THE QUALITY OF ARGUMENTATIONS OF FIRST-YEAR PRE-SERVICE TEACHERS

Leander Kempen, Rolf Biehler

University of Paderborn

In this paper we report our findings from a study, in which 177 undergraduate pre-service teachers had to verify a statement of elementary number theory at the onset of a lecture serving as a bridging course. The answers were categorized to investigate the qualities of the given argumentations. We also separated the results of three different subsets: (1) the of students in their first semester, to get to know their level of arguing after having past their A-Levels, (2) the students, that take part in this course the first time and are in a higher semester and (3) the subset of repeaters to investigate their problems in detail.

INTRODUCTION

One of the greatest problems in the transition from school to university mathematics is the new role of proof at university level. Selden (2012, p. 392) identifies the nature of proof and its increased demand for rigour at University as a major hurdle for many beginning university students. In Germany, this problem got recently even more severe, because the number of school years necessary for going to university was reduced from 13 to 12 years (so called "G8"). Moreover, proof plays a decreasing role in the practice of school mathematics teaching.

The University of Paderborn offers the course "Introduction into the culture of mathematics" specifically designed as a bridging course in the first term in order to help students to successfully accomplish the transition to university. This course is a requirement for the first year secondary pre-service teachers (non grammar schools). Since one of the main focus of the course is on argumentation and proof, the lecture deals with mathematical "research" in the field of elementary arithmetic, different kinds of argumentations (e.g., generic proofs), logic, formalization and formal proofs. The aim of this paper is to report on the study concerning the argumentation skills of the students at the beginning of the course and to identify common gaps or pitfalls in their argumentations.

RELATED RESEARCH

There is a large variety of research on proof competencies of school students shortly before matriculation. In the TIMS-Study in 1998 mathematics students in their final year of secondary school were asked to write a proof. Of all ten countries tested, the German students received the worse results: Only 21% were able to construct a valid proof (Reid & Knipping, 2010, p. 68). Reiss and Heinze (2000) showed that the larger part of German school students in their survey was not able to use deductive arguments

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when trying to construct a proof. Moreover, in the study of Reiss, Klieme and Heinze (2001) only a few students were able to use their mathematical knowledge to build up an argumentation in order to prove a given statement. But this problem is not limited to the German school system. Reiss and Ufer summarize their international research review on students' proving skills as follows: "A coherent result, which is reflected in the empirical studies on mathematical proofs in school mathematics, is the poor performance of pupils concerning reasoning and argumentation" (Reiss & Ufer, 2009, p. 164; translated by the author).

Similar problems are known for undergraduates and pre-service teachers, when trying to prove a statement. Common are problems with mathematical knowledge (definitions, notation, etc.) and a lack of methodological knowledge (e.g., Moore, 1994; Weber, 2001). Biehler and Kempen (2013) investigated students' ability in constructing generic proofs and formal proofs and found serious deficits in the proof production of pre-service teachers concerning the deductive reasoning and the handling of algebra.

RESEARCH QUESTIONS

We are investigating the development of the students' proving skills during the course in the context of the first author's dissertation. Therefore it was necessary to look at the argumentation skills at the beginning of the course. We also wanted to have a look at students' problems when building up an argument. Since the group of participants is heterogeneous, we chose with the following questions:

4. What kind of argumentation skills have the students at the beginning of the course?
5. What are the differences in the argumentations of the students in their first semester at university, of the students in a higher semester that are taking the course for the first time and the students that once failed the final exam of the course and are now doing it for the second time?
6. Are there specific problems in the solutions of the subset of the repeaters? What are these?

METHODOLOGY

In the first session of the course, the students were given a questionnaire with items concerning argumentation and proving, attitudes towards proving, the nature of mathematics and the nature of mathematics teaching. In this paper, we will discuss the analysis of the first item of the questionnaire, which demanded argumentation skills (proving skills) of the students.

Task and task analysis

The first task of the questionnaire, which we will discuss in detail here, is the following:

The sum 11 + 17 is an even number
Is this true for every sum of any two odd numbers?
- Argue convincingly!

