INVESTIGATING STUDENT PARTICIPATION TRAJECTORIES IN A MATHEMATICAL DISCOURSE COMMUNITY

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This paper details the analysis of the participation of individual students in a teaching experiment in which the researcher aimed to facilitate a mathematical discourse community. This involved positioning students as mathematical authorities capable of generating and evaluating mathematical thinking. The extent to which students acted as mathematical authorities was investigated by tracking their participation across a number of lessons. Students' use of discourse community practices such as explaining and justifying thinking, evaluating the thinking of others and asking questions was documented and Wenger's (1998) trajectories of identity were used to describe their participation. The profiles of four students of different achievement levels with contrasting participation practices will be presented and discussed.

INTRODUCTION

In recent years, theoretical and empirical research has amassed which demonstrates the benefits of participation in classroom mathematical discussions (Walshaw & Anthony, 2008). Learning mathematics can be conceived of as becoming a participant in progressive discourse. This follows Sfard's (2001) conception of learning mathematics as developing a discourse and Bereiter's (1994) arguments for science as progressive discourse. Bereiter lists the 'moral commitments' that facilitate progressive discourse. These involve a willingness to work toward common understanding, a willingness to pose questions and propositions so that they can be tested by others, a willingness to expand the set of collectively accepted propositions, and a willingness to subject any belief to criticism in order to advance the discourse. Bereiter also argues that classroom discussions can and should have these characteristics.

One example of his ideas in the context of the mathematics classroom is the Math Talk Learning Community (MTLC) framework (Hufferd-Ackles, Fuson & Sherin, 2004). This was developed as part of a year-long study in an elementary class where the focus teacher successfully implemented reform-orientated, discussion-intensive teaching practice. The framework charts the progress of the class and describes developmental trajectories in the areas of questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility for learning. These trajectories detail changes in teacher and student actions as the class began operating as a discourse community and generally involve devolution of mathematical authority from teacher to students in each of the areas listed above. This results in lessons consisting of community negotiation of mathematical meaning where students' "math sense becomes the criterion for evaluation" of mathematical ideas (Hufferd-Ackles et al., 2004, p. 88).

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The Irish primary mathematics curriculum emphasises mathematical discussion as a key part of a child-centred, constructivist approach (Government of Ireland, 1999). In the Irish context, Dooley (2011) has explored the potential of the discursive approach for harnessing learner agency and devolving mathematical power to students. The aim of my research was to facilitate a discourse community as described by the MTLC framework and to explore the nature of student learning over time in such a community. As such, I was following Mercer's (2008) calls for a renewed focus on the temporal aspects of the teaching and learning process. Analysis of student participation in whole class discussion at group level was conducted using the MTLC framework (Hufferd-Ackles et al., 2004) and has been discussed elsewhere (NicMhuirí, 2012; 2013). This analysis of the teaching experiment showed that I was successful to some extent in devolving mathematical power to students by positioning them as mathematical authorities capable of generating and evaluating mathematical ideas. However, it also became apparent that the nature of student participation varied from pupil to pupil. For this reason, and to better understand how the discourse community worked in practice, analysis was carried out on the participation of individual students. This analysis will be detailed and the trajectories of four students of different achievement levels and participation styles will be presented.

THEORETICAL FRAMEWORK

The research was conducted from a sociocultural perspective and my focus was on what students might learn from the discourse community approach in terms of transformation of participation (Rogoff, 1994). Dreier's (1999) notion of a trajectory of participation in social practice through both time and space was used as a means of a conceiving of individual students' participation in the discourse community over time. The concept of a community of practice (COP) (Lave & Wenger, 1991; Wenger, 1998) was also used. Engagement in a joint enterprise within a COP requires negotiation and "creates among participants relations of mutual accountability that become an integral part of the practice" (Wenger, 1998, p. 78). The concept of identity is central to theories of participation in a COP and Lave suggests that becoming 'knowledgably skilful' (1993, p. 65) and developing an identity as a community member are part of the same process. Because identity is constantly renegotiated in practice, identities form trajectories within and across communities (Wenger, 1998). Identity is developed in participation with others so the teacher and the classroom community are key influences for students (Grootenboer & Zevenbergen, 2008). It can be argued that within a discourse community, there is a different understanding of what it means to be 'knowledgeably skilful' (Lave, 1993) than in traditional mathematics classrooms. In traditional classes, students are often positioned as 'received knowers' who reproduce teachers' methods (Boaler, 2003) but in a discourse community, students are positioned as mathematical authorities capable of generating and evaluating mathematical ideas. For this reason, it was envisaged that participation in a mathematical discourse community might influence students' mathematical identities.

