MAKING SENSE OF WORD PROBLEMS: THE EFFECT OF REWORDING AND DYADIC INTERACTION

Maria Mellone[1], Lieven Verschaffel[2], Wim Van Dooren[2]

[1]Università Federico II di Napoli, Italy, [2]Katholieke Universiteit Leuven, Belgium

In this study we investigated the effect of the request to reword the text of problematic word problems on the occurrence of realistic answers. We proposed the activity of rewording word problems to fifth grade pupils either working individually or in dyads. We found that the rewording the problems while working individually had no effect, while rewording in dyads produced a strong increase of pupils' realistic answers. Moreover we analysed the pupils' reworded texts in order to characterize the kind of information added by pupils.

INTRODUCTION

The well known *l'age du capitaine* (Baruk, 1985) was one of the most popular studies that brought to the attention of international research the phenomenon known as 'suspension of sense-making' when solving word problems (Schoenfeld, 1991). Indeed many pupils' responses to this problem, as well as to other word problems of the same or a similar kind, have shown a tendency to unthinkingly apply arithmetic operations without critically considering the reality that the word problem is referring to. Explanations that have been raised for this phenomenon often refer to the stereotypical nature of the word problems typically used in school and to the implicit and explicit rules which govern educational practices surrounding word problems (the so-called didactical contract, see Brousseau, 1986) (Verschaffel et al., 2000).

In the study reported in this paper, we investigated the effect of inviting pupils to reword a given word problem – individually or in dyads – on the realistic nature of their answer to that word problem. Moreover, we analysed the information added in pupils' reworded problems to get a deeper understanding of their sense making process.

THEORETICAL FRAMEWORK

A word problem can be defined as:

> [...] a text (typically containing quantitative information) that describes a situation assumed as familiar to the reader and poses a quantitative question, an answer to which can be derived by mathematical operations performed on the data provided in the text. (Greer, Verschaffel, & De Corte, 2002, p. 271).

World problem solving still comprises an important aspect of mathematical school life. One of the main goals of word problems is to bring pieces of reality in the classroom in order to let pupils experience different aspects of mathematical modelling and problem solving processes, without the practical inconvenience to a direct contact with real

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world contexts (Verschaffel et al., 2000). In the 90's two pioneering studies (Greer, 1993; Verschaffel, De Corte, & Lasure, 1994) suggested that this goal of bringing mathematical modelling experiences into the classroom is very often not met. In these studies, it was found that upper elementary pupils only very rarely make realistic considerations when solving word problems. This was shown by a contrast between the very good performance on so-called standard word problems (S-items) – that can be solved correctly by straightforwardly applying operations with the numbers given in the word problem – and very low performance on so-called problematic word problems (P-items) – where peculiarities of the everyday life situation described in the word problem need to be taken into account. For example, the P-item *A man wants to have a rope long enough to stretch between two poles 12m apart, but he has only pieces of rope 1.5m long. How many of these pieces would he need to tie together to stretch between the poles?* was answered with "12 : 1,5 = 8 pieces" by virtually all fifth graders from Verschaffel et al.'s (1994) study.

Many authors have argued that this phenomenon can to a large extent be grasped by looking at the processes that occur at the beginning of pupils' word problem solution. Often, pupils seem to decide, based on a very quick and superficial reading of a word problem, which mathematical model leads to the solution. However, for the P-items as described above (and for word problems more generally), ideally there is an intermediate stage between the initial reading of the word problem text and the construction of a mathematical model. This intermediate stage, often described as the creation of a situation model (Kintsch & Greeno, 1985; Verschaffel et al., 2000), consists of representing the key elements and relations in the problem situation. In this stage, one's real-world knowledge about and personal experiences with the situation described in the P-item may help to construct a rich situation model.