We deliberately asked for arguing convincingly, because the demand to “prove” a certain statement implicates for many students the use and handling of algebra. Since the idea here is not to get to know what the students consider to be a “proof” or what constitutes a “proof” for them, but how they construct a “convincing argumentations” for an infinite number of cases for themselves and/or others.

It is possible to answer this question with only basic knowledge of elementary arithmetic and algebra. One may argue without using variables, constructing a narrative proof (describing your valid argumentation with words) or a generic proof (explaining your valid argumentation in a concrete context, i.e. concrete examples or geometric diagrams) or one may use variables to compute algebraic expressions and argue with the final term.

We categorized the students’ answers to investigate the quality of the given arguments. To analyze this aspect more in detail, we also categorized the following aspects: The use of variables, the way of argumentation, the use of examples and the type of gap in the argumentation. Due to the size of this paper, we will only address the main dimension: The quality of argumentation.

Analysis of the data

For analysing the quality of arguments, we looked for appropriate categories in the literature. Bell (1976) identified several levels for categorizing pupils’ proof productions. Using as first division between empirical and deductive areas, he built up a set of categories regarding the quality of argumentation. In the following years proof productions were mainly analysed to identify different *proof schemes* that are describing students’ ability in proving: “A person’s proof scheme consists of what constitutes ascertaining and persuading for that person” (Harel & Sowder, 1998, p. 244). Recio and Godino (2001) adapted the approach of Harel and Sowder to investigate the proof schemes of students starting their studies at University. Since the set of categories of Bell (1976) and Recio and Godino (2001) complete each other, we combined their categories and modified them for our study. We finally came up with the following set of categories.

Set of categories

- **C99: no answer is given.**
- **C0: no argumentation is given.** (See Figure 1.)

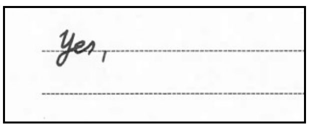


Figure 1: A student answer, which belongs to the category C0.

Empirical argumentations

- **C1: illustration.** The truth of the general statement is illustrated by several examples. (See Figure 2.)
- **C2: empirical verification.** The truth of the general statement is inferred from a subset of examples. (See Figure 3.)

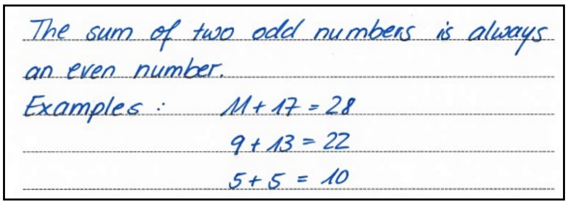

The sum of two odd numbers is always an even number.
Examples : 11+17=28
9+13=22
5+5 = 10

Figure 2: A student answer, which belongs to the category C1.

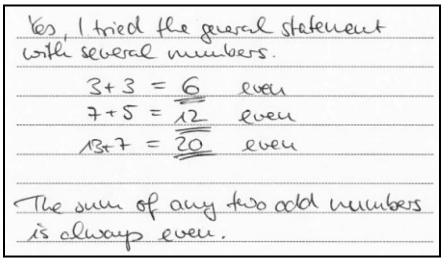

Yes, I tried the general statement with several numbers.
3+3 = 6 even
7+5 = 12 even
13+7 = 20 even
The sum of any two odd numbers is always even.

Figure 3: A student answer, which belongs to the category C2.

Deductive types of argumentations

- **C3: pseudo-verification by just repeating the statement to be proved.** The answer is given by stating the statement that the sum of any two odd numbers is always even. (See Figure 4.)
- **C4: pseudo-verification by pseudo argumentation.** The verification is done by an explanation that merely paraphrases the statement that the sum of two odd numbers is always even. (See Figure 5.)

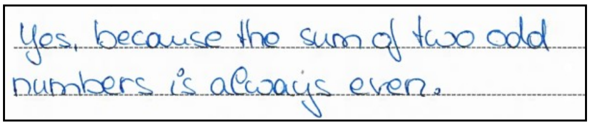

Yes, because the sum of two odd numbers is always even.

