INTERACTIVE CONSTRUCTION OF ATTENTION PATHS TOWARD NEW MATHEMATICAL CONTENT: ANALYSIS OF A PRIMARY MATHEMATICS LESSON

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On the basis of the construct of "discursive focus" by Sfard (2000), this study explores how students' attention is brought to new mathematical content in whole-class interaction between the teacher and the children. In a sixth-grade lesson introducing the concept of constancy of proportion, we analyzed the progression of social interaction in terms of how different foci were presented, problematized, or modified. The results show that the children's vague attention to the constant number was questioned and made an object of examination. The children's attention was then carefully controlled by involving them in building new perspectives, which became the basis for making sense of constancy of proportion. We also point out several significant teaching actions for making this process happen.

BACKGROUND AND PURPOSE OF THE STUDY

Over the past few decades, more studies have been conducted to unpack features of classroom discourse that provides rich learning opportunities. Some researchers study the form and structure of exchanges between teacher and students in terms of hidden classroom-interactive patterns (e.g., Voigt, 1985; Wood, 1998). Many studies also explore the mode and format of classroom communication in which students engage in argument (e.g., Lampert & Blunk, 1998; Krummheuer, 1995). Building on these studies, we have proposed a social interaction pattern to capture interactions in lessons introducing new mathematical content (Koizumi & Hino, in preparation). By examining a primary mathematics lesson conducted by an experienced teacher, this paper proposes to clarify the ways children's attention is brought to new mathematical content in whole-class interactions after their individual activity.

One of two reasons for exploring this type of classroom interaction is that few studies have concentrated on the social interaction pattern that discloses the students' elaboration process for their ideas about lesson objectives. Several proposed patterns show that the teacher's purpose receives more weight than students' thinking (e.g., Voigt, 1985). In the alternative pattern, students take conversational control, and they are responsible for re-explaining their thinking to others (e.g., Wood, 1998). The analysis of classroom episodes mainly concerns how students are helped by the teacher to talk about important mathematical ideas with respect to a solution given by one student; however, learning opportunities would be embedded in various interactional contexts during the lesson. To deepen our understanding of the relationship between social interaction and the development of students' mathematical thinking (Wood et al., 2006), we believe that whole-class interaction directed to new mathematical

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content will offer important information. The second reason is that this type of interaction requires the teacher to fulfill active roles in comparing, integrating, or evaluating varied solutions presented by the students. Walshaw and Anthony (2008), in their literature review on teachers' roles in developing high-quality classroom discourse, repeatedly assert the importance of a teacher who does *not* simply hear and accept all answers, but *attentively listens to the mathematics in students' talk*. In this paper, we intend to concretize the teacher's role by observing and analyzing what an experienced teacher actually does during such interactions.

Thus, this paper addresses two research questions: (i) What are the paths of children's attention to new mathematical content? (ii) What kinds of leadership does the teacher employ to catalyze this process?

THEORETICAL FRAMEWORK

In our investigation, we use the construct of discursive focus by Sfard (2000). Pursuing the construction of mathematical objects from the discourse perspective, Sfard argues that the effectiveness of verbal communication is determined by degree of clarity of discursive focus presented within the communication. In doing so, she distinguishes three components of focus employed to grasp the object of attention. *Pronounced focus* is "the word used by an interlocutor to identify the object of her attention" (p. 304). *Attended focus* is "what and how we are attending—looking at, listening to, and so forth—when speaking" (p. 304). Finally, the *intended focus* is the "interlocutor's interpretation of the pronounced and attended foci"; this component includes "the whole cluster of experiences evoked by these other focal components as well as all the statements he or she would be able [to] make on the entity in question, even if they have not appeared in the present exchange" (p. 304). Although intended focus is less tactile than the other two, its presence can be signaled by particular discursive clues or the speaker's tendency to interchangeably use different names. According to Sfard, this focus indicates an actual, context-dependent discursive occurrence. When these foci relate to some stable, self-sustained entity, *an object is constructed discursively*. The discursive objects come into being (or into the signifier's realization) by the important processes of saming, encapsulating, and reifying (Sfard, 2008, pp. 170-171).

