HOW IS THE FUNCTION CONCEPT INTRODUCED IN TEXTBOOKS?: A COMPARATIVE ANALYSIS

Dae S. Hong, Kyong Mi Choi

University of Iowa

This study compared sections of functions and linear functions from four Korean textbooks and Core Plus Mathematics Project (CPMP). To understand differences and similarities among these textbooks, both horizontal and vertical analyses were conducted. The horizontal analysis results revealed that topics related to functions and linear functions are introduced relatively earlier in Korean textbooks than in CPMP. The vertical analysis results confirmed the findings of previous study (Hong & Choi, 2014), which can be interpreted as "textbook signature".

INTRODUCTION

Reports from international comparative studies such as Trend in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) indicates that East Asian students perform consistently well. Among the various areas of mathematics education research, textbooks play an important role in determining what is taught and what students learn. There are different views of textbooks, however, researchers agree, in various degrees, that an analysis of textbooks can partially explain differences in student achievement (Zhu & Fan, 2006). Although Korea is one of the high achieving countries in international assessments, there are few mathematics education studies that disseminate Korean secondary mathematics textbooks (Hong & Choi, 2014). "Lesson signature" and "textbook signature" are distinctive characteristics across lessons and textbooks in each country (Hiebert et al., 2003; Charalambous, Delaney, Hui-Yu & Mesa, 2010). Such characteristics could partially explain what students in different countries learn. By comparing textbooks, this study will examine features of secondary Korean and American mathematics textbooks. If some features are consistently found, we can say that there is "textbook signature" among Korean and American secondary textbooks. We chose one of the fundamental and central unifying concepts in mathematics, the concept of functions (up to the linear functions) for comparison (Eisenberg, 1992; National Council of Teachers of Mathematics [NCTM], 1989, p. 154). Here are research questions that we attempt to answer:

1. What similarities and differences are observed in the content of function lessons of Korean and standards –based American secondary textbooks?
    i) How are topics of function introduced and developed?
    ii) What practices and contexts are used in the mathematics problems?
2. What types of responses and levels of cognitive demands are required in the textbook problems?

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LITERATURE REVIEW

Students' Learning of Function

Studies show the formal definition of a function, the Dirichlet–Bourbaki definition is introduced often, but there are various misconceptions about a function: a function is defined by an algebraic formula, a function must be continuous, and a split domain will represent more than one function (Hitt, 1998; Mesa, 2004). To overcome misconceptions, multiple representations are often emphasized (Brenner et al., 1997).

Textbook Comparison Studies

Studies show that Asian countries' textbooks (Japan and Taiwan) contain more mathematical topics in one school year and introduce these topics earlier in the sequence of the school year compared to both elementary and secondary American textbooks (Steveson & Bartsch, 1992; Stigler et. al., 1982). In analyses of mathematics problems, studies have presented varying results. Some have shown that in both elementary and secondary mathematics textbooks, Asian countries contain more challenging problems and problems requiring explanation than American standards–based and traditional textbooks (Li, 2000; Son & Senk 2010) while other studies discovered that standards–based textbooks include more challenging problems and problems requiring explanation than traditional and Korean secondary textbooks (Cai, Nie, & Moyer, 2010; Hong & Choi, 2014).

METHODOLOGY

Framework

A two-dimensional framework, *horizontal* and *vertical* analyses, was employed (Charalambous et al., 2010). A *horizontal* analysis provides background information on the textbook development, the number of lessons and grade level placement of certain topics. A *vertical* analysis provides in-depth understanding textbook content including how the lesson begins and concepts are developed (Table 1)

Horizontal Analysis	Vertical Analysis
Education systems	Introduction and development of topics
Number of lessons	Cognitive demands of problems
Number of problems	Type of responses
Grade level placement of topics	Practices used

Table 1: Framework for Textbook Analysis of Content and Problems

The vertical analysis is also used for the problem analysis, which consists of three dimensions – cognitive demands, response type, and practices used. To investigate the cognitive demand of problems, we adopted the *Task Analysis Guide* by Stein, Grover, and Henningsen (1996). Codes of M (Memorization) and P (Procedures without Connections) are considered lower-level demands and PC (Procedures with Connections) and DM (Doing Mathematics) are higher-level demands. Response type of each problem was examined to determine whether the mathematics problem requires students to provide only an answer (numerical, algebraic expressions or graph) or explain or process and justify their reasoning. The last part of the problem analysis is based on various practices described in literature about functions. Symbolic rule, social data, physical phenomenon, and ordered pairs are practices seen in previous studies (Hitt, 1998; Mesa, 2004).

