), and it fulfilled a communicational function (Sfard's definition of gesturing). My purpose here is not to objectify an action but to present the dual functions of dragging and gesturing in the *dragsturing* action for analysing the students' thinking-communicating process. Prior to this, Ana and Tammy had used deictic gestures, complemented by words like "this" and "here", for naming various mathematical objects when discussing a static diagram.

Furthermore, during the first exchange, Tammy used phrases "is increasing" and "is decreasing" to describe the tangent slope. Her utterances were accompanied by her *dragsturing* (dragging/gesture 1s) which was immediately mimicked by Ana (dragging/gesture 2s). The use of the present continuous tense "is [verb]–ing" was a change from their previous discussion over a static diagram, where the girls used the verb form "is [noun]" four times when discussing the same topic. The word use "is increasing" and "is decreasing" were accompanied by dynamic *dragsturing* to *communicate* the change of tangent slope as the point was being dragged. This shows the interplay between *dragsturing*, language and diagrams in the two students' discourse. Thus, in the present episode, dragging and gesturing transformed the way Ana and Tammy communicated about the tangent slope. The verb forms suggest that "something is happening" at the very moment. This analysis is made possible by studying the interplay between *dragsturing*, language, and diagrams in the students' mathematical communication.

Episode Part 2: Engagement in valued mathematical discourse practices

A: <*T presses "show secant" button*> Secant. <*A presses "show secant calculation" button.*>

T: For… <*A starts performing dragging/gesture 3s (Figure 3a)*> if you want to get the secant line… you have to find two points to, to, <*A's dragging/gesture 3s ends and immediately starts performing dragging/gestures 4s (Figure 3b)*> calculate change of *y* and change of *x*.

A: I think, when the two points get closer <*dragging/gesture 4s end*>, the tangent line is… there is less different between the tangent line and secant line. <*T starts performing dragging/gestures 5s (Figure 3c)*>

T: And <*dragging/gestures 5s ends*> they will be together.

A: And if there are the same point, they will be the same, the two lines.

(a) Dragging/gesture 3s: (b) Dragging/gesture 4s: (c) Dragging/gesture 5s:

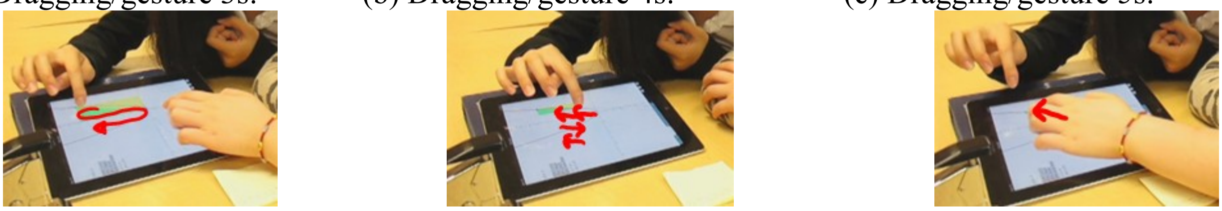


Figure 3(a),(b), and (c): T and A's dragging and gesturing actions in Episode Part 2.

As the episode unfolds, Ana and Tammy began to explore the other two *Hide/Show* buttons, and continued to drag the points. They moved from discussing procedures to talking conceptually about the definition of a derivative. This can be observed through the evolution of different mathematical discourse practices they engaged in. Upon *exploring* the change of tangent slope in the early part of the episode, Tammy suggested that "if you want to get the secant line… you have to find two points to, to calculate the change of *y* and change of *x*" (lines 9 to 11). At this point, Tammy's mathematical discourse practice focused on *calculating*.

However, the students' talk did not end with a formula as observed in the static environment; Tammy's *calculating* was followed by Ana's *comparing*, evident in her word use "closer" (line 12) and "less different" (line 13) to describe the state of the two lines when the tangent approaches the secant. Her comparing led to *predicting* and *generalising* about the tangent line in Tammy's "the two points *will be* together" (line 15) and Ana's "they *will be* the same, the two lines" (line 16). The use of the future tense in "will be" in both statements indicates that both students had moved from a procedural and algebraic way of thinking about derivative to a conceptual and geometric one. Tammy's *dragsturing* (dragging/gesture 5s) at the end to bring the secant line towards the tangent line can be taken as confirming her *generalization* that the two slopes will eventually be the same.

