

# RELATING STUDENT MEANING-MAKING IN MATHEMATICS TO THE AIMS FOR AND DESIGN OF TEACHING IN SMALL GROUP TUTORIALS AT UNIVERSITY LEVEL

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*In a developmental research approach, from a sociocultural position, we address the meanings students make of mathematics in teaching sessions and how this relates to the intentions of the teacher and approaches to teaching. Analyses of data come from small group tutorials of one tutor with first year university mathematics students (n=5). We exemplify using data from one tutorial which addressed concepts in calculus that first year students encounter in their lectures. We explain teaching design and an approach to implementing it, and address issues that arise in practice and how these are related to students' meaning-making of mathematical concepts. Development of 'knowledge in practice' is seen alongside that of knowledge in the public domain.*

## INTRODUCTION

In one UK university, first year mathematics students are expected to attend lectures in calculus and linear algebra. Each student is also a member of a small tutor group (of from 5 to 8 students) that works on the material of these lectures. Lecturers in the modules set problem sheets each week for students to tackle. In small group tutorials (one hour per week), the tutor works with students on material relating to the two modules, often taking questions from the problem sheets. We focus here on the activity of one tutor with her group of 5 students who are in a joint programme of *Mathematics and Sport Science*. Her main aim for tutorials is to support students to understand, or to make meaning of the mathematics of the lectures. In each tutorial the tutor makes a judgement as to which questions to focus on in the tutorial (other tutors might do things differently). For her, these questions should satisfy two conditions:

- a. they should reveal key concepts in the mathematics of the lectures – to some extent, all questions set by the lecturer do this, but the tutor chooses particular ones to highlight key concepts in her judgment;
- b. they should be questions with which students struggle or have difficulties.

A general expectation is that students will work on the problem sheets in their own time and come to a tutorial with their questions. Therefore, in every tutorial the tutor asks students to inform her of questions with which they struggle or would like help. They respond occasionally but largely they do not respond. It often seems as if they have not addressed any of the questions before coming to the tutorial. The tutor does not want to exercise too much pressure on what they have to do before coming, since they are then likely not to come. She would rather they came, so that (she hopes) some 'useful' work can be done. The tutor decides what is 'useful' based on her knowledge

of the mathematics and of her students and what they find difficult. After the tutorial, the tutor reflects on what has occurred, whether her earlier judgments were appropriate, and what alternatives there could have been.

## RESEARCH QUESTIONS

The aim of this research is to study how the practical manifestations of teaching in a tutorial satisfy the aims of *teaching for students' learning* (Jaworski, 1994; 2003b). We wish to discern as far as possible the associated meaning making of the students in a tutorial and how this is (or not) linked to the style of teaching, taking into account the wider social factors of the setting. We have three basic research questions:

1. What is the nature of the teaching manifested in the tutorials?
2. What student meanings can we discern and in what ways?
3. In what ways can we link (1) and (2) and what issues does this raise?

Through this research, we seek also to redress the scarcity of research into the “actual classroom teaching practice” of university teachers (Speer, Smith and Horvath, 2010, p. 99) and extend knowledge of teaching in small group tutorials (Jaworski 2003b).

## MEANING-MAKING IN MATHEMATICS

There is a considerable literature on mathematical meaning making at a range of levels (e.g., Kilpatrick, Hoyles, & Skovsmose, 2005). We set the scene here by drawing on three perspectives. The first links meaning making to *making connections*, both within mathematics and to the world beyond mathematics.

[M]athematical meanings derive from connections: intra-mathematical connections which link new mathematical knowledge with old and extra-mathematical meaning derived from contexts and settings which include – though not uniquely – the experiential world” (Noss, Healey, & Hoyles, 1997, p. 203).

The second suggests that making meaning in mathematics is a process of “socialisation” into the culture and values of “doing mathematics” (Ben-Zvi & Arcavi, 2001). In the third, Nardi (2008, p. 111) refers to students “mediating mathematical meaning through symbolisation, verbalisation and visualisation” suggesting that students experience the tension between the need to *appear to be*, or *to be* mathematical. Thus, making connections, the worlds of mathematics and beyond, and processes of socialisation into culture and values are all central to making meaning in mathematics. Further, students have to get beyond the instrumental use of key processes in learning mathematics to become mathematical, to make meanings at a conceptual level. We draw on all these perspectives in our analyses.

