

SMALL GROUP INTERACTIONS: OPPORTUNITIES FOR MATHEMATICAL LEARNING

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Small group interactions can provide rich conceptual mathematical understandings. This paper reports on the mathematical talk of Māori and Pāsifika students as they participated in small group activity. The findings illustrate that when the students were scaffolded to work collaboratively the talk shifted between focusing on mathematics (mathematizing) and people (subjectifying) and this supported their learning.

INTRODUCTION

Student talk, and the role it holds in mathematics education, has increasingly been explored by mathematics education researchers in recent decades. The general consensus is that students learn richer and deeper mathematical concepts when provided with opportunities to engage in talk and interactions with others during mathematical activity (White, 2003; Wood, Williams, & McNeal, 2006). However, we know that just any student talk or interaction is not sufficient to ensure conceptual learning within productive talk. For example, Cohen (1994) and Mercer and Wegerif (1999) argue the importance of problem solving activity in which students are required to rely on each other and use exploratory talk. Other researchers (e.g., Boaler, 2008; Hunter, 2007) promote the need for teachers to develop productive mathematical talk through activity that requires group members to interact and work collaboratively. Although the importance of productive mathematical discourse is well recognised, what forms it can take are less well-known. Likewise, we do not know what other social forms of talk also support student learning. The focus of this paper is on the mathematical and social talk used within small group interactions. The specific research questions explored in this paper are:

What patterns of interaction did the students engage in during small group activity?

How did the patterns of interaction support or limit individual opportunities for mathematical learning?

The theoretical framework of this study is derived from a sociocultural perspective. From this perspective sociocultural researchers (e.g., Andriessen, 2006; Lerman, 2001) suggest that academic learning is inherently social and embedded in active participation in communicative reasoning processes. This includes attending to the academic and social aspects of the interactions and provides reasons for exploring all forms of talk used in small group activity to explain how the interactions may provide affordances or constraints in the learning process.

CONCEPTUAL FRAMEWORK

Within the commognitive framework proposed by Sfard (2008) student talk is intertwined with mathematical learning. Sfard (2008) outlines how as students engage in activity their mathematical talk draws closer to more academic mathematical discourses. She contends that their participation in the mathematical discourses—that is their talk about mathematical objects—is a needed component for learning to occur. In addition, affordances for a change in the mathematical discourse can only occur if the students are engaged in mathematizing—that is that they are communicating about mathematical objects—and the amount and quality of the mathematizing directly correlates with conceptual learning of mathematics.

Although mathematizing is an essential component for mathematical achievement because learning is social other factors need consideration. When students are engaged in mathematical activity they may talk about mathematical objects but they also talk about other things including themselves and other students. Significant work by a group of researchers (e.g., Boaler, 2008; Cohen, 1994; Hunter, 2007; Mercer & Wegerif, 1999; Webb & Mastergeorge, 2003; Wood & Kalinec, 2012) has examined ways student talk can involve both academic and social ways of participating in mathematical activity, in recognition that particular types of these social interactions also support mathematical learning. For example, Wood and Kalinec (2012) illustrate ways in which students in small groups use different types of talk depending on their focus. These include a focus on mathematical objects (mathematizing), people (subjectifying), or their attributes (identifying). Although mathematical learning occurred Wood and Kalinec (2012) illustrated how opportunities for learning were available differently for different students, according to another group member's vision of their peers, and themselves as the perceived appointed 'teacher'.

However, students can be scaffolded by teachers to learn ways to interact and use both academic and social talk to advance mathematical achievement of all members of the small group. For example, Boaler, (2006; 2008) and Hunter (2007) illustrated the effectiveness of giving student open-ended problems and tasks which support a range of ways for group members to contribute to the group processes. Of key importance in the development of productive group processes and discourse was the emphasis placed on group member's responsibility to each other. Central to the group responsibility was the requirement that the students justify and provide valid reasoning. They were also required to actively engage and monitor their own reasoning and the reasoning of others and when confused ask questions or seek other forms of help. Webb and Mastergeorge (2003) suggest that opportunities for learning for all group members are increased when they effectively seek or provide appropriate help. They contend that help seekers need persistence and precision in their requests and helpers must not only provide clear explanations but also monitor how their response supports the help seeker's understandings. Specific forms of talk also need to be scaffolded. Without specific structuring Mercer and Wegerif (1999) showed that the most common forms of talk children used in small group activity were either disputational or cumulative;

forms of talk which are not productive in mathematical activity. But, when specific guidance is provided students develop exploratory talk; a productive form of talk which supports mathematizing.

