YOUNG CHILDREN'S THINKING ABOUT VARIOUS TYPES OF TRIANGLES IN A DYNAMIC GEOMETRY ENVIRONMENT

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This paper presents preliminary results of longitudinal study on the development of children's geometric thinking in dynamic geometry environments. Here we investigate young children's (age 7-8, grade 2/3) interactions, in a whole classroom setting with an Interactive Whiteboard, with Sketchpad-based tasks involving the use of different types of constructed triangles (scalene, isosceles, equilateral). We use Sfard's discursive approach to show how the children developed a reified discourse on these different types of triangles and how they described the behaviour of these triangles in terms of their invariances (side lengths and angles).

INTRODUCTION

In this paper, we report on an exploratory study conducted with a split class of grade 2/3 children (ages 7-8) working with various types of triangle sketches using *The Geometer's Sketchpad*. The focus of this research is to study how the use of *Sketchpad* affects the children's thinking about triangles, including how they attend to various aspects of the dynamic sketches, how they talk and gesture about the moving objects on the screen, and how they reason about the behaviour of different types of triangles (i.e. scalene, isosceles and equilateral triangle).

CHILDREN'S UNDERSTANDING OF CLASSIFICATION OF SHAPES

Research shows that children have difficulty working with definition when classifying and identifying shapes (Gal & Linchevski, 2010). de Villiers (1994) suggests that classifying is closely related to defining (and vice versa) and classifications can be hierarchical (by using inclusive definitions, such as a trapezium or trapezoid is a quadrilateral with at least one pair of sides parallel – which means that a parallelogram is a special form of trapezium) or partitional (by using exclusive definitions, such as a trapezium is a quadrilateral with only one pair of sides parallel, which excludes parallelograms from being classified as a special form of trapezium). In general, in mathematics, inclusive definitions are preferred. A number of studies have reported on students' problems with the hierarchical classification of quadrilaterals (Fuys, Geddes & Tischer, 1988; Clements & Battista, 1992; Jones, 2000). However, Battista (2008) designed the *Sketchpad*-based *Shape Makers* microworld that provides grade 5 students with screen manipulable shape-making objects. For instance, the Parallelogram Maker can be used to make any desired parallelogram that fits on the screen, no matter what its shape, size or orientation—but only parallelograms. This motivated our research on children's identifying and classifying of different types of triangles.

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THEORETICAL PERSPECTIVE

In previous research, we have found Sfard's (2008) 'commognition' approach is suitable for analysing the geometric learning of students interacting with DGEs (dynamic geometry environments) (see Sinclair & Moss, 2012). For Sfard, thinking is a type of discursive activity. Sfard's approach is based on a participationist vision of learning, in which learning mathematics involves initiation into the well-defined discourse of the mathematical community. The mathematical discourse has four characteristic features: word use (vocabulary), visual mediators (the visual means with which the communication is mediated), routines (the *meta-discursive rules* that navigate the flow of communication) and narratives (any text that can be accepted as true such as axioms, definitions and theorems in mathematics). Learning geometry can thus be defined as the process through which a learner changes her ways of communicating through these four characteristic features. In the context of identifying shapes, Sfard has proposed the following three levels of discourse characterised by different types of routines and word uses, which Sinclair & Moss (2012) use in their study of children's interactions with DG triangles:

- 1st level: the word 'triangle' is used as a proper noun. The routine of identification involves visual object recognition.
- 2nd level: the word 'triangle' is used as a family name, that is, the name of a category of elementary objects; identification is made according to visual family recognition as well as through an informal properties check.
- 3rd level: the word 'triangle' is used as the name of a category of objects, and identification is made through visual family resemblance first, and then verification/refinement of properties.

At the 3rd level, since the condition specified by one definition (i.e. of equilateral) may be an extension of the condition in another (i.e. of isosceles triangle), children can make use of inclusive definitions (term suggested by de Villiers, 1994). We are particularly interested in investigating how the children might move between different word uses, routines, narratives and progress to higher levels of discourse.

METHOD OF RESEARCH

Participants and data collection

This teaching experiment is part of a larger project that involves the study of children's geometric thinking in the primary grades. We worked with grade 2/3 split classroom children from a pre-K-6 school in an urban middle SES district. There were 24 children in the class from diverse ethnic backgrounds and with a wide range of academic abilities. We worked with the children on a bi-weekly basis on a variety of geometric concepts for seven months. Three lessons were conducted on the topic of triangles. Each lesson lasted approximately 60 minutes and was conducted with the children seated on a carpet in front of an interactive whiteboard. Two researchers, and the classroom teacher, were present for each lesson. One researcher (second author) took

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the role of the teacher for these interventions. Lessons were videotaped and transcribed. This paper is focused on the first and second of the three lessons. Previous lessons involved *the concepts of* symmetry and angles, but they had never received formal instruction about classification of triangles before.