Data: Textbook Selection and Background

Two courses of standards-based American mathematics Core Plus Mathematics Project (CPMP) textbooks – Course 1 and Course 2 – were chosen for analysis. For the Korean textbooks, Middle School Mathematics 1 and 2 by Dusan and Jihak Publishings, were selected. In addition, the curriculum guidelines from the Korean Ministry of Education (MOE) and Common Core State Standards – Mathematics (CCSS-M) were also examined. U.S. Department of Education and called CPMP an exemplary mathematics program in 1999. A recent report from the Indiana Department of Education (2011) states that CPMP is aligned well with the CCSS–M.

Reliability of Coding

For coding reliability, the two authors, fluent in both English and Korean, independently coded each problem in the textbooks. Next, a third rater, a doctoral student in mathematics education, randomly chose one textbook from each country and independently coded each problem. When the two authors disagreed, those items were coded based on majority rule using the third rater's codes. There were no items in which all three raters disagreed. The percent agreement of the three raters was between 88% and 93%. In all, there are 459 problems in the four Korean textbooks and 513 problems in the CPMP are analyzed and coded.

RESULTS

Horizontal Analysis

In Korea, there are two kinds of textbooks in Korea − government *published* and government *authorized* textbooks (Korean Textbook Research Foundation [KTRF], 1998). Whether they are government published or authorized, the content in all textbooks are almost identical. Each state or district determines school curricula in the U.S. Instead of government agencies, professional organizations such as NCTM provide curriculum guidelines and standards (NCTM, 1989). During the so–called standards era of the 1990s, the National Science Foundation and other organizations funded many curriculum development projects in the U.S. which resulted in several 'standards-based' mathematics curricula.

CPMP includes nine lessons on the topic of function and linear functions while the Korean textbooks have eleven lessons. Korean textbooks include two more lessons, but it is difficult to say that Korean students learn more topics because all content of these two lessons are included in CPMP. CPMP is designed for grades 9 to 11 (Schoen & Hirsch, 2003b), while the Korean textbooks are for grades 7 and 8. The CCSS–M recommends topics of functions be introduced in grade 8. This means Korean students learn the concept of function relatively earlier compared to CPMP students. In total, there are 459 and 513 problems in the four Korean and CPMP textbooks, respectively: 20.8 problems per lesson in Korea and 57 problems per lesson in CPMP. This result is contrast to a result of Son and Senk's (2010) study comparing elementary textbooks while coinciding with a result from a study comparing secondary school textbooks (Hong & Choi, 2014).

Vertical Analysis

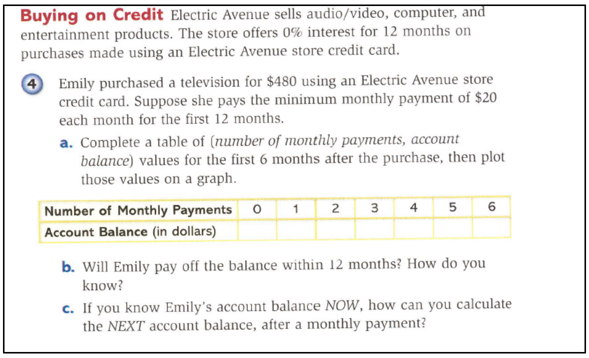
In this section, we briefly how these textbooks introduce the concept of a variable. How textbooks introduce the concept of a variable is closely related to how the concept of function are developed. A variable is defined as *unknown* in textbooks and curriculum guidelines from the Korean MOE while CPMP uses variables to describe linear patterns/functions/equations, meaning that it is something that changes. Korean textbooks' static approach, which is also found in traditional American textbooks, is called a structural approach while CPMP's approach to linear functions is a functional approach (Cai et al., 2010).