Figure 4: A student answer, which belongs to the category C3.

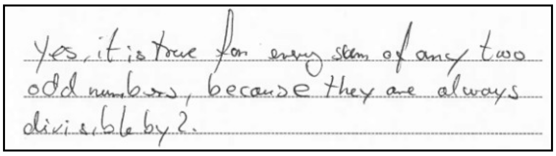

Yes, it is true for every sum of any two odd numbers, because they are always divisible by 2.

Figure 5: A student answer, which belongs to the category C4.

- **C5: pseudo argument, mathematically wrong.** The arguments given are either non relevant for the task or mathematically wrong. (See Figure 6.)
- **C6: relevant details, but fragmentary.** The answer contains relevant aspects that could form part of a proof, but the student fails to build up a coherent argument. (See Figure 7.)

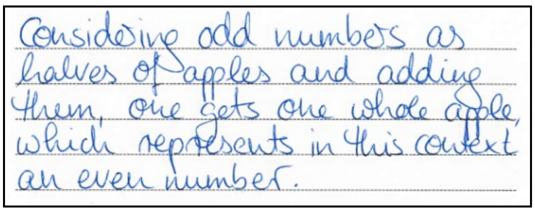

Considering odd numbers as halves of apples and adding them, one gets one whole apple, which represents in this context an even number.

Figure 6: A student answer, which belongs to the category C5.

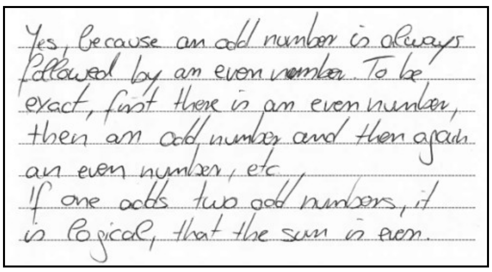
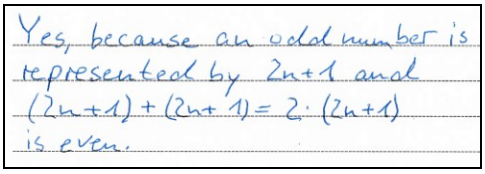

Yes, because an odd number is always followed by an even number. To be exact, first there is an even number, then an odd number and then again an even number, etc.
If one adds two odd numbers, it is logical, that the sum is even.

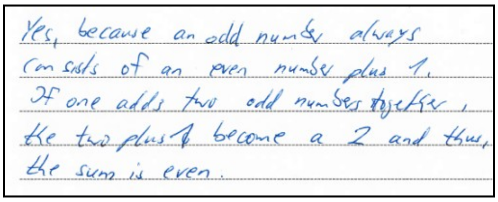
Figure 7: A student answer, which belongs to the category C6.

- **C7: connected arguments with unrecoverable gap.** The student gives a connected argument with explanatory quality, but the argumentation includes an unrecoverable gap. (See Figure 8.)
- **C8: connected argument, but incomplete.** The student gives a connected argument with explanatory quality, but the argumentation is incomplete. – Here one could close the created gap by adding some sentences. (See Figure 9.)



Yes, because an odd number is represented by 2n+1 and (2n+1) + (2n+1) = 2 · (2n+1) is even.

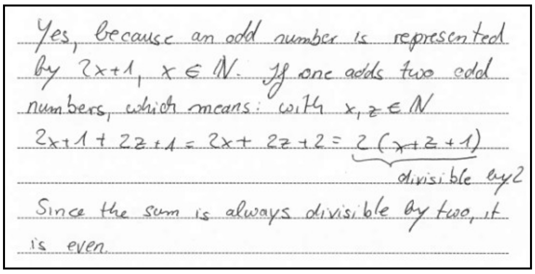
Figure 8: A student answer, which belongs to the category C7.



Yes, because an odd number always consists of an even number plus 1. If one adds two odd numbers together, the two plus 1 become a 2 and thus, the sum is even.

Figure 9: A student answer, which belongs to the category C8.