Dynamic triangle sketches

Along the lines of work of Battista (2008), we developed the Triangle ShapeMakers sketches (see Figure 1) for different types of triangles (scalene, isosceles, equilateral triangles, right triangle). Each triangle type had a different colour (pink for scalene, red for equilateral, blue for isosceles and green for right).

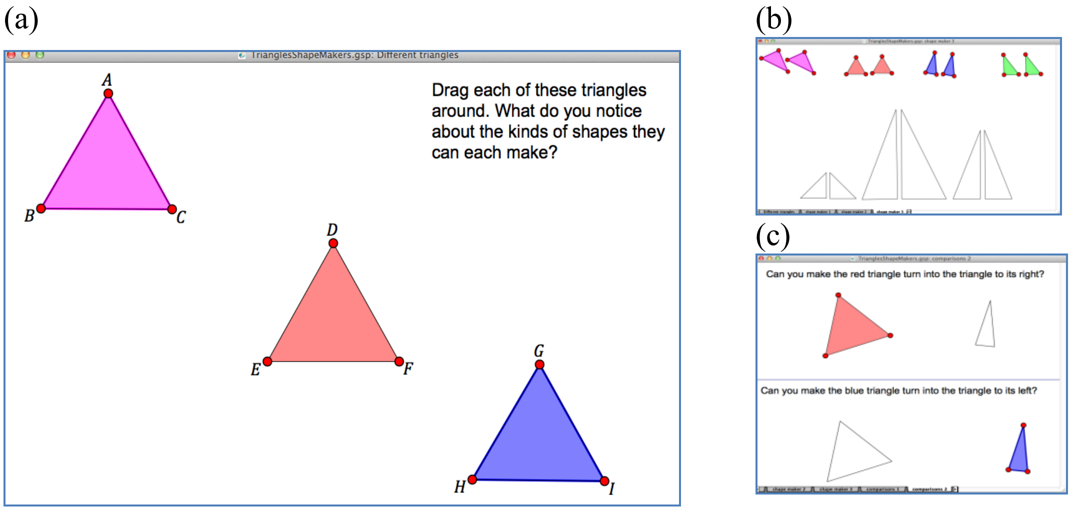

Figure 1(a, b, c): Three different Triangles ShapeMaker sketches

In the sketch shown in figure 1(a) all look like equilateral triangles, but only the middle one is constructed to be so; the bottom right one is an isosceles triangle and the top left is scalene. Students were asked to explore the similarities and differences between the three triangles. For the sketch in figure 1(b), the students were asked to explore which coloured triangles could fit in the given triangle outlines. Note that the equilateral triangles cannot be used, and only the right triangles will fit into the two left-most outlines. The sketch in figure 1(c) focused on exploring whether an equilateral triangle can fit into the given isosceles triangle (top) and whether an isosceles triangle can fit into a given equilateral triangle (bottom).

Behaviour of dynamic scalene, isosceles and equilateral triangles

Although no vertex was labelled in the sketches, we have done so in Figure 1a in order to explain the dragging behaviour of different triangles. In the scalene (pink) triangle, dragging any one vertex (A, B or C) does not move the other two vertices, whereas in equilateral (red) triangle, dragging vertex E or F (which determine the size of the triangle) moves the entire triangle except the vertex F or E respectively; dragging vertex D simply translates the triangle from one place to another. In the isosceles (blue) triangle, dragging vertex (I) does not move the other two vertices, dragging vertex H or G moves the entire triangle except the vertex G or H respectively.

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EXPLORING STUDENTS' LEARNING ABOUT TYPES OF TRIANGLES

To begin, the teacher (second author) appointed three children to drag each of the coloured triangles in the sketch (figure 1a). The children seated on the floor were asked to be "detectives" and to "describe what kinds of triangles can be made", "what can change and what stays the same" in each of the triangles.