The concept of function

Differences between the two countries' textbooks are observed from the very first lesson. At the end of the first unit in CPMP course 1, Patterns of Change, CPMP defines a function in the following way:

> In mathematics, relations like these – where each possible value of one variable is associated with exactly one value of another variable – are called functions (Hirsch et al., 2007a, p. 69).

This definition is given immediately after a description of a bungee cord being attached to one weight, real-world application. Figure 1 shows problems in the lesson. Aside from problem 4-a, these problems require students to reason, think, and explain their thoughts and reasoning, simple algorithms or computations will not be enough.



Buying on Credit Electric Avenue sells audio/video, computer, and entertainment products. The store offers 0% interest for 12 months on purchases made using an Electric Avenue store credit card.

4. Emily purchased a television for $480 using an Electric Avenue store credit card. Suppose she pays the minimum monthly payment of $20 each month for the first 12 months.

   a. Complete a table of (*number of monthly payments, account balance*) values for the first 6 months after the purchase, then plot those values on a graph.

Number of Monthly Payments	0	1	2	3	4	5	6
Account Balance (in dollars)							

   b. Will Emily pay off the balance within 12 months? How do you know?

   c. If you know Emily's account balance *NOW*, how can you calculate the *NEXT* account balance, after a monthly payment?

Figure 1: First few problems about in CPMP (Hirsch et al., 2007 a, p. 152).

The following is the definition found in Korean textbooks:

> For two variables x and y, if there is one corresponding y value for x value, y is called a function of x and it can be written symbolically in the following way, $y = f(x)$. (Lee et al., 2008, p. 131)

After the definition of a function, a formal definition of domain and range of a function is followed (Figure 2) and these terms are not introduced in CPMP. Formally defining function and introducing domain and range suggests that the Korean textbooks' approaches are structural.

함수 $y=f(x)$에서 변수 x의 값을 나타내는 수 전체의 집합을 **정의역**이
라 하고, 변수 y의 값을 나타내는 수 전체의 집합을 **공역**이라고 한다.

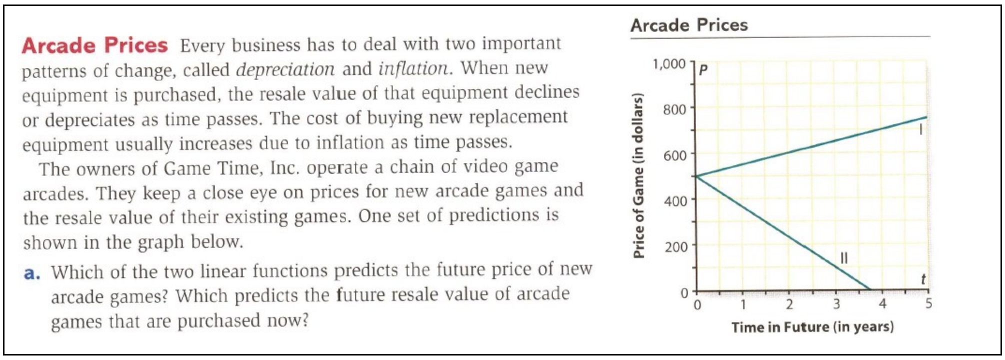
Translation: For a function $y = f(x)$, the collection of all possible x values is called the **Domain** and the collection of all possible y values is called the **Range**.

Figure 2: Domain and Range in Korea textbook (Lee et al., 2008, p. 133).

Rate of change, slope and ordered pairs

After solving five real-life context problems, CPMP introduces the concept of slope using physical phenomenon to show rate of change. On the other hand, the Korean textbooks maintain their focus on symbolic and pure mathematical approaches. Their next topic are ordered pairs, coordinate plane and sketching the graph of $y = ax$. On the last lesson about linear function in Middle School Mathematics 1, the Korean textbooks introduce some real-life applications, but differ from the CPMP because the problems require procedures of finding an equation and computing a number for a "real-life" value without requiring explanations.

CPMP's emphasis on real-life applications can be seen in Figure 3. For example, part b requires students to explain the meaning of a positive or negative slope in a realistic context of depreciation and inflation.


Arcade Prices Every business has to deal with two important patterns of change, called *depreciation* and *inflation*. When new equipment is purchased, the resale value of that equipment declines or depreciates as time passes. The cost of buying new replacement equipment usually increases due to inflation as time passes.