METHOD

The data presented in the paper is part of one of three consecutive studies which spanned six years. In the design research approach (Cobb, 2000) used, a Communication and Participation Framework (CPF) (See Hunter & Anthony, 2011) was collaboratively constructed (in the first project) and employed across all the projects. The Framework provided the teachers with a flexible and adaptive tool to map out and reflectively evaluate pathways of pedagogical actions to use, to guide the students' development of academic discourses (exploratory talk) and other forms of social talk which support mathematical learning.

The teacher reported on in this paper was involved in the second Project and was an experienced teacher. The students were largely of Pāsifika (South Pacific) ethnic groupings, their ages ranged from 8-12 years. The study was conducted in New Zealand low income urban primary schools.

The following data analysis table was adapted from Wood and Kalinec (2012, p. 113)

On-task codes	Mathematizing		Any utterance about a mathematical object.
	Subjectifying	Action oriented subjectifying	Any utterance that focuses on a person's on-task actions rather than on the person as such.
		Identifying	Any utterance about who a person is or his/her features.
	None of the above		Any utterance that was on task, but did not fall into any of the other on-task categories.
Off-task codes	Subjectifying	Action oriented subjectifying	Any utterance that focuses on a person's off-task actions rather than on the person as such.
	Identifying	Identifying	Any utterance about who a person is or his/her features.
		Blazing	Any utterance that is an exaggerated negative identification of another person or members of another's family.
	None of the above		Any utterance that is off task, but did not fall into any of the categories above.

Table 1: List of codes and descriptions

Data collection over one year included teacher and student interviews, classroom artefacts, field notes, and a large collection of video recorded lesson observations. The data reported on in this paper is based on transcriptions of the entire video recorded

lesson observations. The transcripts were split into each speaker’s turns and then further split into one or more utterances based upon the focus of the talk. To analyse the data we adapted and used parts of the coding structure employed by Wood and Kalinec (2012). We drew on the categories they used (see Table 1) to code the utterances and used this to analyse the data to provide both a quantitative and qualitative view.

This paper reports on two separate lesson observations. The first transcript is of a lesson which occurred in the second month of the study and the second transcript occurred in the tenth month of the study. Both lessons are representative of small group mathematical activity and the academic and social talk the same group of students were engaged in, in response to the on-going scaffolding provided by the teacher.

			Utterances
On-task codes	Mathematizing		41%
	Subjectifying	Action oriented subjectifying	30%
		Identifying	1%
	None of the above		2%
Off-task codes	Subjectifying	Action oriented subjectifying	
	Identifying	Identifying	14%
		Blazing	2%
	None of the above		10%

Table 2: Type and frequency of utterances during lesson 5

			Utterances
On-task codes	Mathematizing		53%
	Subjectifying	Action oriented subjectifying	41%
		Identifying	1%
	None of the above		1%
Off-task codes	Subjectifying	Action oriented subjectifying	
	Identifying	Identifying	3%
		Blazing	
	None of the above		1%

Table 3: Type and frequency of utterances during lesson 14

RESULTS AND DISCUSSION

Table 2 and Table 3 summarise the type and frequency of utterances in the two lessons. In this section we elaborate on the different categories of interactions the students engaged in and explore how these supported or limited learning of different students. We look at what was learnt and how the talk changed across the year (represented by the two lessons).

Mathematizing and Action-oriented subjectifying talk

As explained in Table 1 mathematizing talk focuses on mathematical objects while subjectifying talk focuses on people and what they are doing as part of their on-task actions. In both lessons mathematizing talk and action-oriented subjectifying talk were a clear feature of the on-task behaviour of the students. In the first transcription (See Table 2) 41% of the talk was mathematizing and 30% was subjectifying talk. The students were engaged in on-task talk 74% of small group activity. In the second transcription (See Table 3) 53% of the talk was mathematizing and 41% was action oriented subjectifying talk. The students were engaged in on-task talk 96% of the small group activity. However, there was a clear difference in the ways both forms of talk were used in the two lessons. This contributed significantly to the learning of one all students but one in particular we named Viliami.

In the excerpt which follows (of the first lesson transcription) the students used both forms of talk to make sense of what was required in the task, construct a cumulative solution strategy or to develop an explanation of the strategy they were using. On-task action oriented subjectifying talk was used by different group members to support the mathematizing.

Excerpt 1: Constructing an explanation cumulatively

The students are solving a problem that involves adding 899 and 156.

Timoti: Oh yeah, we could put 800...

Viliami: 800 plus 100

The students are mathematizing but they are doing this using cumulative talk (Mercer & Wegerif, 1999). Without listening and exploring the reasoning of other members each student adds the next step they think will work.

Timoti: Yeah but we've got to tell how we added it.