Comparing the dragging of scalene and equilateral triangle

The first child Neva dragged the pink (scalene) triangle into various sizes and orientations i.e. skinny and long, small and big triangles. Then Adil dragged the red (equilateral) triangle and the teacher asked if he could make it long and skinny. Observing the dragging patterns, some students said no. The teacher asked the students that why is it not possible to make the red triangle long and skinny.

Egan: Because the red one, it's different than that (*pointing to the pink triangle*) and I think it can only go by a perfect triangle.

The teacher asked what Egan meant by "perfect triangle", to which Rabia responded:

Rabia: Because the other triangle (*pointing to the pink triangle*) can move at a point but this one (red triangle) can move bigger or smaller differently.

Another student described the behaviour of a perfect triangle as below:

Jace: Everything moves with it except one point.

Teacher: (*Dragging the equilateral triangle*). Even when it is getting bigger and smaller, is there anything that stays the same as I make it bigger and smaller? (Many children put their hands up) Neva?

Neva: The angles.

The children started to notice the changes in the red triangle as Adil dragged one of the vertices. Egan's statement "it's different than that" shows that he started to notice the differences between the red and pink triangles, even though they initially looked the same in their static configurations. Egan's response 'only go by a perfect triangle' is based on his visual recognition of similarity to previously seen prototypical triangles, thus using a first level of discourse. Further, in Rabia's description "other triangle can move at a point" and in Jace's statement "everything moves with it except one point", the action words 'move at a point', 'everything moves' shows that the students are paying attention to the particular kinds of movement depicted by each triangle. The dragging tool initiated this kind of reasoning, so that the transforming triangles functioned as new visual mediators. Rabia's statement "move bigger or smaller differently" shows that she was noticing the different kind of size changing behaviour of the equilateral triangle as compared to the scalene triangle. The use of words 'red one', 'it', 'the other triangle', 'this one' to address the triangles show that the children are talking about one particular triangle as opposed to the family of those triangles. In addition, Neva noticed that 'angles are staying' the same under dragging. Thus, the children started to notice the informal properties (based on dragging behaviour) as well as formal properties (invariance of angles in red triangle) of the different triangles.

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This shows that many of them are using a mixture of 1st and 2nd level discourse around types of triangles.

Exploring the overlapping of one triangle over the other fixed triangle

After the students explored the dragging of different triangles, the teacher asked them if they could fit the blue (isosceles) triangle over the pink (scalene) triangle (without touching the pink triangle). Rabia first matched the two vertices of the blue triangle to one side of the pink triangle (fig 2a) and then tried to drag third vertex (shown by green dot in fig 2b) in upward direction and concluded that she couldn't fit the blue triangle onto the pink one.

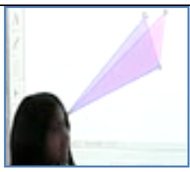
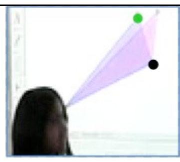
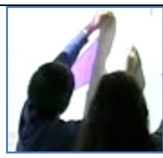
2a, 2b: Matching of two vertices by Rabia	 	2c: Jory's stretching gesture	

Figure 2a, 2b, 2c: Snapshots of Rabia's overlapping attempt & Jory's gesture

Teacher: You think you can't? How come you can't?

Rabia: Because I think if I move that one (*placing marker at vertex <black dot> in fig 2b*), that one also moves (*placing marker at green dot in fig 2b*)

The teacher asked for other arguments, and called on Dale and then Jory:

Dale: Because the blue one can only move symmetrical.

Jory: So, this one (*placing right index finger at green dot (fig 2b)*) wherever you move it, then this one (*placing right index finger at black dot (fig 2b)*) moves with this (*placing left index finger at green dot (fig 2b)*), so when you move, it will go that way (*stretching his arms upwards along the two longer sides (figure 2c)*).

Rabia's statement "if I move that one, that one also moves" shows that she is paying attention to the causal type of movement relationship between the different parts of the triangle. While Dale's explanation "it can only move symmetrical," suggests that he has noticed invariance in the isosceles triangle, either holistically, or as a function of the movement of the congruent sides. The systematic dragging of the vertex of one of the longer sides of the isosceles (blue) triangle by Rabia acted as a visual mediator and seemed to help Dale see the property of symmetry. Jory's use of the words "wherever you move it, then this one moves with this" and "so when you move, it will go that way" along with the stretching arms gesture shows that he is also thinking about the simultaneous change in length of two arms of the isosceles triangle. The teacher labelled Dale's reasoning "a symmetry argument" and Jory's a "stretching argument". Most of the students agreed with these arguments by raising their hands or nodding their heads. Overall, the students came up with three arguments: (1) dragging vertex argument (dragging one point moves the other point) (2) symmetry argument (blue one can only move symmetrical) (3) Stretching side argument (with arm stretching gesture

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where two arms act as two sides of triangle). These arguments show a 2^{nd} level of discourse because they refer to informal/formal properties of the isosceles triangle.

