FROM KNOWLEDGE AGENTS TO KNOWLEDGE AGENCY

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In this report we further develop the notion of knowledge agent and analyse knowledge agency in an 8th grade mathematics classroom learning probability. By knowledge agency we mean the many ways and variations in which knowledge agents act. We also observe the teacher as an orchestrator of the learning process who as such invests efforts to create a learning environment that enables students to be active and become knowledge agents. In our previous work we have identified mainly a single student who acted as knowledge agent. Here we show how four students acted as a group of knowledge agents and that knowledge agency may appear in different forms: as one student and his followers, as two students, and as group of students.

INTRODUCTION

For several years now we have been investigating the mechanism of knowledge shifts in mathematics classroom. We combined two approaches/methodologies that are usually carried out separately: The Abstraction in Context approach with the RBC+C model (Dreyfus, Hershkowitz & Schwarz, in press) and the Documenting Collective Activity (DCA) approach with its methodology (Rasmussen & Stephan, 2008). This combination revealed that some students functioned as knowledge agents, where a knowledge agent is a member in the classroom community who initiates an idea, which subsequently is appropriated by other member/s of the classroom community. Knowledge agents are active in shifts of knowledge that are downloaded from the whole class discussion into a group's work or uploaded from a group's work to the whole class discussion, or stayed horizontally within the whole class discourse or the small group discourse. (Hershkowitz, Tabach, Rasmussen & Dreyfus, in press; Tabach, Hershkowitz, Rasmussen & Dreyfus, 2014). We refer the reader to these references for descriptions of the two approaches/methodologies.

In the present research report we focus on empirical examples of knowledge agency in a mathematics classroom learning probability. By knowledge agency we mean the many ways and variations in which knowledge agents act. We also observe the teacher as an orchestrator of the learning process who as such invests efforts to create a learning environment that enables students to be active and become knowledge agents. Thus, our study is aimed at expanding the idea of knowledge agent to knowledge agency based on an empirical bottom up approach.

THEORETICAL FRAMEWORK

Identifying and understanding the processes governing shifts of knowledge in inquiry mathematics classrooms is a big challenge (Saxe et al., 2009). Hence, questions

2014. In Oesterle, S., Liljedahl, P., Nicol, C., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 3, pp. 281-288. Vancouver, Canada: PME.

regarding the ways that knowledge evolves and moves between and within individuals, groups and the whole class community became very important. These questions are linked to the construct of knowledge agency.

Muller, Yankelewitz & Maher (2012) characterize agency in the classroom in the sense of the interplay of mathematical ideas in the mathematics environment (p. 373). Other researchers take a more explicit stand, where agency is considered mainly as taking the initiative (Pickering, 1995; Wagner 2004), when one or more students create their own mathematical idea or extend an established idea.

Our view regarding knowledge agency in the classroom is longitudinal. It starts from focusing on the student/s who is/are the first to raise a new and relevant mathematical idea when constructing new knowledge. They are knowledge agents for us only if other student/s in the class appropriate this knowledge and use it, that is if knowledge shifts are actualized.

The teacher's role in relation to knowledge shifts, knowledge agents and knowledge agency as a whole in an inquiry classroom is quite delicate. She orchestrates the whole process of learning, but without directly acting as a knowledge agent. Her task is to encourage students to act as knowledge agents within the learning process. Our lens on knowledge agency therefore focuses on this delicate role of the teacher.

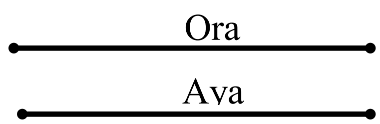
THE STUDY

The data for this study were collected by video recording in an 8th grade class engaged in learning probability. The camera was focused either on the whole class discussion or on a focus group. A unit consisting of a sequence of problem situations was carefully designed to offer opportunities for constructing and consolidating knowledge and practices in classroom. The unit included ten lessons.

