AN ANALYSIS OF MATHEMATICAL PROBLEM-POSING TASKS IN CHINESE AND US REFORM TEXTBOOKS

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This study analyzed the problem-posing tasks in Chinese and U.S. elementary mathematics textbooks. Significant differences were found between the Chinese and U.S. textbooks in the presentation of problem posing activities. By analyzing problem posing in textbooks, we gain insight into how reform ideas are reflected in the mathematics curriculum. With respect to problem posing itself, it would appear that the curriculum reform has moved problem-posing tasks into greater prominence, but great effort is needed to make problem posing a reality in both curriculum and instruction. In fact, our analysis shows that even in these reform textbooks, the proportion of problem posing tasks is very small.

INTRODUCTION

In the past several decades, there have been efforts around the world to incorporate problem posing (PP) into school mathematics at different educational levels (e.g., Brink, 1987; Chinese Ministry of Education, 1986; Hashimoto, 1987; Healy, 1993; Keil, 1964/1967). In recent years, there appears to have been a high level of interest among many researchers and practitioners in making problem posing a more prominent feature of classroom instruction (Singer, Ellerton, & Cai, 2013).

If problem-posing activities are to play a more central role in classrooms, they must be more prominently represented in curricula. Similarly, if teachers are to engage students in problem posing in the classroom, they must have sources for problem-posing activities. In fact, education reform movements have recommended that problem-posing activities be included in mathematics curricula themselves. Internationally, school mathematics reforms have recommended that students be able to "formulate interesting problems based on a wide variety of situations, both within and outside of mathematics" (NCTM, 2000) and that instructional activities should emphasize learning problem-posing skills.

Similarly, reforms to curriculum standards in China have increased the prominence of problem posing. The 9-year compulsory education mathematics curriculum standards call for providing students opportunities to pose problems, understand problems, and apply the knowledge and skills learned to solve real-life problems (Chinese Ministry of Education, 2001). Similarly, the curriculum standards for senior high school mathematics also call for developing students' abilities to pose, analyze, and solve problems from mathematics and real life (Chinese Ministry of Education, 2003). Indeed, in the reform standards, students are encouraged to discover and pose problems in order to prepare them to think independently and be inquirers.

2014. In Oesterle, S., Liljedahl, P., Nicol, C., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 3, pp. 393-400. Vancouver, Canada: PME.

However, the implications for the inclusion of problem posing in the curriculum are not necessarily clear. This ambivalence is reflected in the available research on problem posing and curricula. Although reform movements have called for problem posing activities to be included in mathematics curricula, there has not yet been a substantial body of research examining whether and how curricula incorporate problem posing. The purpose of this study is to analyse problem-posing tasks included in a Chinese reform elementary school curriculum and a U.S. reform elementary school curriculum.

There are at least three reasons why we undertook this study. First, we simply wanted to know if textbooks included problem-posing tasks and the kinds of problem-posing tasks that were included. Second, problem-posing activities are usually cognitively demanding tasks (Cai & Hwang, 2002). Whether it involves generating new problems based on a given situation or reformulating an existing problem, problem posing often requires the poser to go beyond problem-solving procedures to reflect on the larger structure and goal of the task. As tasks with different cognitive demands are likely to induce different kinds of learning (Doyle, 1983), the high cognitive demand of problem-posing activities can provide intellectual contexts for students' rich mathematical development. Such activities can promote students' conceptual understanding, foster their ability to reason and communicate mathematically, and capture their interest and curiosity (NCTM, 1991). Thus, an analysis of problem-posing tasks in textbooks would provide one perspective to show the learning opportunities for students through problem posing. Third, problem-solving processes often involve the generation and solution of subsidiary problems (Polya, 1957). Thus, the ability to pose complex problems should allow for more robust problem-solving abilities (e.g., Cai & Hwang, 2002). Encouraging students to generate problems is therefore not only likely to foster student understanding of problem situations, but also to nurture the development of more advanced problem-solving strategies.

SELECTION OF TEXTBOOKS AND ANALYSIS

We chose *Investigations in Number, Data, and Space* (TERC, 2008a, 2008b, 2008c, 2008d, 2008e, 2008f) as the U.S. reform textbooks. This textbook series was developed based on the NCTM Standards (NCTM, 1989) with the support of the U.S. National Science Foundation. We chose the elementary mathematics textbook series published by Beijing Normal University (BNU) (BNU, 2001 to 2006), which was developed based on the Standards published in 2001 (Chinese Ministry of Education, 2001).

In analysing the problem-posing tasks, we first identified all of the tasks involving problem posing in both textbook series. As long as the task required students to pose a problem based on a situation or an operation, we identified it as a problem-posing task. Then we analysed these problem-posing tasks along three dimensions: (1) grade levels; (2) content areas; and (3) types of problem-posing tasks. We describe these dimensions in detail in the results section. In addition to these three aspects, we also paid attention to whether there was the inclusion of pictorial, graphical, or tabulated (PGT) representations in the PP tasks and whether sample questions were included for students in the PP tasks.

