INFLUENCE OF EARLY REPEATING PATTERNING ABILITY ON SCHOOL MATHEMATICS LEARNING

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Recent studies in early mathematics education highlight the importance of patterning abilities and their influence on mathematics learning and the development of mathematical reasoning in young children. This paper focuses on young children's repeating patterning abilities and reports results from an ongoing four-year longitudinal study that investigates the development of early numeracy understanding of 408 children from one year prior to school until the end of grade 2. The analyses in this paper reveal a significant influence of young children's repeating patterning abilities one year prior to school on their mathematical competencies at the end of grade one.

INTRODUCTION

Mathematics has often been defined as science of pattern (e.g., Davis, & Hersh, 1980). It is also widely acknowledged that a general awareness of mathematical pattern and structure is important for mathematics learning at all stages (e.g., Mason, Stephens & Watson, 2009; Mulligan, & Mitchelmore, 2009). In this paper we focus on patterning abilities in early mathematics and adopt a differentiated understanding of pattern. We particularly focus on the question what different types of patterns and what kind of patterning activities might influence the development of which key mathematical concepts and processes in early years mathematics learning and later on.

REPEATING PATTERNS AND THEIR IMPORTANCE ON EARLY MATHEMATICS LEARNING

With Mulligan and Mitchelmore (2009) we define a mathematical pattern as any predictable regularity. In the work with Kindergarteners and primary school children, where our research is based, we distinguish three main types of mathematical patterns: spatial structure patterns, repeating patterns, and growing patterns. Examples for spatial structure patterns are spatial dot patterns and grids like the twenty field, both used in the early years to visualize numbers. Repeating patterns consist of a sequence of elements (the unit of repeat) that is repeated indefinitely (e.g., ABCABC…). In growing patterns a sequence of elements changes systematically (e.g., 1, 3, 5, 7, …).

In this paper we draw the focus exclusively on repeating patterns and their importance for mathematics learning.

Patterning activities with repeating patterns are supposed to develop general mathematical concepts in children such as ordering, comparing, sequencing, classification, abstracting and generalizing rules and making predictions (see e.g.,

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Threlfall, 1999). These concepts then lead to the development of mathematical reasoning in young children (English, 2004; Mulligan, & Mitchelmore, 2009, 2013). It is mostly in the area of algebra (or pre-algebra) that repeating pattern work is seen as a conceptual stepping stone (Threlfall, 1999). National curriculums often consider repeating patterns (together with growing patterns) as a precursor for functional thinking and algebra (NCTM, 2000; Queensland Studies Authority, 2008). Mulligan and Mitchelmore (2009, 2013) highlight repeating patterns as important for measurement (which involves the iteration of identical spatial units) and as critical to the development of counting and multiplicative thinking (which involves the iteration of identical numerical units). However, it is important to note that these assumptions have been mainly derived from either observation, a experience, or are theoretical considerations. From the empirical perspective, in the last decade there is a substantial body of research, mainly qualitative studies, focusing on patterning strategies and looking at the level of students' awareness of or attention to pattern and structure (see e.g., Mulligan, & Mitchelmore, 2009; Papic, Mulligan, & Mitchelmore, 2011; Radford, 2010; Rivera, 2013; Warren, & Cooper 2006, 2008). Few studies however have tried to quantitatively measure the significance of patterning abilities in the early years for later mathematics learning.

FINDINGS FROM RECENT QUANTITATIVE STUDIES ON YOUNG CHILDREN'S PATTERNING ABILITIES

Mulligan and Mitchelmore (2009) tested 103 Australian Grade 1 students (5.5 to 6.7 years) on 39 pattern and structure items. They found a nearly perfect correlation between young students' general mathematical understanding and their pattern and structure competencies. A German study (Lüken, 2012) with 74 school starters (5.8 to 7.2 years) showed a significant correlation on a medium level between patterning competencies and early mathematical competencies and a slightly lower correlation with the mathematical achievement at the end of grade 2. Van Nes (2009) interviewed 38 Dutch Kindergarteners (four- to six-year-olds) on tasks on counting, subitizing, repeating and spatial structure patterns. As she used a small sample van Nes only very carefully suggests a correlation between a child's pattern and structure competencies and its mathematical competencies. However, all three studies either lack the use of statistically reliable instruments, or base their conclusions on rather small samples. Above all, all three studies did not discriminate between the different types of patterns. Thus, it is yet to be specified, if each of the three types of patterns in early childhood separately correlates with mathematical competencies and which key concepts and processes they effect.

To underpin the importance of patterning abilities regarding *repeating* patterns this paper focuses on the question, whether a child's ability to reproduce, extend, and explain a repeating pattern has a statistical effect on its mathematical competencies in kindergarten and the transition from kindergarten to school. Hence, the paper addresses the following questions:

- Is there a significant effect of young children's repeating patterning abilities on their mathematical competencies?
- To what extent do their repeating patterning abilities influence young children's mathematical performance one year prior to school, immediately before school entry and at end of grade 1?