The three foci have helped make transparent the teacher's support and guidance in the interaction progress for an introductory lesson to new mathematical content (Koizumi & Hino, in preparation). In the present paper, we further analyze the interaction in another sixth-grade lesson conducted by the same teacher. Comparing this lesson with the previous one, we found that children in this lesson struggled more in the presented task by the teacher.

RESEARCH METHOD

In January and February 2009, ten consecutive lessons were implemented and recorded in a sixth-grade classroom in a public primary school in Japan. The lesson topic was *proportional relationship*. When the data were collected, the teacher had 30 years of teaching experience and occupied the school's position as head of mathematics curriculum and instruction.

These lessons were recorded according to the *Learners Perspective Study – Primary* data-collection procedure by revising the *Learners Perspective Study* methodology (Shimizu, 2011). In the classroom, three cameras (focused on the teacher, target children, and the whole class) video recorded each lesson. After each lesson, the target children were interviewed about what they studied in the lesson and what they thought was important. The teacher was interviewed twice about her thinking and emotions during the lesson. In addition, she was asked to write the goal of each lesson, along with her personal reflections.

In these lessons, children were introduced to the concept of proportional relationship mainly through tables. With tables, proportional relationship was defined on the basis of co-variation between two quantities, as shown in Lesson 2 (L2) of Table 1, below. The relationship between two quantities, △ and ○, was also formulated in the equation "○ × fixed number = △ and △÷○ = fixed number" (L5). Graphical representation of a proportional relationship was also introduced by plotting several points and observing their arrangement as a straight line traveling through the point where both quantities are zero (L6, L7).

Lesson	Topic	Lesson	Topic
1	Exploring the relationships of two quantities varying together.	6	Representing the relationships of two quantities with graphs.
2	Definition of proportional relationship	7	Exploring the features of the graph.
3	Checking whether two quantities are in the proportional relationship.	8	Appreciating the value of graph. Exercises.
4	Making tables and checking whether two quantities are proportional.	9	Exercises (Using digital material)
5	Finding constancy of proportion in the relationship of two quantities.	10	Challenging exercises.

Table 1: Topics of the Ten Lessons

In this paper, we use the data on L5 intended to introduce new mathematical content to the children. Analysis was qualitatively conducted to capture the teacher's methods of eliciting and organizing the children's thinking when introducing new mathematical content. In the first stage of analysis, we identified the phases and activities in the transcripts of "public" talk by the children and the teacher. Using Sfard's three foci in the second stage, we discerned specific instances of the teacher's support and guidance during the interaction process. Our interest especially concerns how the focus is modified or a new focus is built and what role the teacher plays. We corroborated some of our interpretations with the data from the teacher and the targeted children.

CLASSROOM EPISODE AND INTERPRETATION

Lesson 5 aimed to find constancy of proportion in the relationship of two quantities and to express it in the form of an equation. Since L2, the class had been studying the *horizontal* (co-variation) relationship in a situation of pouring water into a tank using a table showing various amounts of time and the corresponding depths of water in the tank. In L5, the teacher used the same task, but this time she intended the children to *vertically* (correspondence) look at the table to derive constancy of proportion.

<table>
<tr><th>Phase</th><th>Activity</th></tr>
<tr><td>Proposing the problem</td><td>The teacher presented the task. She distributed the worksheets below to children.