Out of the sociocultural view, the vocabulary perspective would criticise Tammy for incorrectly stating that "*tangent line* is increasing... and from here to zero, it's decreasing," (line 2) in the earlier part of the episode when it is really the *tangent slope* that is changing. Likewise, the multiple meaning perspective would point to Ana's inability to grasp the meaning of "function" later in the episode. Hence, neither perspective would view Ana and Tammy as engaging in valued mathematical discourse practices like *comparing*, *predicting* and *generalising*.

Since gestures are taken as communicational acts in Sfard's term, it was interesting to observe that the girls incorporated gestures in responding to each other. For example, while Tammy talked about the two points on the secant line, Ana was *dragsturing* the points on the secant line around, which seemed to be responding to Tammy's utterance. Then, the two exchanged roles when Ana suggested that the secant line will get "closer" to the tangent line. Tammy seemed to have responded by her *dragsturing* to bring the lines "together". These gesture-utterance correspondences were noted in the analysis of other pairs of bilingual learners' conversational pattern involving dynamic sketches as well.

DISCUSSION

The detailed analysis provides strong evidence that bilingual learners utilised a variety of resources, including language, gestures and visual mediators in their mathematical communication—with gestures taking on a prevalent role. These included deictic gestures accompanying static visual mediators as well as dynamic gestures for communicating temporal relationships such as the "change of *x*". Moreover, a new form of gesture emerged in the touchscreen dragging action with the dynamic diagrams. These *dragsturings* fulfil the dual function of dragging and gesturing.

The presence of dragging and gestures transformed word use. As illustrated in the episode, Ana and Tammy resorted to verb forms that imply motion while they used dragging to change the tangent slope. This was a change of verb-form from their earlier discussions around the static diagrams, where the students used the "is [noun]" form to communicate a static sense of calculus ideas. In a sociocultural view, the bilingual learners engaged in significant mathematical ideas on both static and dynamic environments, but they participated in *different* mathematical discourse practices. With the static diagrams, the students communicated about calculus procedurally by defining mathematical objects and developing a formula for tangent slope. With dynamic diagrams, their communication was characterised by comparing, predicting and generalising practices, as shown in the episode. The analysis is made possible by studying the interplay between word use, dragging, gestures and diagrams. I argue that these elements must be accounted for in the full set of resources that bilingual learners utilise in mathematical communication. As Sfard (2009) explains, utterance and gestures take on different roles in mathematical communications. I would go further in suggesting that language, gestures, and diagrams serve *complementary* functions in mathematical communications.

New conversational patterns were introduced by the students in the current episode. With a static visual mediator, the students mainly communicated with utterances accompanied by deictic gestures. This conversational pattern evolved in the presence of *dragsturing* over a dynamic visual mediator, where gestures-gestures and gestures-utterances sequences were observed in the conversation. This observation supports that bilingual learners make use of gestures as important forms of communication, and in this case, to respond to each other in mathematical communications. Also in the study, I observed one person *dragsturing* simultaneously as the other spoke; this allowed the two students to communicate simultaneously without interfering with each other. Using Sfard's communicational framework–which defines gestures as communicational acts–is especially useful for understanding the mutual *communications* involved in these new kinds of conversational patterns.

It could be said that the design of the dynamic sketches has a significant role in facilitating students' mathematical communications. The *Hide/Show* buttons allowed the students to talk about their ideas gradually one button at a time, while the *dragging* affordance enabled them to attend to dynamic relationships and connect algebraic with geometric representations of calculus. In tune with previous studies on DGEs-mediated student thinking (Falcade, Laborde and Mariotti, 2007), the students may have communicated about derivatives geometrically and conceptually as they exploited the functionalities offered in the sketch. As Chen and Herbst (2012) contend, "the constraints of diagrams may enable students to use particular gestures and verbal expressions that, rather than using known facts, permit students to make hypothetical claims about diagrams" (p.304).

CONCLUSION

In this paper, I showed that bilingual learners utilise language, gestures, dragging and diagrams as a full set of resources to communicate mathematically. I also addressed the interplay between these resources for uncovering bilingual learners' competencies engaging in significant calculus ideas. In my analysis, *dragsturing* emerged as a new, significant form of communication which gave rise to new conversational patterns. This study points to an expanded view of bilingual learners' communication that includes gestures, dragging and diagrams. In particular, future research should consider examining the kinds of gestures and the interplay of resources, which are situated in the mathematical activities, in order to identify mathematical competencies for bilingual learners.

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