METHODOLOGY

Design research has been applied to the classroom in the form of the 'classroom design experiment' (Cobb, Gresalfi & Hodge, 2009) which has grown from the teaching experiment research approach. The aim of this classroom design experiment was both the facilitation of a discourse community and the study of this instructional design. Fractions, decimals and percentages were chosen as focus areas as these have been identified as problematic in Irish primary classes (Eivers et al., 2010). The experiment was carried out at fifth class level (10 – 11 years old) with 18 students in a designated disadvantaged boys' school in which the researcher taught fulltime. Schools are designated as disadvantaged by the Department of Education based on indicators of socio-economic status in the population of parents. Lessons were recorded using a digital voice recorder based on their perceived potential for interesting classroom discourse. 'Interesting' should be understood to mean relevant to the research because of predicted participation patterns of students in whole-class discussion. Digital records of board work from the interactive board were also saved. In all, 31 recordings were collected over the course of a school year. Thirteen recordings were transcribed so as to be representative across mathematical topics and over time. The ethical issues of conducting teacher-research in ones' own classroom are complex (NicMhuirí, 2012) but the university mandated guidelines were followed at all times.

There are methodological issues relevant to examining the student experience through time and suitable data interrogation techniques were not easily identified. Mercer asserts that "the same act repeated cannot be assumed to be 'the same' act in repetition, because it builds historically on the earlier event" (2008, p. 36). This creates problems about using coding schemes that do not acknowledge the temporal nature of discourse. This is problematic when considering one student's participation over time but particularly problematic when considering the influence of the community on the individual. The contribution of any individual student may build on the historical contribution of a different student. For this reason, it proved unmanageable to devise a systematic coding scheme that circumvented the embedded nature of student contributions in specific times and contexts. Instead, the transcripts were interrogated to discover to what extent students engaged in practices of the discourse community.

Boaler defines classroom practices as "the recurrent activities and norms that develop in classrooms over time" (2003, p. 3). The key student practices in a discourse community were extrapolated from the MTLC framework (Huferd-Ackles et al., 2004), rather than the empirical study because the community of the classroom design experiment might not exhibit all the practices that were envisaged in the design. The key practice that was envisaged for students was to act as mathematical authorities by engaging in generating and evaluating mathematical ideas as discussed earlier. When investigating the participation of individual students in lesson transcripts, attention was paid to how the student came to speak i.e. invited or unprompted. Similarly the nature of their contributions were studied to determine whether the contribution was mathematically correct or incorrect; the degree to which it was confidently and

coherently stated; whether it built on the solutions of others or came to be built on by others; whether it contained a question for me or for another student; and whether it gave any indication of ability or emotion. In this way, a description of the nature of student participation over time was created.

The resulting participation profile was then examined with reference to Wenger's (1998) trajectories of identity within a COP. These trajectories reflect the positioning of the person within the COP. Wenger presents five types of trajectories: peripheral, inbound, insider, boundary and outbound trajectories. A peripheral trajectory suggests less than full participation in community practices. An inbound trajectory may indicate current peripheral participation but a commitment to future full participation. An insider trajectory indicates full participation in community practices. A boundary trajectory indicates that identity is located in the nexus of communities of practice and an outbound trajectory indicates outward movement from one community to another. The participation of 10 out of the 18 students was investigated. These students represented different achievement levels according to standardised test results.