METHODOLOGY

Context of this paper is a longitudinal study investigating the development of number concept development of 408 children from one year prior to school entry (5-year olds) until the end of grade 2. The study seeks to identify children that struggle with respect to their mathematics learning after the first (and second) year of school and compare their performance with their number concept development one year prior to school as well as immediately before school entry (i.e., grade 1).

Hence, the data collection involves four measuring points (MP1 – MP4) i.e., one year prior to school, immediately before school entry, at the end of grade 1 and grade 2 (which will be conducted in June 2014). At each measuring point the children performed on both a standardised test on number concept development that is suitable for their respective age (OTZ, DEMAT 1+ / 2+) as well as on a task-based one-to-one interview (EMBI-KiGa, EMBI). Table 1 provides an overview of the study design.

Measuring points	Instruments	Participants
June 2011 MP 1	OTZ	children participating in the study (n = 538)
	EMBI-KiGa	children participating in the study (n = 538)
June 2012 MP 2	OTZ	children participating in the study (n = 495)
	EMBI-Kiga	children participating in the study (n = 495)
June 2013 MP 3	DEMAT 1+	all grade 1 classes with children participating in the study (n = 2250)
	EMBI	children participating in the study (n = 408)
June 2014 MP 4	DEMAT 2+	all grade 2 classes with children participating in the study
(to be conducted)	EMBI	children participating in the study

Table 1: Measuring points, instruments and number of participants in the study

At MP3 and MP4 the whole learning group of children in the study is tested in order to compare the children's performance to their peers' and to diminish intra- and inter-group effects. When available, the instruments chosen for the data collection had been developed and trialled in international settings.

The OTZ (*Osnabrücker Test zur Zahlbegriffsentwicklung*) is a German adaptation of the "Utrecht Numeracy Test" (van Luit, van de Rijt, & Pennings, 1994; van de Rijt, van Luit, & Pennings, 1999) – a standardized individual test aiming to measure children's number concept development that involves logical operations based tasks as well as counting related items (van Luit, van de Rijt, & Hasemann, 2001).

The EMBI (*Elementarmathematisches Basisinterview*) is the German version of the Australian "Early Years Numeracy Interview" (DEET, 2001) developed by Doug Clarke an his colleagues in Melbourne – a task- and material-based one-on-one interview assessing children's developing mathematical understanding in the four areas counting, place value, addition/subtraction strategies, multiplication/ division in grade one and two (Peter-Koop, Wollring, Grüßing, & Spindeler, 2013).

The EMBI-Kiga (*Elementarmathematisches Interview Kiga*; Peter-Koop, & Grüßing, 2011) corresponds with the "Detour for children starting the first year of school" of the Early Years Numeracy Interview (ibid, 24–26), that is also recommended for children in grade 1 and 2 who demonstrated difficulty in counting a collection of 20 objects. For a detailed description of the items and their development see Clarke, Clarke, and Cheeseman (2006).

The DEMAT 1+ (*Deutscher Mathematiktest für 1. Klassen*; Krajewski, Küspert, & Schneider, 2002) and the DEMAT 2+ (Krajewski, Liehm, & Schneider, 2004) are German curriculum based standardized paper and pencil tests to be conducted at the end of the school year with the whole class.

One instrument only, the EMBI-KIGA, uses an item on repeating patterns. We used this item at MP1 as a measure for the children's repeating patterning abilities one year prior to school. The repeating pattern in this item is an ABCC pattern. The children are asked to reproduce, to extend and to explain the pattern. Figure 1 shows the complete item. The material used is coloured plastic teddies (counters).

Now watch what I do with the teddies.

The interviewer makes a ABCC pattern with the teddies (green, yellow, blue, blue, green, ...).

a) I have made a pattern with the teddies. Please make the same pattern.

(If the child's pattern is a correct copy, point to it.
If not, point to your pattern.)

b) Please make the pattern go on a bit more.

c) How did you decide what came next in the pattern each time?

Figure 1: Repeating pattern item from the EMBI-Kiga/Early Numeracy Interview (DEET, 2001, 24-25)

For the data-analyses first a comparison of means in form of a one-way analysis of variance (one-way ANOVA) was conducted, because the item on children's patterning abilities, which serves as the independent factor variable, can take three values (0, 0.5, and 1) and thus defines three separate groups based on the children's performance on the item. Based on these groups the mean scores in all mathematics tests at all measuring points have been searched for significant differences between the groups. With the results of the one-way ANOVA the partial eta^2-values have been calculated in order to approximate the amount of variance in the mathematics tests that can be explained by the children's performance on the patterning item. As a last step the linear correlation (Pearson's r) between the item-performance and the mathematics test performances have been calculated to illustrate the linear dependencies of the two variables.