- **C9: complete explanation (a) - with minor (formal) inaccuracies.** The student derives the conclusion by a connected argument and from generally agreed facts of principles. Just because of minor (formal) inaccuracies the explanation is not a perfect verification. (See Figure 10).
- **C10: complete explanation (b).**The student derives the conclusion by a connected argument and from generally agreed facts of principles. (See Figure 11.)

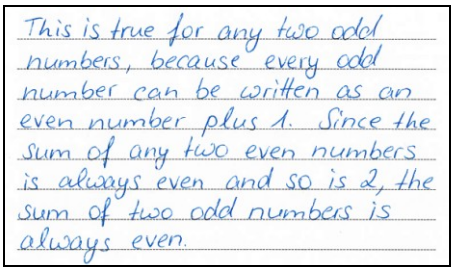


Yes, because an odd number is represented by 2x+1, x ∈ N. If one adds two odd numbers, which means: with x, z ∈ N

$2x+1 + 2z+1 = 2x + 2z + 2 = \underbrace{2(x+z+1)}_{\text{divisible by 2}}$

Since the sum is always divisible by two, it is even

Figure 10: A student answer, which belongs to the category C9.



This is true for any two odd numbers, because every odd number can be written as an even number plus 1. Since the sum of any two even numbers is always even and so is 2, the sum of two odd numbers is always even.

Figure 11: A student answer, which belongs to the category C10.

RESULTS

Apart from analyzing the answers of the whole group [n = 177], we also looked at three different subgroups: (1) the subset of the students in their first semester at university [n = 69], (2) the students, that take part in this course the first time and are in a higher semester [n = 58] and (3) the subset of the repeaters, the students that have failed the final exam in a previous semester and now have to do the course again [n = 50]. Thus, it is possible to get an overview of the argumentation and proving skills of all participants, to evaluate the competencies of the first-year students, to investigate the problems of the students that once failed the exam and also to have a look at the students, that are in a higher semester, but take the course for the first time. Figure 12 shows the quantitative results, clustered in the following way: "emp." combines the empirical argumentation subcategories [C1+C2]; "pseudo" combines the pseudo argument subcategories [C3+C4+C5]; “v.a.” (valid arguments) combines subcategories with valid arguments, but without a complete explanation [C6+C7+C8], whereas “c.e.” with a complete explanation [C9+C10].

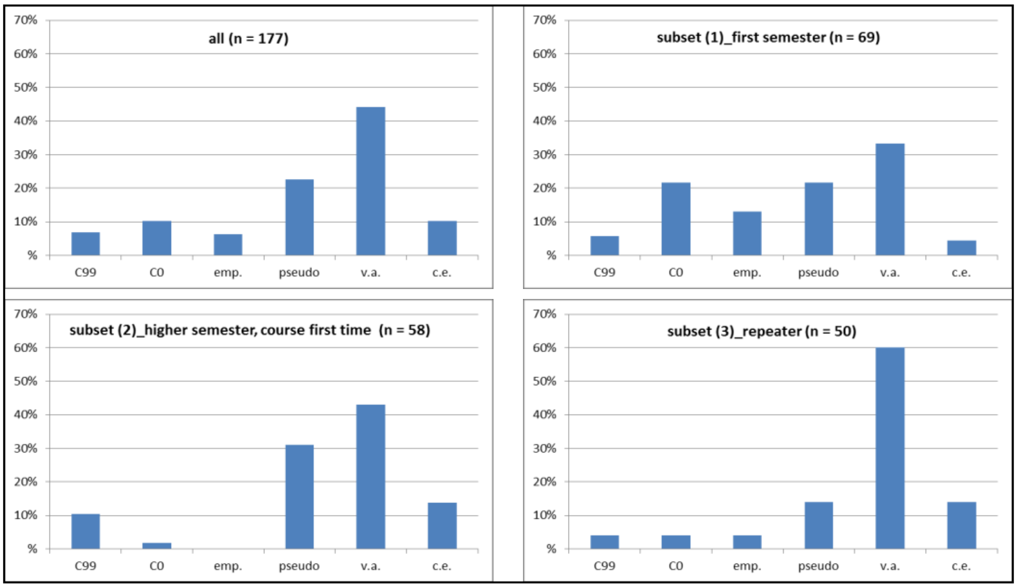


Figure 12: Frequencies of answer types.