STUDENT PROFILES

The participation profiles of four students of different achievement levels and participation styles will be presented here. Pseudonyms have been used.

Darragh

While all higher achievers were active contributors to class discussion, none contributed to quite the same extent as Darragh. He consistently, and from the very beginning of the teaching experiment, contributed significant mathematical ideas and vocabulary to class discussions that other students later used. For example, he used the terms 'simplify' and 'equivalent' before I did and explained them to his peers when questioned. His many contributions were confidently and coherently stated and he was mathematically correct more often than he was incorrect. He regularly commented unprompted on the solution efforts of his peers, sometimes building on their suggestions (5 transcripts). In fact, he sometimes interrupted me or other students to share his thinking. He questioned students about their strategies (2 transcripts) and also directly questioned me in two lessons. Questioning of any kind was not a common student practice. It appeared that Darragh had an awareness of his own role and ability and once questioned his peers on whether they understood a mathematical explanation he had offered. On another occasion, he referred to not wanting to "confuse people." He appeared to be alert to the nature of our mathematical activity, commenting when we had discussed at length whether 1/25 is equivalent to 25%, that it had been "a big discussion for a little question." On his own initiative, he once described mathematical links between the activities in different lessons. On another occasion, again unprompted, he followed up on a question I had posed to another student. Darragh appeared to act as a mathematical authority, regularly contributing ideas and engaging in determining what was mathematically correct. In Wenger's (1998) terms, the nature of Darragh's participation could be described as an *insider trajectory*.

Jake

Jake's score on the standardised test placed him in the middle of the range of class achievement. He generally contributed to discussions on my invitation. His contributions, though often mathematically correct, were at times faltering and hard to follow. This is particularly true of his contributions to lessons early in the school year. His apparent increased ability to articulate his thinking after this may be due to the experience gained in teaching experiment lessons. It may equally reflect a greater competence with the later lesson topics. Jake often referenced other students' work sometimes to agree with it or to suggest a new approach when a peer made an error (3 transcripts). On other occasions, he shared significant ideas that went against contributions previously made by his peers (2 transcripts). Some of Jake's contributions were significant both in relation to their mathematical content and their role in shaping the classroom discussion (2 transcripts). He also appeared to be willing to make an attempt at solution and share his ideas when faced with challenging problems. For example, on one occasion the class were attempting to convert 23/25 to a percentage and Jake suggested unprompted that it might be 22 10/10 %. While this contribution shows a gap in his knowledge of percentages, it also shows that he was willing to take risks and attempt to apply some of his previous knowledge about fractions to the new situation. This pattern of participation suggests a genuine effort to act as a mathematical authority. It is tempting to describe the nature of Jake's participation as indicative of an inbound trajectory because of his growing confidence observable across the course of the year. However, he engaged in the practices of the discourse community from the first lesson of the teaching experiment. In this lesson, he displayed high levels of responsibility for learning when he disagreed with previous contributors to present his own understanding of the problem situation. This suggests that a description of *insider trajectory* is more suitable.

Kevin

Kevin was a lower achieving student who regularly displayed a willingness to contribute when invited to do so. His contributions were generally comprehensible but were not always mathematically correct. On a number of occasions, he used language which lacked mathematical precision. For example, he once suggested an alternative solution to sharing pizzas by saying, "You can put one slice in a half to get the same way… but not like one big half." In this case it seemed that he was referring to cutting a half in half but lacked either the mathematical knowledge or language to identify what the result would be. He sometimes commented on the ideas of others, generally to agree with them rather than disagree (3 transcripts). On two occasions his contributions appeared to influence the ideas of others. On the first occasion, he described a pattern he had noticed in a group of equivalent fractions which another student built on in later contributions. On the second occasion, he correctly identified the largest of a group of decimal numbers and when another student suggested that it may be a different number, Kevin successfully explained his reasoning to him. Though Kevin's ideas sometimes lacked the mathematical complexity and precision of some of his peers, his

pattern of participation indicates a genuine engagement in the community. However, the extent to which Kevin acted as a mathematical authority is debateable, particularly in regard to his evaluations of the thinking of others. Although he sometimes was observed to agree with the ideas of others, he was not observed to disagree with his peers or ask questions of their methods or my explanations in whole class discussion at any time. Such actions could be useful learning practices for all children and valuable next steps to a fuller participation in the discourse community for Kevin. I would argue that there is not enough evidence to describe Kevin's participation as an insider trajectory. Instead I would suggest that he may have been on an *inbound trajectory*.