RESULTS

Number of Problem-Posing Tasks

Grade	Chinese – BNU		US -- Investigations	
	Total tasks	% PP	Total tasks	% PP
1	570	5.96	490	0
2	549	5.65	741	1.62
3	541	2.77	832	0.96
4	561	2.85	760	2.24
5	619	2.91	726	3.17
6	545	3.12	-- [1]	--
Total	3,385	3.87	3,549	1.69

[1] The Investigations series does not have Grade-6 textbooks.

Table 1: Percentage of problem-posing tasks in each grade

The Chinese textbook series has a total of 131 PP tasks, while the U.S. textbook series has 60 PP tasks. The total number of problems and the percentage of PP tasks in the two textbook series are shown in Table 1. For both the Chinese and U.S. textbook series, the percentages of PP tasks are quite small. However, there is a larger percentage of PP problems in the Chinese textbook series than that in the US textbook series (3.87% vs. 1.69%; $z = 5.54, p < .001$). There are similar trends between the two textbook series for first grade ($z = 5.50, p < .001$), second grade ($z = 3.98, p < .001$), and third grade ($z = 2.56, p < .05$). There is no difference in terms of the percentages of problem-posing tasks in the fourth and fifth grades. There are also some observable differences in terms of percentages of PP tasks across grade levels. For the *Investigations* series, the fifth grade has the highest percentage of PP tasks, but the Chinese series has the highest percentage in the first grade.

Problem-posing Tasks in Content Areas

In this part of the analysis, we focused on the 131 PP tasks in the Chinese textbook and the 60 PP tasks in the U.S. textbook. We coded the tasks in terms of the five content strands: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. In the Chinese textbook series, there are some PP tasks which do not neatly fit into a content area, such as “what mathematical problems could you find in your life?” About 18% of the 131 PP tasks are of this kind. However, in the U.S. textbook series, there are none of this type of PP task. Table 2 shows the percentage distributions of the PP tasks in the five content areas. While the majority of problem-posing tasks are related to Number and Operations in both series, there is a significant difference between the two textbook series with respect to the percentage distribution of problem-posing tasks in the five content areas (χ^2 (5, N=191) = 31.67, $p < .001$). In addition, the percentage of the PP tasks in Number and Operations in the U.S. series is significantly higher than that in the Chinese textbook series ($z = 2.35$, $p < .05$). For the Chinese textbook series, the second highest percentage of problem-posing tasks was related to data analysis and probability, which was significantly higher than that in the US series ($z = 2.07$, $p < .05$). For the US textbook series, the second highest percentage of problem-posing tasks was related to algebra, which was significantly higher than that in BNU series ($z = 3.64$, $p < .001$). Very few PP tasks were related to geometry and measurement.

Content Area	Chinese (n=131)	US (n=60)
Numbers and Operations	61.07	78.33
Algebra	3.05	18.33
Geometry	2.29	0
Measurement	2.29	0
Data analysis and probability	12.98	3.33
Undetermined	18.32	0

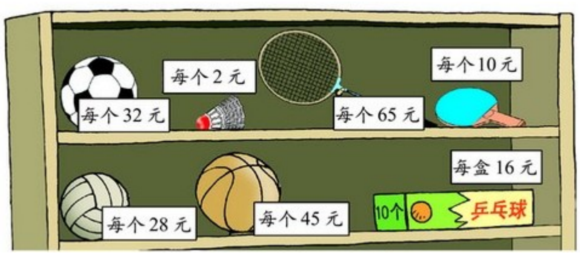
Table 2: Percentages of PP tasks in content areas

Types of Problem-Posing Tasks

Five types of PP tasks were identified. Each of these types of PP tasks along with an example is given below.

Type I (Reformulation of a given problem). Students are asked to pose a similar problem based on a given problem. For example: *If 6 people share 3 apples, each person will get ½ of an apple. Make up a problem about equal shares so that each person gets one fourth of something* (TERC, 2008c, Unit 7, p. 35).

Type II (Posing additional problems). Students are asked to pose additional problems for a given problem. One example from Chinese textbook series is shown below (BNU, 2003, 3A, p. 43).



(1) If we want to buy 5 volleyballs, how much do we need to pay?
(2) If we bought three footballs, and paid the cashier 100 dollars, how much can we get for change?
(3) If I want to buy one badminton racket and 10 badminton shuttlecocks, how much do I need to pay?
(4) Please pose two more questions and answer them.

Figure 1

Type III (Posing problems with given operations). Students are asked to make up a word problem that can be solved with a given operation. For example: *Write a story problem for* 65×35*. Then solve the problem and show how you solved it* (TERC, 2008d, Unit 8, p. 29).

Type IV (Posing a problem through supplementing information and questions). In order to pose a problem, supplementing information and questions are needed. For example, *Four children (A, B, C, and D) are practicing Chinese typing. In the following table is their practice time every day and their records on a test where each of them could select an article to type. Based on the data source, please pose two questions and try to answer them* (BNU, 2014, 4A, p. 72).

		A	B	C	D
Practice time every day (in minutes)		20	30	35	60
Test records	Time (Minutes)	12	19	18	13
	No. of words typed	384	931	846	728

Figure 2

Type V (Describe a situation to match a given mathematical representation). For example, *Write a story to match the graph shown below* (TERC, 2008C, Unit 6, p.19).