<u>Results concerning all students [Figure 12, top left]:</u>

Regarding all tests 12 students did not answer the task. 18 students (10.17%) did not argue why the statement is true [C0] and 11 (6.21%) used an empirical approach [emp.]. In 40 answers (22.60%) there were only pseudo-arguments mentioned [pseudo]. 96 persons (54.24%) gave correct arguments [v.a.+c.e.] and 18 argumentations of these (10.17%) were rated as “complete explanations” [c.e.].

<u>Results of subset (1) [Figure 12, top right]</u>:

Considering the subset of students in their first semester, 15 solutions (21.74%) did not contain any argumentation [C0] and an empirical approach was used by 9 persons (13.04%). Pseudo-arguments were given by 15 students (21.74%) and out of the 30 answers (43.48%) with correct arguments [v.a.+c.e.] there are 3 (4.35%) “complete explanations”.

<u>Results of subset (2) [Figure 12, bottom left]:</u>

In this group only one student gave an answer without argument and no one used an empirical approach. A pseudo-argument was given by 18 students (31.03%). Out of the 33 answers (56.90%) with correct arguments [v.a.+c.e.], we rated 8 (13.79%) as “complete explanation”.

<u>Results of subset (3) [Figure 12, bottom right]:</u>

In the subset (3) “repeater”, only 2 (4.00%) persons used empirical considerations and 7 students (14.00%) gave wrong pseudo-arguments. In 28 answers (56.00%) we found a serious gap in the argumentation [C7] and only 7 students (14.00%) achieved a “complete explanation”.

Comparison of the argumentations given:

The answers of the students in their first semester displayed a variety of arguments combined with different types of argumentations (e.g. narrative proof, generic proof, etc.). 13 of these students, and also 10 students in a higher semester, who took the course for the first time, argued with properties of even and odd numbers, without using algebra. In the subset of repeaters all students, who argued with correct arguments used formalization and algebra for their argumentation.

Specific problems in the solutions of the subset of the repeaters:

As mentioned above, all repeaters, that gave correct arguments, did this by formalization and algebra. But in 28 of all 37 cases, the students only used one variable for representing any two odd numbers and therefore failed to verify the statement.

DISCUSSION

To sum up, only 6.21% of all answers contained a purely empirical approach. This result is inconsistent with many studies: In the survey of Barkai et al. (2002) about 52% of elementary teachers offered an empirical argument when asked to justify a statement of elementary number theory (see also Reid & Knipping, 2010, p. 68).

In the subset of the first-year students, about 13% tried to verify the statement by empirical arguments and 43.48% of these students were able to argue with valid arguments. But only 3 of them gave an argumentation we could consider as valid verification. Since the given task is a basic (nearly trivial) theorem of elementary number theory, which is easy to verify, this result is distressing. We have indications that the mathematical education at school in Germany does not provide future students for secondary teacher studies in mathematics with many skills to work on a proving task. However, we have not taken a representative study. But the results reinforce the need of our bridging course.

The problems of the repeaters are distinct, too. In this elementary task, only 37 students (74%) gave a valid argument in their argumentation. All these 37 answers used formalization and algebra, but 28 of these failed to represent any two odd numbers, because of using only one variable. It seems obvious, that the problems in using variables and algebra lower the argumentation skills of many students. This finding is in line with the literature (e.g., Epp, 2011).

One can identify several challenges for the teaching of arguing and proving at university level: In our study, these first-year students in Germany are not equipped with argumentation skills which are a requirement for learning to prove. It seems, as if mathematics at school does not provide the future students with adequate heuristics for problem-solving and basic proving skills. These findings underline the importance of courses like the "Introduction into the culture of mathematics". Therefore bridging courses have to start with basic skills for arguing and proving. Here, it is important to emphasize the meaning of informal arguments in order to stress the quality of a given argumentation. If we highlight the possibility to formalize an informal argument we also underline the function and value of using algebra and variables in mathematics.

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