The present paper focuses on lesson 8 of the unit. During lesson 4, the chance bar as a tool for describing probability in 1-dimensional spaces was introduced and used. Lesson 8, like many of the other lessons, started with a whole class discussion (WCD) followed by small group work, during which we followed the work of a focus group (FG). In the WCD, the teacher initiated a discussion on the *Arrows Problem* (see below), which dealt with a 2 dimensional sample space with un-equal probabilities, represented by a square area model (two orthogonal chance bars).

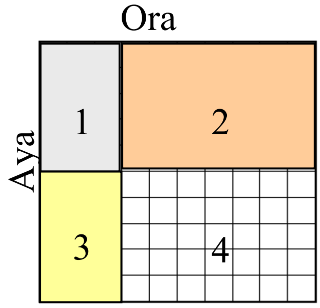
The Arrows Problem

Ora and Aya each shoot one arrow aimed at the target.



a. The probability of Ora hitting the target is 0.3. Mark this approximately on the chance bar.
b. The probability of Aya hitting the target is 0.5. Mark this approximately on the chance bar.
c. Let us draw a square using both chance bars. (The length of the square's side should therefore be 1.) [An empty square was provided.]

d. Use your marks on the chance bars to divide the square **approximately** according to the girls' chances of hitting or not hitting the target.



e. Within each of the four rectangles created, write down what its area expresses.
f. What is the area of the entire square?
g. Inside each rectangle, write down its area.
h. What is the probability of **both girls hitting** the target?
i. Color a rectangle corresponding to Oar's chances of hitting the target.
   What is the area of this rectangle?
j. What is the probability of at least one of them hitting the target? Color the appropriate area.

A priori analysis of the Arrows Problem

The following Knowledge Elements (KEs) were intended to be constructed while engaging in solving the Arrows Problem.

Es – Building a square model for the probabilities of a given 2-dimensional sample space problem.

Em – Understanding the meaning of a rectangle in the square model as representing the (two dimensional) event.

Ep – (event probability) The rectangle measure (area) equals the probability of the event represented by it according to Em.

Ec – The whole process can be checked by summing all probabilities to one (100%).

These KEs have a hierarchical structure. Em cannot be achieved without Es, and Ep cannot be understood without constructing Em. Ec is built on the previous three.

ANALYSIS AND FINDINGS

The lesson included a WCD, followed by FG work. The WCD was divided into 4 episodes, presented and analysed below. Notation: T – Teacher, S(s) – Student(s).

Episode 1: Chance bar for one dimensional sample space (1-7)

1	T:	We have this: 'Ora and Aya shoot an arrow at a target. … The probability that Ora will hit the target is 0.3. Mark approximately on the chance bar'. What is the question? What to mark approximately on the chance bar?
2	T:	Remind me what is there at the ends of the chance bar? Orly, what is there at its ends?
3	Orly:	0 and 1
4	T:	0 and 1. Now we would like to mark Ora, whose chance to hit the target is 0.3. Would you like to come and mark? You remember the issue of chance bar? [S marks on the chance bar.] He marked the chance; do you think he is correct?

5 Ss: Yes!

6 T: Good. Now the probability that Aya will hit is 0.5. Where to mark Aya?

7 Ss: In the middle!

The teacher reads the problem and encourages the students to be involved (1-2). It seems that the idea of marking the probability of an event on a chance bar, which was introduced in lesson 4, *functions as if shared* in the class (see Hershkowitz et al., in press). The above short episode provides evidence that presenting the probability of a simple event on a chance bar has been consolidated by at least some students.

Episode 2: Building a square model (8-27)

8 T: 0.5 is in the middle. So this is Aya and this is Ora [pointing to chance bars]. Now we will draw a square using the two chance bars. So the length of the side of the square is 1. Why does the length of the square equal 1?

9 S: Because this is the length of the chance bar.

10 T: Because this is the length of the chance bar. The chance bar was from 0 to 1, right? So we turn one of the lines to build a square from it. I will turn Aya's line and put it here. 'Divide the square approximately according to the chance, the probability of the girls to hit or miss the target'. Does anybody understand what this means? Yes?