The fact that pupils often do not succeed to create a(n extended) situation model of P-items and therefore fail to solve these problems realistically, may partly be explained by the scarcity of information available in the word problems themselves. As Zan (2011) suggested, word problems may show "narrative ruptures" when the question and the information needed for the solution are not consistent from the point of view of the narrated story. Voyer (2011) distinguished three kinds of information that may affect the extent to which this situation model is actually constructed: *Solving information* consists of the essential numerical data, the order of presentation of these data and the size of the numbers; *Situational information* is information which plays a role in the development of a context that anchors the mathematical question in a real life situation, like the initiating events, the setting details and temporal information; and *Explanation information* makes the relationships among the various pieces of information found in the text more explicit. Voyer (2011) posed different versions of the same frame problems to a sample of pupils to understand the relationship between the information presented in the problem text and the constructed situation model. He found that adding information that was non-essential for the mathematical solution of the problem, but still relevant to the problem context, had a positive influence on

pupils' performance. Similarly, Palm (2008) observed that when information was added to word problems to make them more authentic (i.e., more closely simulating the real-life situation in which the problem occurred), a larger proportion of children makes proper use of their real-world knowledge in the problem-solving process.

RATIONALE AND RESEARCH QUESTIONS

So far, research has shown that reworded problems that provide more background information lead to better performance on word problems in general (Voyer, 2011), and to more realistic considerations vis-à-vis P-items specifically (Palm, 2008). In the current study, we did not make use of reworded problems in which we added information ourselves. Instead, we asked pupils to reword these problems by themselves and looked whether this would positively affect the realistic nature of their answers to these problems. Indeed, pupils may also be able to add to the word problems the various situational elements that were suggested by Voyer (2011). Using the argumentation by Zan (2011), the impact may even be stronger than when giving word problems with the information already added: One of the crucial differences between a real problem solving situation and a school word problem is that the latter is typically *hetero-posed*, i.e. the person posing the problem is someone different from who has to solve it (Zan, 2011). Therefore any word problem has to be expressed by a (generally written) text, to communicate to the solver *what* he/she has to solve, through an explicit request. Our proposal to ask pupils to reword the text of the problem may partially recover this loss of authenticity, and therefore positively affect the realistic considerations they make. In particular we wanted to see if this rewording helps pupils to consider aspects of reality in their situation model of the problem and consequently react more realistically to these items.

In order to strengthen the possible effect of asking pupils to reword given P-items, we asked some pupils to do this in dyads rather than working individually. Indeed we weren't sure that the rewording would be as effective if pupils work alone. Working in dyads creates a condition in which pupils are forced to make their proposals for rewordings (and possibly even the considerations that lead to them) explicit, and discuss to arrive to an agreement. Working in groups has been shown successful in eliciting realistic reactions (Verschaffel et al., 2000) and thus this kind of condition could make the rewording work more effective and the pupils react more realistically to the P-items.

So, in the current study, we investigated whether asking pupils to reword given P-items would lead them to solve these items more realistically afterwards, and if so, whether this would be more effective when pupils do this assignment in dyads rather than individually. Based on our theoretical framework, we expected a very low number of realistic responses in pupils who solve the task individually, a moderate increase in realistic responses when pupils were asked to reword the problems or could work in dyads, and the highest number of realistic responses in pupils who were asked to reword the problems while working in dyads.

METHOD

A total of 179 fifth graders (88 female and 91 male) were involved in this study. They came from three schools of Naples (South Italy, a region that is well-known for its weak results for mathematics both in national (INVALSI) and international (PISA) assessments). All pupils were randomly assigned to one of four conditions:

- IS condition (n = 19): Pupils individually (I) worked on solving (S) P-items.
- DS condition (n = 62): Pupils worked in dyads (D) on solving (S) P-items.
- IR condition (n = 38): Pupils individually (I) worked on solving P-items after being asked to reword (R) them.
- DR condition (n = 60): Pupils worked in dyads (D) on solving P-items after being asked to reword (R) them.