Steven

Steven was a lower achieving student who contributed regularly to whole class discussion both unprompted and by invitation. His contributions were coherently stated but were often not correct and on a number of occasions he struggled with mathematical language (4 transcripts). He frequently admitted to not understanding explanations of mine or of his peers (5 transcripts). In fact, he asked questions in the whole-class setting in 8 out of the 13 transcribed lessons. In one lesson, Steven asked 7 out of the 14 recorded student questions. These questions tended to consist of requests for restatements of explanations rather than questions probing the mathematical content. As mentioned above, this mode of participation is different from other students, the majority of whom posed few if any questions. It appeared that Steven did not attach meaning to the commonly perceived social risk of asking questions or making mistakes. On one occasion, toward the end of the year, despite questions and hints from his peers, he persisted for a number of minutes in attempting an incorrect fraction-subtraction method on the whiteboard. He appeared to appreciate the attention of the class but did not engage with their comments about the mathematics involved in his method. In this sense, Steven's contributions to discussions often appeared to be made with the aim of gaining the attention of the class, rather than with the aim of developing mathematical understanding.

Despite his regular contributions, the nature of Steven's participation appeared to be limited. Like Kevin, the extent to which he acted as a mathematical authority is debateable. While the admissions of incomprehension suggest that he was following the mathematical discussion and self-monitoring for understanding, they also suggest that he may have been struggling with mathematics at the class level and may indicate that he was appealing for more explicit direction in an effort to lessen the cognitive load. His actions suggest that he did not view participation in whole class discourse as a community effort to negotiate mathematical meaning. In fact, there is little evidence to suggest that his identity is invested in future full participation, a necessary condition of an inbound trajectory (Wenger, 1998). For this reason, a description of his participation as a *peripheral trajectory* may be best.

DISCUSSION

The analysis of participation trajectories shows variation in the extent to which students engaged in discourse community practices. Those on insider trajectories, like Darragh and Jake, may have demonstrated positive practices to their peers as their participation styles consisted of many of the desirable student actions for a discourse community. Such students might be considered as 'old timers' (Lave & Wenger, 1991) from whom other students may have learned ways of acting in the community. The role of the teacher cannot be described in the same way as teacher actions in the discourse community are not necessarily suitable models for student actions. For example, the teacher will often refrain from evaluating mathematical contributions. Instead students are expected to take on this role. Boaler (2006) discusses how some effective teachers explicitly draw attention to and promote valuable learning practices in whole class discussion. It is likely that the incorporation of this teaching practice within my own approach would have improved students' use of discourse community practices, particularly students like Kevin and Steven. It is possible that such a teaching practice would result in more dynamic trajectories of identity.

The analysis of individual lessons (NicMhuirí, 2013) showed students appeared to be positioned as 'active knowers' (Boaler, 2003) and used their own agency to generate mathematical ideas. The analysis presented here attempts to go beyond a simple snapshot of a student's actions in any one lesson, and is focused on how the relationship between the student and the discipline is developing over time irrespective of the particulars of mathematical topic or content. Although the teacher and the classroom community are key influences, the involvement of the teacher is central only for a limited time in a student's life. It is a student's identity and relationship with the discipline of mathematics that will remain an influence on the student's learning over time (Grootenboer & Zevenbergen, 2008). The participation trajectory analysis gives some insight into that relationship and also gives an illustration of mathematics learning as participation in progressive discourse (Bereiter, 1994).

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