The owners of Game Time, Inc. operate a chain of video game arcades. They keep a close eye on prices for new arcade games and the resale value of their existing games. One set of predictions is shown in the graph below.

a. Which of the two linear functions predicts the future price of new arcade games? Which predicts the future resale value of arcade games that are purchased now?

Figure 3: Slope in a real-life context (Hirsch et al., 2007 a, pp. 158-159).

Cognitive Demands of Problems

CPMP includes more problems with higher level cognitive demands (PC and DM) than the Korean textbooks – 25.7 % in CPMP and 7.4% in Korean. The majority of problems – 74.3% of problems for CPMP and more than 90% for the Korean textbooks – require low cognitive demand (M and P).

Response Type

Over 90% of problems in the Korean textbooks require number, expression, or graph-type responses. One remarkable difference is that more than 40% of problems in

CPMP require explanations compared to only 6.5% of problems in the Korean textbooks. These results, cognitive demands response type, are consistent with previous study (Hong & Choi, 2014), which can possibly be interpreted as “textbook signature.”

Practices Used

Symbolic practice is the most frequent type for both Korean (43.2%) and CPMP (37.4) textbooks, which was also found in a previous study (Mesa, 2004). The notable difference between the two countries’ textbooks is the portion of real-life problems (Social Data and Physical Phenomena): 56.1 % in CPMP and 16.5 % in Korean textbooks.

DISCUSSION AND CONCLUSIONS

This article compared sections of functions and linear functions from four Korean textbooks and CPMP. The horizontal analysis results revealed that topics related to functions and linear functions are introduced relatively earlier in Korean textbooks than in CPMP, which confirms findings in previous studies (Steveson & Bartsch, 1992). The vertical analysis results demonstrate that CPMP places strong emphasis on real-life applications rather than pure mathematics, a functional approach while the Korean textbooks emphasize symbolic and algebraic representations, a structural approach. A majority of problems in both the CPMP and Korean textbooks only require lower level cognitive ability in contrast to the results of previous studies about textbooks of East Asian countries (Charalambous et al., 2010; Son & Senk, 2010). However, this outcome corresponds with a finding from Hong and Choi (2014) and these characteristics can be considered “textbook signature” among Korean secondary mathematics textbooks.

This study yields results, which both contradict and confirm previous findings: CPMP, the American standards–based curriculum, offers more opportunities for students to solve, explain, and reason about higher level cognitive demanding mathematics problems than Korean secondary textbooks. This conflicts with American students’ mediocre performances on international comparative studies because CPMP students can possibly engage in more meaningful and interesting tasks when learning mathematics. Our results confirm the well-known fact that the link that connects what textbooks potentially offer and what students learn is what teachers do and how they implement textbook content in their classes. Further studies on how teachers in different countries implement and use curriculum materials may explain the gap between what textbooks offer and students’ performances. Because of contradicting results, Hong and Choi (2014) state that textbooks may not be the reason for American and Korean students’ performances on international comparative studies. Our results confirm their assertion that CPMP students have several opportunities to reason, solve, and think about mathematics problems. Although we cannot generalize our results, properly implementing these textbooks in mathematics classes may be the key to improving students’ performance in international studies.

In terms of the contents of these textbooks, each country's textbook may need to equally include various function practices. Korean students may struggle with real-life contexts of functions while CPMP students may have difficulty with symbolic and algebraic representations of the function concept. Textbook publishers and authors should consider distributing other representations and practices equally.

Further studies to measure "lesson signature" and "textbook signature" may be interesting as well. A previous criticism of American mathematics curriculum is the incoherent mathematics curriculum among different states (Schmidt et al., 2001). If there exists "textbook signature" among other standards–based textbooks, we can have a more coherent mathematics curriculum.

References

Brenner, M., Mayer, R., Moseley, B., Brar, T., Durán, R., Smith Reed, B., & Webb, D. (1997). The role of multiple representations in learning algebra. *American Educational Research Journal*, *34*(4), 663-689.

Cai, J., Nie, B., & Moyer, J. C. (2010). The teaching of equation solving: Approaches in standards-based and traditional curricula in the United States. *Pedagogies: An International Journal*, *5*, 170-186.