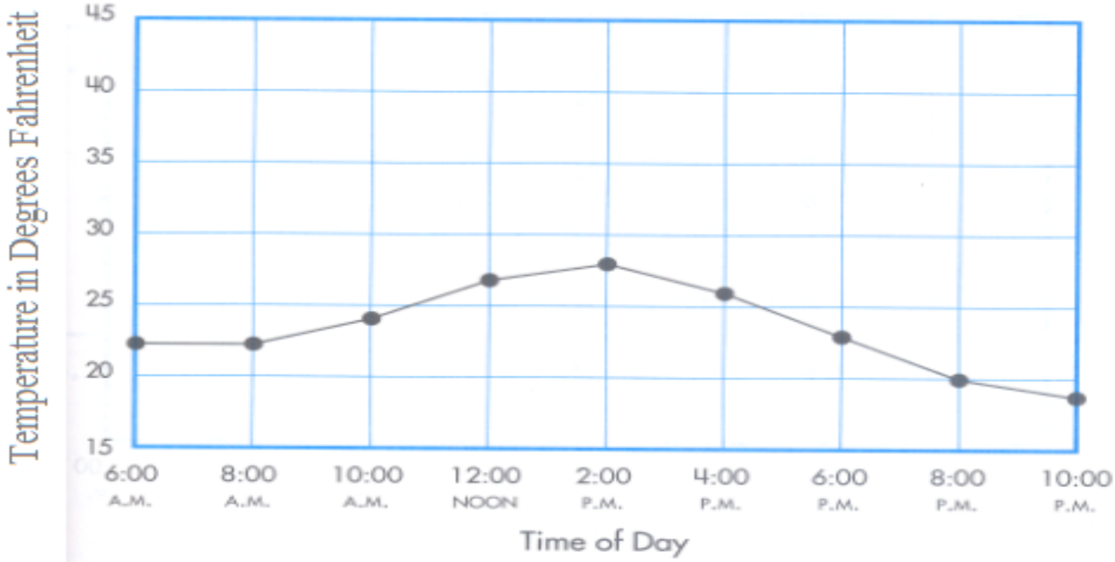


Figure 3

Table 3 shows the percentages of the five types of PP tasks in both textbook series. Overall, the percentage distribution of the five types of PP tasks was significantly different between the two textbook series (χ^2 (4, N=191) = 131.50, p < .001). The majority of PP tasks in the Chinese textbook series were type II tasks, which was significantly higher than that in the U.S. textbook series (z = 7.12, p < .001). However, for the U.S. textbook series, the majority of PP tasks were type III tasks, which was significantly higher than in the Chinese textbook series (z = 10.21, p < .001). For the Chinese textbook series, the second and the third highest percentages of PP tasks were type I and IV tasks, which were also significantly higher than in the U.S. textbook series (Type I: $z = 2.03$, $p < .05$; Type IV: $z = 3.11$, $p < .002$). However, in the U.S. textbook series, there was a much higher percentage of type V PP tasks than in the Chinese textbook series ($z = 3.40$, $p < .001$).

PP Types	I	II	III	IV	V
China (n=131)	24.43	55.76	3.82	14.50	1.53
US (n=60)	11.67	1.67	73.33	0	13.33

Table 3: Percentage of Each Type of Problem-posing Tasks

Inclusion of PGT Representations and Sample Problems

We also analyzed the PP tasks to examine if there was the inclusion of pictorial, graphical, or tabulated (PGT) representations in the PP tasks and whether sample questions were included for students in the PP tasks. Nearly 80% of the PP tasks in the Chinese textbook series included pictorial, graphical, or tabulated (PGT) representations, but only 20% of the PP tasks in the U.S. series included PGT representations. This result is somewhat suprising as Chinese students are less likely to use PGT representations than U.S. students in problem solving (Cai, 1995).

With respect to whether sample questions were included for students in the PP tasks, in 57% of the PP tasks from the Chinese textbook series, a sample question was given. However, a sample question was given in only 15% of the PP tasks in the U.S. series.

CONCLUSIONS

Curriculum reform has often been viewed as a powerful tool for educational improvement because changes in curriculum have the potential to change classroom instruction and student learning (Cai & Howson, 2013). There is a lack of research examining problem-posing in the mathematics curriculum. The research presented here sheds new light on the inclusion of problem posing in the mathematics curriculum. By analyzing problem posing in textbooks, we gain insight into how reform ideas are reflected in the mathematics curriculum. With respect to problem posing itself, it would appear that curriculum reform has moved problem-posing tasks into greater prominence, but great effort is needed to make PP a reality in both curriculum and instruction. In fact, our analysis shows that even in the so-called reform textbooks, the proportion of PP tasks is very small. In order to truly make problem posing prominent in classroom instruction, curriculum developers and textbook writers must increase the coverage of PP tasks in textbooks so that teachers can draw from the resource and teach students problem posing. Only then can students have rich opportunities to learn mathematics through problem posing.

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