RESULTS

The results of the one-way ANOVA reveal significant ($p < 0.001$) differences in the mean values of the test and interview performances between the groups. Furthermore, the item on children's repeating patterning abilities shows substantial influences on their performances in all mathematics tests at each measuring point (see Table 2), i.e. one year prior to school, immediately before school entry as well as at the end of grade 1.

							Repeating patterning item one year prior to school	
			df	Mean Square	F	Sig.	Partial Eta^2	Pearson Correlation
Mathematics tests one year prior to school	OTZ total	Between Groups	2	2207,314	53,638	,000	,209	,457**
		Within Groups	405	41,152				
	N = 407	Total	407					
	EMBI-KiGa total	Between Groups	2	361,898	85,767	,000	,301	,547**
		Within Groups	399	4,220				
	N = 401	Total	401					
Mathematics tests at school entry	OTZ total	Between Groups	2	1293,345	43,497	,000	,177	,420**
		Within Groups	405	29,734				
	N = 407	Total	407					
	EMBI-KiGa total	Between Groups	2	60,155	26,151	,000	,114	,335**
		Within Groups	405	2,300				
	N = 407	Total	407					
Mathematics tests at the end of grade 1	EMBI total	Between Groups	2	214,795	18,515	,000	,085	,289**
		Within Groups	400	11,601				
	N = 402	Total	402					
	Demat 1+ total	Between Groups	2	1502,274	26,534	,000	,116	,340**
		Within Groups	405	56,617				
	N = 407	Total	407					

Table 2: One-way ANOVA results, Partial Eta2 and Pearson correlation (**correlation is significant on the 0.01 level)

One year prior to school the children's performance on repeating patterning abilities explains about 21% of the variance on the overall mathematics test performance (OTZ) and shows a significant medium correlation with $r = 0.457$. This also holds true for the performance on the EMBI-Kiga, where the influence is slighty stronger (30.1% explained variance, Pearson's r = 0.547), which can be explained through the inclusion of the item in the interview.

At the second measuring point immediately before school entry the item shows a medium but significant correlation to the children's performances on the OTZ (Pearson's r = 0.42) and still explains 17.7% of the overall mathematics test-performance (OZT). For the EMBI-KiGa performance the item-performance demonstrates similar effects and explains 11.4% of the variance with a significant correlation of Pearson's r = 0.335.

At the end of grade 1 the item-performance still explains 11.6% of the variance of their performance on the standardised DEMAT 1+ and shows a low but significant correlation (Pearson' r = 0.34). For the EMBI interview the children's repeating patterning abilities explains 8.5% of variance of the overall interview performance and correlates with Pearson's r = 0.289.

DISCUSSION

With respect to the question if each of the three types of pattern (see above) in early childhood separately correlates with mathematical competencies, this effect could be shown for repeating pattern abilities. Those children, who manage to solve the EMBI-Kiga item on repeating patterns one year prior to school, i.e., they can reproduce, extend and explain a repeating pattern of the form ABCC, are the children who demonstrate elaborate number concept development in kindergarten and who achieve best in a standardised mathematics classroom test at the end of grade 1.

This relationship appears to be stable over a period of two years and can be shown with different measuring instruments, i.e. individual (OTZ) and group tests (DEMAT 1+) as well as one-on-one interviews with a focus on strategies (EMBI-Kiga/EMBI). The explanation of variance for mathematics test performance provided by the pattern item as expected decreases until the end of grade 1 (a period over 2 years), but remains at a substantial level.

Looking closer at the mathematical concepts and processes of the applied instruments (see Table 1), significant positive linear correlations are found between repeating patterning abilities and computation skills (DEMAT 1+ and EMBI), i.e. children who demonstrate elaborate repeating patterning ability prior to school also show elaborate computation skills with respect to addition and subtraction at the end of grade 1. In addition, the data reveal significant positive linear correlations between repeating patterning abilities prior to school and addition and subtraction strategies other than counting (i.e., counting all, counting on and counting back) at all measuring points (EMBI). Furthermore, we cannot draw any conclusions with respect to other mathematical abilities or mathematical content areas e.g., geometry.

However, the question whether repeating patterning ability is a predictor for the development in specific domains of early numeracy learning yet remains open also due to limitations of the item used to assess patterning abilities in the EMBI-Kiga. Additional items considering different levels of difficulty with respect to repeating patterns as well as the documentation and analysis of children's explanations of the pattern would be necessary to further investigate that impact.

In summary the study reported in this paper indicates that it is important to differentiate the rather broad concept of pattern with respect to early mathematic learning (Papic et al., 2011). A correlation could be shown for repeating patterning abilities, but still needs to be investigated for growing patterning and spatial patterning abilities. Hence, a further large scale longitudinal study that involves several items on each repeating, growing, and spatial patterning abilities in order to increase reliability is desirable in the future.

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