The DS-condition was added as an extra control condition in order to disentangle the effect of rewording the problems on the one hand and working in dyads on the other hand.

All pupils (or dyads) received a booklet with four word problems adapted from Verschaffel et al. (1994) and were asked to solve them. In the IR and DR conditions pupils were asked to first reword the problems, and they received detailed instructions to do so. More specifically, they were asked to rewrite each word problem by adding details that could help to figure out the underlying situation as in a story (who is involved, what is happening and why, where does the question come from?), while making sure that the operations and the answer to the problem would remain the same.

Pupils working in dyads (in the DS and DR conditions) received only one booklet per dyad and were instructed to work together on this assignment, to talk to each other and to negotiate about the rewriting of the word problem (in the DR condition) and to write down an answer only after agreement had been reached.

Due to space limitations, we will focus on one of the four P-items included in the study, namely the buses problem, adapted from Verschaffel et al. (1994): *450 soldiers must be bused. Each army bus can hold 36 soldiers. How many buses are needed?*

ANALYSIS

The key variable in this study is whether pupils' reactions to the P-problems were non-realistic (NR) or realistic (RR). For the buses problem above, answers were considered NR when the numerical answer was the mere reporting of the result of the operation 450 : 36 (i.e. "12.5 buses", or "12 buses remainder 18"), without any comment about the problematic nature of the problem and/or the given answer. Answers were considered RR when they showed some realistic consideration, either by rounding up the number of buses to the next number ("13 buses are needed"), or by adding any comment that indicated that realistic considerations were made (e.g., "12 buses are needed, and 18 soldiers are left", or "12 buses and perhaps a smaller one").

In addition to the nature of pupils' reactions to the word problems, we also did a deeper analysis of the reworded texts by the pupils in the IR and DR conditions. This analysis could reveal to what extent pupils had followed the instruction to reword the problem to a more meaningful one, and what kind of information pupils had added to do so. For this reason, we coded for every reworded problem in the IR and DR condition the number of information elements present in the reworded problem that were not yet in the original problem text, separated in descriptive (D) information (names, objects, places), intentional (I) information, action (A) information, temporal (T) information, and causal (C) information.

The latter categorization was based on Voyer's (2011) distinction among the different kinds of information that can be added in a word problem text, as well as on another study about the effect of different kinds of rewording of word problem texts on pupils' performance (Vicente, Orrantia, & Verschaffel, 2007). Here is an example of a pupil's reworded text together with our analysis:

> The head of the Italian army decided to make a war against the U.S. army. To do this, the Italian army has to train, so it must be transported in a proper military camp. The soldiers are 450 and must be transported by bus to 36 soldiers each. How many buses are needed?

D: 'The head of the Italian army', 'U.S. army', 'proper military camp';

I: 'he decided to make a war';

A: 'to make a war', 'to train';

T:

C: 'they have to train'

RESULTS

As expected, there was a very low number of RRs to the word problem in the IS condition, where pupils worked individually and were not asked to reword the problem. Only 1 out of 19 pupils (5.3%) gave a RR. The same was true when pupils worked in dyads (DS condition): None of the 31 dyads gave an answer that could be considered realistic. Moreover, contrary to our expectations, we also did not find a positive effect of asking individual pupils to reword the problems: Only 2 out of 38 pupils in the IR condition (5.3%) gave a RR. However, asking dyads to reword the problems led to a spectacular result, as 22 of the 30 dyads in the DR condition (73.3%) gave a realistic answer. So, only the combination of rewording and working in dyads led to a dramatic increase in the number of RRs to the buses item.

As explained above, we also looked more carefully at the reworded texts that were produced by pupils in the IR and DR conditions, to get a better understanding of the effect of our experimental manipulations. Table 1 summarizes the mean number of elements added in the reworded texts for these two conditions.