Let's examine in more detail the relationship in which depth of water is proportional to time. Let's find the fixed number that does not change by vertically looking at the table.
<table><tr><td>Time (min)</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>Depth (cm)</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td><td>14</td><td>16</td><td>18</td><td>20</td></tr></table></td></tr>
<tr><td colspan="2">Individual activity</td></tr>
<tr><td>Eliciting children's ideas</td><td>Children presented their ideas.
AO: I think that 2 x of time is the depth of water.
IT: I found that if I divide the depth of water by 2, it becomes the time. This can be said to all the values, well…, if I used 4÷2, then it becomes 2, which is the time. Therefore, I think this [2] can be said to be the number not moving. Therefore, I think that the depth of water equals the time divided by 2.
The teacher pointed out that if we apply IT's idea to the equation, it becomes 2=1÷2. TA proposed the equation depth of water÷2=time. Then, several children talked about 0.5 as a constant number. Finally, NA spoke that depth of water divided by time becomes 2 all the time.</td></tr>
<tr><td>Focusing on the object of examination</td><td>The teacher proposed that the fixed number should be 2 based on the logic that the depth of water increases 2 cm every time 1 min.</td></tr>
<tr><td>Formulating the result on the basis of the object</td><td>The teacher said that the proportional relationship can be expressed by ○×2=△ and △÷○=2 using ○ as time and △ as depth of water. She also mentioned that the fixed number is 2 this time and that the number can vary according to the proportional relationship in the situation.</td></tr>
</table>

Table 2: Phases and Activities in Lesson 5

When presenting the task, the teacher clearly stated the lesson's goal: "Today, I want you to find the vertical relationship." When she explained the worksheet distributed to the children (see Table 2), she said more about the vertical relationship: "I stressed the point of finding the fixed number that does not change at all when you look at the vertical relationship in the table." Then, as usual, the teacher spent some time allowing the children to work on the task in their own ways. The task was not easy for many of the children. In particular, they were observed to have difficulty in formulating the equation (depth÷time=2), which was the objective of L5. In the following section, we describe the whole-class interaction after the individual activity, especially focusing on the phase "eliciting children's ideas."

Correcting a Mistake by a Child

Two children, AO and IT, presented their findings on the blackboard (Figure 1).

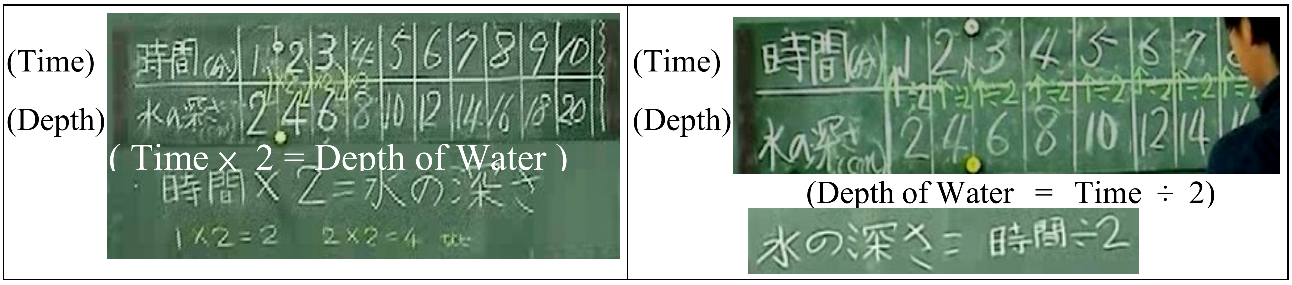

Figure 1: Work presented by two children: AO (left) and IT (right)

Although IT's equation was not correct, other children raised their hands to show their agreement with IT's work. Then the teacher questioned the correctness of his equation:

01	T:	By the way, IT, if we put *1* in the *time* [in your equation], then it becomes *1÷2*. This makes the *depth of water* strange, don't you think?

After the teacher's comment, TA proposed the equation *depth of water ÷ 2 = time* by explaining her reasoning:

02	TA:	For example, if the *depth of water is 2* and time is *1*, then *2 ÷ 1* is *2*, and if the depth is *4* and time is *2*, *4 ÷ 2* becomes *2*.

By interrupting IT when he was trying to erase his equation, the teacher continued the conversation.