Charalambous, C., Delaney, S., Hsu, H.-Y., & Mesa, V. (2010). A comparative analysis of the addition and subtraction of fractions in textbooks from three countries. *Mathematical Thinking and Learning*, *12,* 117-151.

Eisenberg, T. (1992). Functions and associated learning difficulties, In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 140-152). Dordrecht: Kluwer Academic.

Fan, L., & Zhu, Y. (2007). Representation of problem-solving procedures: A comparative look at China, Singapore, and US mathematics textbooks. *Educational Studies in Mathematics, 66*, 61-75.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J. K., … Stigler, J. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 videostudy* (NCES 2003-013 Revised). Washington, DC: U.S. Department of Education, National Center for Education Statistics.

Hirsch, C. R., Fey, J. T., Hart, E. W., Schoen, H. L., & Watkins, A. E. (2007). *Core-Plus Mathematics: Contemporary mathematics in context, Course 1*. Columbus, OH: Glencoe/McGraw-Hill.

Hitt, F. (1998). Difficulties in the articulation of different representations linked to the concept of function. *Journal of Mathematical Behavior, 17*(10), 123-134.

Hong, D., & Choi, K. (2014). A comparison of Korean and American secondary school textbooks: The case of quadratic equations. *Educational Studies in Mathematics*, *85*(2), 241-263.

Indiana Department of Education. (2011). *Mathematics – Integrated mathematics I-III curricular materials*. Retrieved from http://www.doe.in.gov/achievement/curriculum/mathematics-integrated-mathematics-i-iii-curricular-materials

Kang, O., et al. (2009). *Joonghakyo Soohak 1* [Middle School Mathematics 1]. Seoul, Korea: Dusan Publishing Co.

Korean Textbook Research Foundation. (1998). *A study on making list of all textbooks published in Korea since 1945 and studying how to collect these textbooks*. Seoul, Korea.

Kramer, S. L., & Keller, R. (2008). A brief report: An existence proof: Successful joint implementation of the IMP curriculum and a 4 x 4 block schedule at a suburban U.S. high school. *Journal for Research in Mathematics Education*, *39*, 2-8.

Lee, K., Wang, K., Song, K., Lee, K., & Ahn, I. (2008). *Joonghakyo Soohak 1*[Middle School Mathematics 1]. Seoul, Korea: Jihak Publishing Co.

Li, Y. (2000). A comparison of problems that follow selected content presentations in American and Chinese mathematics textbooks. *Journal for Research in Mathematics Education*, *31*, 234-241.

Mesa, V. (2004). Characterizing practices associated with functions in middle school textbooks: An empirical approach. *Educational Studies in Mathematics*, *56*, 255-286.

Ministry of Education in Korea. (2007). *The 7th national mathematics curriculum*. Seoul, Korea: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H. C., Wiley, D. E., Cogan, L. S., & Wolfe, R. G. (2001). *Why schools matter: A cross national comparison of curriculum and learning.* San Francisco, CA: Jossey-Bass.

Son, J., & Senk. S. (2010). How standards curricula in the USA and Korea present multiplication and division of fractions. *Educational Studies in Mathematics*, *74*, 117-142.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in standards classrooms. *American Educational Research Journal*, *33*, 455-488.

Stevenson, H. W., & Bartsch, K. (1992). An analysis of Japanese and American textbooks in mathematics. In R. Leetsman & H. J. Walberg (Eds.), *Japanese educational productivity* (pp. 103-134). Greenwich, CT: JAI Press.

Stigler, J. W., Lee, S.-Y., Lucker, G. W., & Stevenson, H. W. (1982). Curriculum and achievement in mathematics: A study of elementary school children in Japan, Taiwan, and the United States. *Journal of Educational Psychology*, *74*, 315-322.

Vinner, S., & Dreyfus, T. (1989). Images and definitions for the concept of function. *Journal for Research in Mathematics Education*, *20*(4), 356-366.

Zhu, Y., & Fan, L. (2006). Focus on the representation of problem types in intended curriculum: A comparison of selected mathematics textbooks from mainland China and the United States. *International Journal of Science and Mathematics Education*, *4*, 609-626.