Timoti has begun to use action oriented subjectifying talk. He focuses the group on their need to explain their actions which increases all their opportunities to learn.

Timoti: See we shouldn't do that fast one, dah. 800 plus 156

Timoti is describing a different solution but the students are not actively engaged in exploring the reasoning. However, he has invited his peers to think about their shared reasoning and as a result Viliami engages in mathematizing

Viliami: That's um... what is this called? 800 plus 100. Is that place value? Like then 900 plus 90 plus 50? Where do the zeroes go now?

The students continued to talk past each other and although they were all focusing on the mathematics they did not successfully solve the problem. This excerpt illustrates the need for the students to not only mathematize but also to engage in the reasoning being used by other members of the group.

Excerpt 2: Using exploratory talk to conceptualise a mathematical explanation

In the following excerpt (from the second lesson transcription) (see Table 3) both mathematizing and action oriented subjectifying talk are used to engage and progress the reasoning of all members of the small group. The students are solving a problem which requires them to multiply 24 by 5. They start by sense-making what they are required to do using both mathematizing and action oriented subjectifying talk.

Viliami: 24 x 5 equals... I know, I know what to start with 20 x 4

Sela [Pointing at the numbers]: Do you understand where you got these numbers from?

Timoti and Viliami together: Yes

Timoti to Sela: Do you want me to explain where we got the numbers from?

Sela has used action oriented subjectifying talk to question the other group members and open up the talk so they can all share their understandings of the problem. This supports them to work towards a common mathematical goal.

Timoti: We got the 24 from how many corn plants Sione's Dad wanted to plant and we got our 5 from how many corn plants he planted in each row. Does anyone disagree or..?

Timoti uses the context of the problem to make the numbers experientially real which makes the problem accessible for all members of the group.

Viliami: I agree that's right

Sela: I agree too

Timoti: But you've got to say why. Why do you agree?

Sela: Because it says on the problem, cos that's, because that's how much Sione's dad wants to plant, 24 rows of corn plants. He wants 5 corn plants in each row.

Timoti presses further using action oriented subjectifying talk. Through this he establishes the responsibility of all group members to actively engage in sense making. They begin to construct an explanation using mathematizing talk. Each member uses exploratory talk to examine the ideas being constructed.

Sela [recording]: Hey why not split up the 20 from the 24 into 4 lots of 5. Do you get it? You need to be able to explain why we do one more of...

Timoti and Viliami: The 5 x 4

Viliami [pointing at the extra 5 x 4]: What does that mean?

Sela: And we have one more because of the 24 x 5. Can you show where that is so you can explain...

Viliami: I know what that means. I can see it on here because we split the 20 and then we had 4 more. But why did we split the 20...why...

Sela and Timoti together: You know because we do not know our 20 times so 5 times is the easiest.

The group continue to construct and record a conceptual explanation. Sela and Timoti push Viliami to use what Webb and Mastergeorge (2003) describe as effective help seeking behaviour. Sela directs a question at Viliami

Sela: Why did we really have to repeat five times 4?

Viliami: What's the answer?

Timoti: Come on, if you don't know the answer that means you don't know what we're doing. Do you need help? Say how we can help you.

Timoti uses action oriented subjectifying talk to model help seeking behaviour. Viliami is supported to question specifically and extend his mathematical discourse.

Viliami: Can you explain the first part to me please?

Timoti: Cos we're splitting up the 20 from the 24 into 4 lots of 5. Do you get it? You need to be able to explain why we do one more of...

Viliami [Pointing at the last 4 x 5]: What does that mean?

Sela: And we have one more because of the 24 x 5 and can you show where that is so you can explain...

Viliami [Points at the section of the recording as he speaks]: I know what that means. I can see it on here because we split the 20 and then we had 4 more.

Subsequently in large group sharing Viliami shared the strategy. He explained and justified each step of the process to the class. His learning had been durable and he now had access to a discourse to provide a conceptual explanation and justification.

CONCLUSION

Evidence is provided in this paper of the positive outcomes for mathematics learning which can result from small group activity. In both lessons the students spent a significant time mathematizing and using socially based action oriented subjectifying talk. This form of talk was used by group members to ensure that they were all able to engage in mathematizing. The increased use of this on-task social talk illustrates the importance of teacher actions to ensure students can talk and work collaboratively as suggested by other researchers (e.g., Boaler, 2008; Cohen, 1994; Hunter, 2007; Mercer & Wegerif, 1999). The effects of teacher actions to increase both mathematizing and action oriented subjectifying talk were evident in the second lesson where the students were engaged in the mathematical discourses for more than 96% of the small group activity. The findings of this study support Sfard's (2008) contention, that participation in mathematical discourses is essential for conceptual learning.

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