03	T:	Let's look at what the differences are [in these two equations].
04	T:	Very good, they gave us very good examples [to consider]. We'd better substitute them [the word in the equation] with different numbers, I mean numbers. If we change the *time* to *1* in the equation made by IT, if we make *time* into *1*, then it eventually becomes *1 ÷ 2*. Don't erase it. Please write it above [the equation]. It's *1 ÷ 2*. Please write *1* above the time and write *÷2*.
05	T:	[IT wrote above his equation, as directed.] Yes, that's right. Let's write "*1 ÷ 2*" there.
06	S:	It's *0.5*.
07	T:	And then, what is the answer?
08	S:	*0.5*.
09	T:	It becomes *0.5*,... it looks odd. It doesn't become the *depth of water*, does it?
10	S:	Oh… no, it doesn't.
11	T:	Are you OK? OK? Let's see about TA's [equation]. How about TA? If we put *4*, *4*, in the "*depth of water*" [in her equation], *4* divided by *2* is… does it become *time*?
12	S:	Yes, it becomes *time*.
13	T:	Does everyone understand? Are you all right with this?

<u>Interpretation</u>: When IT presented his work (see Table 2), he provided a focus with respect to the constancy of proportion in the table. He expressed it as *the number not moving* (pronounced focus). It accompanied attended focus with arrows and *÷2* in all

of the corresponding cells in the table (see Figure 1). He further explained his reasoning with his hand moving, which served as the attending procedure. However, his pronounced and attended foci were concerned only with the table. It is likely that his intention to identify the common number resulted in relating two numbers in the upper and lower rows. Weak focus on the equation was also observed in other children, as evidenced by some who agreed with IT. Noticeably, TA showed similar focal behavior even though she developed the correct equation (line 02).

Then, the teacher focused the children's attention on the differences between the two equations by comparing them in relation to the corresponding table (lines 03-12). During the interaction, the teacher closely looked into the two equations by connecting each word and symbol in the equation with the numbers in the table. She provided an attending procedure, i.e., *dividing the number in the upper row by 2 in the equation and checking whether the answer is the number in the corresponding lower row* (line 04). This was the first time that the class explicitly attended to the table in relation to the equation. This procedure involved the children in the process, rather than employing teacher's explanation. As a result, several children vocalized their understanding in line 10. In line 11, they applied the same attending procedure to TA's equation.

Children's Proposing New Equation

Then, several children began to talk about *0.5* as a constant number:

14	S:	Teacher. Well… These people thought…
15	S:	All of them are *0.5*.
16	S:	It is *0.5*.
17	S:	If we divide *time* by *2*, all of them can become *0.5*…
18	T:	Oh, well. If we divide *time* by *2*…
19	S:	They all become *0.5*.
20	S:	Yes, you are right.
21	T:	Oh…, *time* divided by *2*. Yes.
22	S:	Let's see. … All are *0.5*, aren't they!
23	T:	Oh, well, but, *time* divided by *2*, what? If we divide *time* by *2*, then, let's see… what? What do you mean by *time*? Do you mean to divide [all of] 1, *2, 3, 4, 5, 6, 7, 8, 9*, and *10* by *2*? What do you mean?

Here SU raised his hand and conveyed his thinking about the object of discussion:

24	SU:	Yes. Well, I mean what these people were saying before. I would say to divide *time* by *depth of water*; then, it becomes *0.5*. I think it will become *0.5* if we do *1÷2*, *10÷20*, *7÷14*, or *5÷10*.

<u>Interpretation</u>: The children actively stated, *all of them are 0.5* (lines 15, 16). *All of them* (pronounced focus) again lacked clarity, and this triggered a child to vocalize an attending procedure (line 17). Because the child attended only to the table, the teacher intervened by questioning the validity of *time divided by 2 is 0.5* (line 23). She specifically mentioned the location of values that should be caught by careful attention.

At this moment, she provided another important attending procedure to construct focus on constancy of proportion, i.e., *to check the equation by not only one pair of the first two numbers (1 and 2), but also multiple pairs of numbers in the table*. Then, SU clearly provided this attending procedure when she justified her equation (line 24).

A Child's Proposing Another Equation

Another child, NA, raised her hand and proposed her equation:

25 NA: In my case, I did the *depth of water* divided by *time* and the *constant number…* (She went to the blackboard and wrote *depth of water ÷ time = 2*.) Well, I used this [equation] for every [number in the table]. I did the calculation *depth of water÷* for the numbers in other places, and they all become *2*.