11 Mike: We divide the square into 4 parts.

12 T: Into 4 parts according to the marks. Aya was marked on half, so we will mark it here. Ora we had 0.3, so we mark it here. There, we got 4 regions. Now let us see if we understand what each region means? For example, we have here 4 regions, lets name them: this is region 1, 2, 3, and 4. What does region 1 mean? What does it mean?

13 Nitzan: Region 1 is that…[silence]

14 T: Is there another region, one whose meaning you know?

15 Nitzan: The regions, this is divided to half and this a third.

16 T: A bit less than a third, right. What does each region describe? What does region 1 describe, Noam?

17 Noam: That Ora and Aya both hit the target.

18 T: It says they both hit. Let's see why it says they both hit. Because here from 0 to a half, this part means Aya hits and this that she missed. OK, if you shoot to the target in Aya's case, the chance that it will hit the target is half. And the same that it will not hit, it is also half. So the chance bar divides into: Yes, will hit the target and No, will not hit the target. OK? The same for Ora, only for her the chance to hit the target is smaller, she might be a less good shooter. So the part here says that Ora hit and there is a larger chance that Ora missed. Is this clear?

19 Liana: So what is region 3?

20	T:	What is region 3, good question, who can answer? Who will tell us what is region 3? Yes, Alon.
21	Alon:	That neither will hit.
22	T:	3 says they both miss, this can also happen. What does region 2 say?
23	S:	That only Ora hit.
24	T:	This is a region that only Ora hit. What will region 4 say?
25	S:	That Aya hit.
26	T:	That only Aya hit. Is it clear what each region means? Is it clear to you?
27	Ss:	Yes!

The teacher leads a WCD for constructing Es, the idea of the *Square Model*. This episode has a 'procedural flavour', but at the same time the teacher "floods" her students with questions concerning the meaning of the model as a whole and its partial regions in particular. The teacher is aiming at constructing Es and Em. Initially, the meaning of each partial region is not clear to the students, as can be seen from Nitzan (13, 15), a student who already understands the meaning of the chance bar for a one dimensional sample space, but cannot yet combine two chance bars together to create a meaningful sample space in two dimensions. Various students contribute to the construction of the meaning of the rectangles: Noam (17), Alon (21) and two additional students (23, 25). As a group, these four students potentially act as *knowledge agents* by providing their fellow students an opportunity to share with them the knowledge element Em. We say potentially as we do not yet have evidence that other students followed them. Liana in 19 shows interest in the meaning of region 3, and perhaps she is the first “follower”.

The way the teacher is orchestrating the discussion is similar to what van Zee and Minstrell (1997) characterized as tossing, meaning that she takes students’ questions and "tosses" them back to the class (e.g., 19-20). By doing so, she is moving the responsibility of meaning making and hence learning back to her students.

Episode 3: Building Ep (28 – 61)

28	T:	OK. Now how do I know, I am looking at question h. 'What is the probability that both hit the target?' Both hit. Which rectangle is it?
29	Ss:	Rectangle 1.
30	T:	Rectangle 1. How from this, from this drawing can I answer the question: 'What is the probability they both hit the target'? Adi?
31	Adi:	I think that…
32	T:	I prefer that you will tell me a computation and not a result, by the way.
33	Adi:	Ah…
34	T:	Does this help you?
35	Adi:	No.
36	T:	No. Ayelet?

37 Ayelet: To calculate the area of the rectangle.

38 T: Ayelet says that she would like to know the area of this rectangle, in which they both hit. How can we find the area of this rectangle?

39 Ss: Side times side.

40 T: Side times side, what is the length of this side? Half. The event they both hit will be half times what?

…

47 Guy: In fact the area that only Ora hit is also 15%, because these are the same measures, 0.5 and 0.3.

48 T: OK, you say, only Ora hit, it is easy to calculate because accidentally, as Aya has 0.5 chance to hit, at the same time she has 0.5 to miss. The calculation is the same calculation, so I need to calculate something which I already know the answer to. And if we would like to calculate others, how are we going to do it?

…

56 T: Can someone say without calculating or with calculating in a different way what will be here? [Points to the area that was not yet computed].