## METHODOLOGY

We take a sociocultural perspective in which knowledge is seen to develop in social settings as part of which individual sense-making develops (e.g., Wertsch, 1991). Teaching and teaching resources are seen to have a central mediating role in the

development of mathematical meanings by students. People make sense of mathematics in relation to the worlds of which they are a part; these ‘worlds’ capture local and more global situations and contexts surrounding human activity (Holland, Lachicott, & Skinner, 2001), the wider social issues mentioned above.

Our methodology is developmental: we use research as a tool to promote development as well as a tool to observe and analyse development (Jaworski, 2003a). We are two researchers: one researcher is also the tutor, whose job is to teach the students – to enable their mathematical understanding. She wants to *promote* meaning making and, at the same time, to *discern* meaning making: related aims which might potentially be in tension. The other researcher observes activity and collects data through audio recording<sup>1</sup> and note taking. In discussion with the tutor, she enables the tutor to reflect critically on the teaching process and together they seek evidence of students’ meaning making (audio-recorded). An expectation of this relationship is that the tutor, through acting as a researcher, develops knowledge in practice which feeds back into the design of teaching. Thus two kinds of knowledge are generated – knowledge in practice which informs the teaching process, and knowledge which can be communicated in the wider research community (for example through this paper).

We collected data from a series of tutorials (10 in all) in Semester 2 of the academic year. At the time of writing, analysis is in its early stages; we expect findings to develop as analysis proceeds. Briefly, the data from a tutorial is first split into episodes in which an episode is a section of the tutorial which has some completeness in itself (e.g., the work of the group on a given problem). We undertake a grounded analysis of the data, episode by episode, coding and categorising (Corbin & Strauss, 2008).

We demonstrate our analytical process through a case of one episode taken from a tutorial from Week 6 (of 12 weeks). Four (out of 5) students are present plus the tutor, and the co-researcher as observer. Lectures are currently focusing on multivariable calculus. The group works on questions from the lecturer’s problem sheet involving differentiation of functions of two variables. We focus on an episode of 10 minutes from close to the beginning of the tutorial. Our analysis is both particular to this episode and also related to analysis of other tutorials and episodes. Codes emerge continually and it is necessary to keep revisiting earlier codes in order to rationalise them with new insights. In particular we recognise the emergence of tensions in the process of teaching development. Analysis is ongoing and we expect to set these observations against those emerging from other data.

## ANALYSIS OF TEACHING

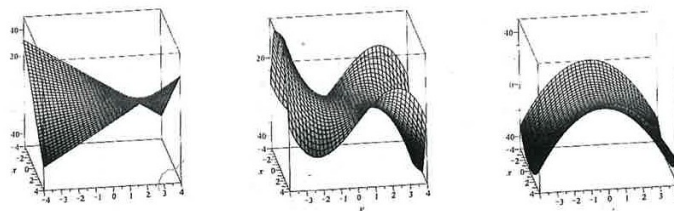
The tutor needs to find out as quickly as possible what the students already know and can do: if basic questions are answered quickly/readily, they can move on to more demanding questions. Students are usually able to tackle procedural questions, but

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<sup>1</sup> Although video data would be valuable it is considered that use of a video camera would be too disturbing for the students.

those demanding conceptual insight cause more difficulty. She chose her first question to encourage students to *make sense of connections* between symbolisation of partial derivatives and their graphical representations, as follows:

The three graphs below show a function  $f$  and its partial derivatives  $f_x$  and  $f_y$ . Which is which and why?



As also recognised in analysis of other tutorials, the tutor employs a questioning style. Analytical codes used previously have included ‘TQ-probing’ and ‘TQ-prompting’ to identify ‘tutor questions’ which “probe” (seek out students’ meanings) or “prompt” (suggest particular meanings). In this episode, almost every ‘turn’ of the tutor includes a question or questions to the students, so this has required a finer coding of questions. The tutor says that she is trying to find out what students know and can express, which she believes will give her insight to their understanding (or meaning-making) in mathematics. In addition she expects their responses to prompt their fellow students to think about the concepts and provide alternative or clearer answers to the questions. Thus, she hopes to encourage students’ engagement both individually and with each other. Her probes/prompts are designed to provide opportunities for students to think, express and articulate what they see and understand, and to reveal what they are not clear about. Such revealing of students’ lack of clarity or inability to express clearly, leads to the successive questions that she asks. Analysis shows the following kinds of questions being asked most frequently as prompting or probing questions:

**Meaning Questions** (Qm) or (Qmw) – *overtly* seeking students’ expression/articulation of meaning, often in response to the question “why?”

**Inviting Questions** (Qi) – asking students to respond; (Qig) – offering the question *generally* (to all students) or (Qid) *directly* to one student (named). The question can be a specific question (Qigs or Qids) or non-specific question (Qig or Qid) where ‘specific’ means that it refers to a specific mathematical item. Often these questions also seek meaning, but more implicitly.