Through teacher-led discussion, the students identified the properties of different sides staying the same or changing length in different triangles. After these invariances were stated explicitly, the teacher introduced the "special names" equilateral, isosceles and scalene for the red, blue and pink triangles, respectively.

In the second lesson, the children worked on the sketch in Figure 1(b). An attempt to fit the equilateral (red) triangle into the right-angled isosceles outline (figure 1b) was unsuccessful, whereas the scalene (pink) triangle fit in that outline without any difficulty. When asked why the equilateral triangle could not fit, Thom said:

> Thom: It's because mostly that won't work because that one of them is seemed to be paralysed or something and doesn't want to move from its seat. But the pink one... the scalene …can move anywhere it wants and the only one. I think it's the only one that can get inside the shape. I think it's only the green and pink that can make that shape, but the others are just paralysed.

Thom used the words 'paralysed', 'doesn't want to move' for equilateral and isosceles triangles, whereas for scalene triangle he used the words 'can move anywhere', 'can get inside the shape'. Clearly, this vocabulary emerged after observing the free and restricted movements of different dynamic triangles and prompted him to make connections with real life experiences of the restrictive mobility of humans. Later, during the exploration of the third sketch (figure 1(c)), after the students had successfully placed an isosceles triangle into an equilateral outline, but not an equilateral triangle into an isosceles outline, the teacher asked:

> Teacher: Why can we turn isosceles into equilateral, but we can't turn equilateral into the isosceles, Lida?
>
> Lida: Isosceles can be turned into equilateral because two sides have to be the same, but that doesn't mean that all three sides can't be the same. At least two sides should be same.

In another overlapping task of scalene and equilateral triangle, the teacher asked

> Teacher: How come scalene can make equilateral triangle?
>
> Jory: Because scalene…um…they can create any shape of triangles.

Lida's statements "At least two sides should be same" and "that doesn't mean that all three sides can't be the same" give evidence of her use of inclusive definitions. Jory's statement "they can create any shape of triangles" for scalene triangles shows his description of the behaviour of all scalene triangles as opposed to one particular scalene triangle. Also this description of scalene being able to create any shape of triangles makes inclusion of isosceles and equilateral triangles evident. Thus, Lida and Jory's arguments show 3^{rd} level of discourse.

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DISCUSSION AND CONCLUSION

Our preliminary analysis shows that, during the teacher-led explorations and discussions with dynamic sketches, children's routines moved from description of tool-based informal properties to formal properties as well as from particular (1^{st} order) to more general (2^{nd} order) discourse about 'triangle'. Children's reasoning started with describing the movement patterns like "Everything moves with it except one point", "If I move that one, that one also moves", "wherever you move it, then this one moves with this" and then eventually shifted to formal properties "angles are staying same", "moves symmetrical". Dragging the vertices acted as a visual mediator and helped the children to develop the routine of looking at movement behaviour and eventually shifting towards formal geometrical properties. Jory used the embodied visual mediators (arms as sides of isosceles triangle) for justifying why isosceles triangle can't fit into the outline of a triangle whose sides are different. The use of action verbs "go", "moves", "staying", "paralysed", "getting bigger or smaller" and 'if-then' statements shows children's propensity to reason in terms of motion in case of classification of triangles, which was clearly initiated by the dynamic and temporal elements of DGE. Thom's use of word 'paralysed' for isosceles and equilateral triangle is quite interesting, and clearly emerged after looking at the restrictive type of movement shown by these triangles, which reaffirms Healy and Sinclair's (2007) claim that the temporality of dynamic mathematical presentations offers striking opportunities for narrative thinking.

Also, Lida and Jory's use of inclusive definitions (Villiers, 1994) emerged as a result of dynamic actions of dragging during an attempt to superimpose one triangle over another. This study reaffirms the results of Sinclair & Moss (2012) by providing the evidence of how dynamic environment can help students to move to higher levels of discourse. This study also provides initial evidence that the teaching of concepts like symmetry and angles in early years can lead to whole set of new possibilities of geometric reasoning about shape and space for young children.

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