<u>Interpretation</u>: NA explained her equation by clearly mentioning that the equation is valid for every corresponding number in the table. She explicitly offered different pronounced foci "constant number," "every," "other places," and "all." They consistently suggest her intended focus on constancy of proportion, in which not only the table, but also the equation is assigned an important position.

DISCUSSION

In the previous section, using the three foci, we illustrated how the children's attention shifted to new mathematical content. The children's vague attention to the constant number was repeatedly questioned and made an explicit object of examination. In this process, the children's attention was carefully controlled by involving them in building new attending procedures, which became the basis for making sense of constancy of proportion. Sfard (2000) argues that the lack of equilibrium between the focal ingredients (pronounced, attended, and intended foci) impels discursive growth. In our analysis, we also observed similar disequilibrium triggering the necessity of well-defined attended focus that guides the communicator's interpretations. Through these processes, the children's focus became clearer and more consistent, a process closely connected to developing comprehension of new mathematical content.

Importantly, the objective of L5 included a mathematical equation as the symbolic means for expressing constancy of proportion. For the children, expressing regularity in the form of an equation was a novel experience. It caused a certain perplexity, but at the same time, it enabled the participants to talk about the validity of different proposals on the constancy of proportion. The children and the teacher proposed, questioned, supplemented, or justified their ideas to shape a clear, precise focus on constancy proportion for the equation. Relying on the children's previous experiences in the lessons, different symbolic means contributed both as metaphor (table) and as rigor (equation), two important discursive steering forces (Sfard, 2000).

Furthermore, the results demonstrate that teacher played a significant role in successfully conducting this process. In attentive listening to the children's talk, the teacher carefully assessed the mathematics behind their talk. Moreover, she purposefully sustained the interaction by providing the foci necessary for them to make sense of new mathematical content. Here two observations should be noted. First, the teacher knew when to intervene, i.e., intentionally to "step in and out" (Lampert & Blunk, 1998) of the interactions. The teacher intervened in certain pronounced foci, especially weak focus on the constancy of proportion in the equation, and made it a target of examination. Second, her supportive method of providing the foci was responsive rather than directive (Walshaw & Anthony, 2008). The teacher provided two important attended foci and procedures of relating the equation to numbers in the upper and lower rows of the table. Both were provided in the middle of interactions with the children, rather than in sole, advance explanation by the teacher.

It should be noted that these actions are closely linked to the teacher's conscious lesson objectives (Koizumi & Hino, in preparation). Therefore, children's paths to new mathematical content become clearer when they are examined throughout the phases of a lesson, and furthermore, in the sequence of lessons. At the same time, since our results are based only on a case study, we need additional analyses of classroom interactions to identify more and other teacher actions in providing children with focus building and refining activities when presenting new mathematical content.

References

Funahashi, Y., & Hino, K. (in press). The teacher's role in guiding children's mathematical ideas toward meeting lesson objectives. *ZDM*.

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 229-269). Hillsdale, NJ: Lawrence Erlbaum Associates.

Lampert, M., & Blunk, M. L. (Eds.). (1998). *Talking mathematics in school: Studies of teaching and learning*. Cambridge, UK: Cambridge University Press.

Sfard, A. (2000). Steering (dis)course between metaphors and rigor: Using focal analysis to investigate an emergence of mathematical objects. *Journal for Research in Mathematics Education*, *31*(3), 296-327.

Sfard, A. (2008). *Thinking as communicating*. Cambridge, NY: Cambridge University Press.

Shimizu, Y. (Ed.). (2011). *Cross-cultural studies of mathematics classrooms from the learners' perspective: Research report of grants-in-aid for scientific research by Japan Society for the Promotion of Science* (No. 19330196) [in Japanese]. Tsukuba, Japan: University of Tsukuba.

Voigt, J. (1985). Patterns and routines in classroom interaction. *Rechèrches en Didactique des Mathématiques*, *6*(1), 69-118.

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research*, *78*(3), 516-551.

Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 167-178). Reston, VA: NCTM.

Wood, T., Williams, G., & McNeal, B. (2006). Children's mathematical thinking in different classroom cultures. *Journal for Research in Mathematics Education*, *37*(3), 222-255.