*Do the students make sense of the particular notation?*

- 3: T: So, first of all what are these things  $f_x$  and  $f_y$ ? Alun. What is, what do you mean, if you write  $f_x$  and  $f_y$ ? [Qm] [Qids]
- 4: S:(Alun)  $df/dx$
- 5: T: And how would you write it? [Qid]
- 6: [He indicates with his hand the partial derivative symbol,  $\partial$ ]

- 7: T: Yes *partial*  $df/dx$  and similarly  $f_y$  is *partial*  $df/dy$ . When you say  $df/dx$  it's not clear, so you want to be clear. We would say here *partial*  $df/dx$  and *partial*  $df/dy$  [She writes on the board  $\partial f/\partial x$  and  $\partial f/\partial y$ ]
- 8: T: So in the question then, we have three graphs; one of them is a function  $f$  and the other two are the *partial* derivatives  $df/dx$  and  $df/dy$ . Now, which is which? [Qig]

At turns 3 and 5 we see direct and specific questions to Alun, who responds. At turn 8, there is a general question to the group as a whole. As well as the tutor's questions here, we draw attention to her emphasis on terminology and symbolism [7]. Previous tutorials have revealed the importance of ensuring that students are clear about terms and their meanings. From this interchange she sees that Alun is aware of the meanings of  $f_x$  and  $f_y$  as shown by his words and gestures. Her reiteration, at turn 7, can be seen as emphasis for the other students.

*What sense are the students making of what they see?*

- 12: T: ... OK, how about you Erik? [Qid]
- 13: S: (Erik) not really sure but I guess that, er,  $f$  will be the middle one.
- 14: T: OK, *why* do you think that? [Qmw] [Qid]
- 15: S: (Erik) because it is got the, er, the slants of the first one, and the...
- 16: T: so you're seeing a relationship between the one in the middle and the other two. What do you mean by the *slants*? [Qm] [Qids]
- 17: S: (Erik) er, I don't know, just the, the gradient there.
- 18: T: *if* you're right and the function is the middle one, erm, before we go any further, Alun, do you think the function is the middle one or would you say one of the others? [Qids]
- 19: S: (Alun) it looks like the more complex

Here we see direct questions to Erik and Alun [12 & 18] and a *why* question to Erik [14]. Erik offers the key word 'slants' which the tutor asks him to clarify. It is 'key' because it is suggestive of meaning, which the tutor seeks to clarify so that it gains more general meaning for the group. As result of further questions, Alun offers the term 'complex', which the tutor goes on to pursue, in a similar style in turns 20 to 32. We pick up the dialogue again at turn 33 where the questioning continues.

- 33: S: (Brian): Well, I guess when you differentiate, you're almost simplifying it to your next .[inaudible]
- 34: T: OK, so if what we have got is, in some sense a polynomial, then when we differentiate a polynomial we get a lower degree. [Pause – looking at students] So is that what you meant by 'simplifying'? So is everybody agreed then that the middle one is the function? OK. It is!! It is. So look to the one on the right, Erik, and tell me how the one on the right fits with what you see in the middle. Is that going to be the partial derivative  $f_x$  or is it going to be the partial derivative  $f_y$ ? [Qids]
- 35: S: (Erik) erm, derivative of  $x$  [inaudible]

- 36: T: Can you say why? [Qmw]
- 37: S: (Erik) aah.. because, I dunno, it looks as if [inaudible] along  $x$  and the other as if it kind of moving up and down  $y$  [inaudible]
- 38: T: so you are changing your mind? [they laugh] How about you Alun? [Qid]
- 39: S: (Alun) erm [long pause] I dunno.
- 40: T: Well, let's suppose that is  $f_x$  as Erik said. What does it mean for it to be  $f_x$ . I mean, if I have a function such as *this* [points to the middle graph], and I am looking to find  $f_x$ . What do I *do*?. It is as if *what*? [Qig]
- 41: S: (Alun) You fix  $y$

## ADDRESSING RESEARCH QUESTIONS

Question 1 has been addressed (briefly) above. In relation to Question 2, analysis points to lines 6, 15, 17, 37 and 41 as indicative of student meaning. The articulation (or gesturing) gave clues to students' insights in relation to the problem. Students' difficulties to express their thinking in articulate forms meant that meanings were hinted at rather than uttered with clarity. We might say there is evidence of students linking the nature of the first and third graphs to the one in the middle and using informal language to express meaning (e.g., 16: *slants*; 37: *as if it kind of moving up and down y*); the need to "fix  $y$ " in order to find  $f_x$ . At this stage in the process of meaning-making, nothing formal was expressed or written down.