	Total elements added	**Descriptive**	**Intentional**	**Action**	**Temporal**	**Causal**
IR	4.03	1.76	0.34	0.85	0.11	0.86
DR	6.32	2.38	0.16	2.55	0.19	1.03

Table 1: Mean number of elements added in the reworded texts in the IR and DR conditions

Some differences could be noted. Pupils working in dyads added on average 6.32 elements to the word problem, whereas pupils working individually only added 4.03 elements, which was a significant difference, $t(67) = 3.15$, $p = .001$. Additional tests showed that this was due to a difference in the number of added descriptive elements (1.76 vs. 2.38 on average, $t(67) = 1.79$, $p = .039$) and action verbs (0.85 vs. 2.55, $t(67) = 4.09$, $p < .001$).

We found many interesting and rich reworded texts built by pupils who answered in a realistic way. For example, the reworded text *450 soldiers must be transported in military bus where they will travel to go to a ceremony. The soldiers must be distributed by 36 in each bus. On the bus the soldiers eat tomato pizza and the generals talk. How many buses are needed?*, which was built by pupils who worked in dyads, shows the presence of some 'useless' details from the mathematical point of view, like "the soldiers eat tomato pizza", but that likely contributed to support pupils in imagining the situation, and afterwards giving the realistic answer "They used 13 buses, and in one of them there will be 18 soldiers". Other interesting considerations can be made regarding those pupils who decided to convert the story text in something closer to their own life. For example, *In the fifth grade classes of Madonna Assunta [pupils' school] there are 450 children who have to go to school camp in Puglia. The teachers have ordered the buses that can hold up to 36 children. Children pose themselves the problem of how many buses will be used to transport the 450 children*. The dyad who built this text answered "12 buses are needed and 18 children do not go to school camp". This kind of behaviour was consistent with the considerations developed in Davis-Dorsy et al.'s study (1991), where a problem personalization (i.e., personalizing the standard version of the problem with children's favourite food, and/or their friends' names) led to better results. In our study, these two pupils themselves proposed a personalization of the problem (they indeed just returned from a school camp), and the re-contextualization of the problem together with the personalized way of solving the realistic problem situation showed a flexible understanding of the arithmetical structure in the problem.

CONCLUSIONS AND DISCUSSION

Mathematical word problems are still an important aspect of mathematical school life (in terms of classroom activities, textbook exercises, evaluation tools, and so on). Undoubtedly, the stereotyped nature of word problems used in school leads pupils to routinely apply arithmetical operations based on superficial text processing, leaving in the shadow the mathematical modelling, in particular the building of a rich situational model. This has been evidenced by previous research about pupils' non-realistic answers to problematic word problems.

In this study, we attempted to deepen fifth graders' construction of a situational model of problematic word problems by asking to reword the word problem text. We also investigated the impact of working individually or in dyads. Combining these two manipulations resulted in four experimental conditions. We did not find any effect for the rewording activity when working individually (IR condition). However, asking dyads to reword the problems (DR condition) led to a spectacular result with almost three quarters of the dyads giving a realistic answer. Therefore, these results stress the importance to frame word problem solving processes as social activities. It would be interesting to investigate if this rewording experience in dyads can affect pupils' individual behaviour also afterwards.

Moreover we analysed the reworded texts produced by pupils in the IR and DR conditions. We found a major number of added descriptive elements and action verbs in the reworded texts of pupils who worked in dyads. Together with the previous results, this makes us to hypothesize that the descriptive elements and the action verbs are important elements in the building of situational models that carries pupils to develop mathematical correct and realistic considerations.

It has been argued that, when solving word problems, sometimes "*too much attention to the story will distract pupils from the translation task at hand, leading them to consider "extraneous" factors from the story rather than concentrating on extracting variables and operations from the more mathematically-salient components*" (Gerofsky, 1996, p. 37). While this may be true for standard word problems, the present study revealed that, as far as P-items are concerned, asking pupils to reword the problems and add various elements may lead to substantially more realistic reactions, at least if they are put in a meaningful communicative setting.

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