57 Itamar: Both miss?

58 T: Yes, both miss

59 Itamar: Also 35%.

60 T: Because?

61 Itamar: Because it is the same size.

Here the episode continues with the teacher leading the emergence of Ep (identifying the measure of each partial area with the probability of the event it represents). The discussion unfolds so that first the meaning is constructed (37-40), and then the procedural knowledge of computing the areas (not presented). The teacher was well aware of students' lack of procedural knowledge regarding multiplication of decimal numbers. She also represented the area by percentages, perhaps trying to create an additional scaffold for some students.

The discussion between Guy and the teacher (46-47) provides evidence that Guy constructed Em and Ep. It also provides evidence that other students acted as knowledge agents concerning Em and Ep. At the same time, he is the first student to express the meaning of the links between the two knowledge elements.

The discussion regarding the probability of each of the events is at a procedural level. Also, the calculation of the probability of each event is done more efficiently and quickly than the previous one. Specifically, Itamar provides evidence for both knowledge elements – Em and Ep.

Episode 4: critical thinking, control (62- 66)

62 T: Right, because it is the same size. Now how can we check that we don't have a mistake?

63 Yael: 15% + 15% + 35% + 35% = 100%

64	T:	Why does it have to be 100% when adding all these?
65	Itamar:	Because 100% is the whole.
66	T:	Because this is the whole, and here we describe all 4 cases that can happen when two people each shot an arrow. Do you understand this task? Including those who did not understand it before?

Here the teacher initiates critical thinking, in order to check the correctness of the probability calculations done. Yael (63) provides data (the probability of each of the four events) and a claim (the sum of the probabilities is equal to 100%). Itamar (65) provides the warrant. In this episode, Yael functions as knowledge agent and Itamar follows her by completing the argument. Together they act as potential knowledge agents for Ec. We do not have any evidence that Yael and Itamar have followers, nor that anyone objects to this argument.

During the following FG discussion, Yael, Noam and Rachel worked on similar problems. In their discussion we have identified traces of knowledge agency, that is their discussion included elements from the WCD.

DISCUSSION

Our study aimed at expanding the idea of knowledge agent to knowledge agency based on an empirical bottom up approach. The combined (RBC and DCA) analytic approach allowed us to document the evolution and the shifts of mathematical ideas in the classroom, and the main roles individuals play in these processes. As defined above, a knowledge agent is a student who, according to researcher observations, first initiates an idea within one classroom setting, which later is appropriated by others in the same or another classroom setting. This means that in addition to the students who act as knowledge agents, there are students who are qualified enough or have adequate ability to be inspired by the new idea and to appropriate it. We call the raising of a new idea and its appropriation by another student a *shift of knowledge*.

In our previous work we have identified mainly single students who acted as knowledge agents. Here, in Episode 2, we have four students who acted as a group of knowledge agents, together putting forward Em (the meaning of each rectangular part of the square model). In Episode 3, we have evidence (29) that other students followed this idea, hence we can say that the four students acted as knowledge agents. We can see that knowledge agency may appear in different forms: as one student and his followers, as two students (Hershkowitz at el., in press), and as a group of students. Later on in this lesson during FG work we have evidence of additional followers. All the above evidence shows mechanisms of knowledge agency in the classroom, which initiates knowledge shifts in the class.

The role of the teacher in any classroom includes responsibility for the knowledge learned. In an inquiry classroom, this responsibility is expressed in an indirect way, meaning that the teacher's task is to create a learning environment in which knowledge agency may flourish. The teacher in this lesson created such an environment by the tasks and by the way she orchestrated the whole class discussion and the lesson as a whole. Particularly, in this lesson the knowledge includes procedural processes concerning the use of the area model for calculating the probabilities of two dimensional sample space events. She also succeeded to include critical thinking (Episode 4), and encouraged knowledge agency (episodes 2-3-4).

In the future, we intend to further elaborate on knowledge agency and knowledge shifts in inquiry classrooms, as well as on the role of the teacher in building and sustaining a learning environment in which knowledge agency and knowledge shifts are a powerful and integral part of the learning activity.

Acknowledgment

This study was partially supported by the Israel Science Foundation under grant number 1057/12.

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