In a presentation of the above data in a seminar in the UK, it was suggested that the tutor is *funnelling* the discussion (Bauersfeld, 1988), prompting students so that they are giving her what they perceive she wants, and that in fact the students have little understanding of the concepts involved. Such interpretation points to the Topaz Effect (Brousseau, 1985) or Didactic Tension (Jaworski, 1994; Mason, 2002) in which a teacher's questions lead students to give correct answers without the understanding the teacher wants. Thus, discerning meanings is important to judging such interpretation.

By conducting a finer (discourse) analysis (of the third short extract above, taking it turn by turn), we show how we try to address such issues concerning meaning making and the influence of the teaching style. After Alun's statement at 19, the tutor pressed the students to say what it means for the function to appear 'more complex', to say "WHY" it would be more complex. The other students agreed with Alun's statement about complexity, but could not say why. At 33, Brian offers a new idea, that differentiation simplifies a function, resulting in a function *simpler* at the next stage. The tutor picks up and extends this idea [34] recognising that such simplification could happen in differentiating a polynomial function to obtain a function of lower degree. This is tutor input; she knows that the students are familiar with these concepts and seeks to remind and consolidate. Her question "So is that what you meant by 'simplifying'?" is somewhat rhetorical: she looks at them all and judges their response to it through body language and facial expressions which the audio data cannot capture. This leads to her acknowledgement that they are right about the middle graph

representing the required function. There is also a sense of pace, and of needing to move on. Her next question is a direct prompt to Erik, challenging him to declare which derivative the right hand graph represents. He suggests  $f_x$ , and the tutor again asks ‘why?’. He struggles to answer, but speaks of ‘movement’ along  $x$  or ‘up and down’  $y$ . These words suggest meaning to the tutor. She asks another student (Alun) to comment, and he is unable to do so. She then follows up [40] with more direct questions, building on Erik’s suggestion. This might be seen as ‘funnelling’. However, there is an issue of what to do at this point – she could just tell them the answer, giving her own explanation; instead she pursues the questioning approach. Alun’s response, “*You fix y*”, seems to be prompted by her “*What do I do*”. The use of pronouns, I and you, makes the situation more personal. They have talked about fixing  $x$  or  $y$  in order to get partial derivatives on an earlier occasion, so there are shared meanings. She knows that he knows about fixing a variable, so the fact that he brings in the idea at this point (albeit in response to her prompt) suggests to the tutor that he is starting to make sense of the various ideas (we see elements of verbalisation and visualisation although not yet the formal stage of symbolisation – Nardi, 2008).

There are dangers in a teacher analysing her own discourse with students, it being tempting to read more into events than was actually evident. However, meaning making for the tutor, trying to make sense of her students’ meanings, is informed by the wider social setting: nuances of tone, gesture and body language more generally as well as historical common experiences and wider social perception. She has had conversations with individual students (for whom she is personal tutor) about their work, progress and social activity. The fact that these students are sport scientists as well as mathematicians brings additional factors to consider – they have less time to give to their mathematics than students who study only mathematics, and have given many indications previously of struggling with mathematical concepts.

## THE NATURE OF KNOWLEDGE

The case explored above offers some early insights into a relationship between students’ meaning making and the teaching approach. Students respond only tentatively to the tutor’s questions; responses are not articulate; it is hard to gain insight to what they understand. It is important for students to be able to explain not only *what* they see, but *why* it is so. Repeatedly asking ‘why’ is a form of *socialisation*: in mathematics we need to be able to explain what we do in conceptual terms. However, we need words to explain difficult ideas – expressing informally can lead to more formal articulation. Students are unused to such expressing. Creating opportunity for them to think and express is an important part of the questioning approach. In this episode, unlike some others, there was little dialogue between students. Analysis shows the tutor where such dialogue would be valuable and prompts consideration of where and how it could have been achieved. The tutor recognises tensions: in some cases it might seem more appropriate to offer her own explanations from which students can gain insights; however, students’ reliance on the tutor giving explanations may inhibit further their willingness to try for themselves. Such growth of awareness

for the tutor is the basis of *knowledge in practice* which informs future action. As we analyse further we expect to be able to crystalize elements of, for example, the questioning approach and its relations to meaning-making. This can contribute to a broader awareness of how we encourage students' meaning-making and the issues and tensions involved. Such shared knowledge can lead to more informed practice widely.

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