

# WHAT ASPECTS OF MATHEMATICAL LITERACY SHOULD TEACHERS FOCUS ON FROM THE STUDENT'S POINT OF VIEW?

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*This study employed latent class cluster analysis to explore students' perceptions of what aspects of mathematical literacy, composed of mathematics competencies and attitudes, teachers should focus on. The sample included 1,219 Taiwanese senior high school students and 59 mathematics teachers. Three profiles were identified for mathematics competence, which were characterized as comprehensive, test-oriented, and limited thought-oriented. Regarding mathematics attitudes and mathematics learning attitudes, three profiles were identified and characterized as: broad, math-interior oriented, and mind-focused. Students and teachers differed in their perceptions on the importance of some aspects of mathematical literacy.*

## INTRODUCTION

Developing students' mathematical literacy has been a critical issue in both the academic study and practice of mathematics education. As early as 1986, the National Council of Teachers of Mathematics established the Commission on Standards for School Mathematics; the central tenet of these standards was the cultivation of mathematical literacy among students (Romberg, 2001). The Program for International Student Assessment, an international comparison study with more than 70 participating countries, also set mathematical literacy as a main focus of its survey.

Many researchers have focused on the structures and connotations of mathematical literacy. Kilpatrick (2001) identified "five strands of mathematical proficiency"—conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The first four pertain to mathematics competence, and the last is related to mathematics attitudes and mathematics learning attitude. These two categories of literacy also correspond to the goals of the Taiwan national mathematics curriculum (Ministry of Education, 2010).

In Taiwan, the 12-year compulsory education program is nearly launched. In the program, mathematics classes will have a big difference from present which will be composed of students at various levels of mathematics, who have various perceptions about mathematics. Thus, probing into senior high school students' perceptions regarding what aspects of, and/or how, mathematical literacy teachers should focus on, is beneficial. Because mathematical literacy is constituted of numerous factors, major profiles were identified to allow teachers to easily understand the results of this study. The main research questions were:

1. What are the profiles that portray senior high school students' perceptions of which mathematics competencies teachers should focus on?
2. What are the profiles that portray senior high school students' perceptions of which mathematics and mathematics learning attitudes teachers should focus on?
3. What are the commonalities and differences between students' and teachers' perceptions of what aspects of mathematical literacy teachers should focus on?

## **RESEARCH METHOD**

### **Conceptual framework**

The conceptual framework for mathematical literacy in this study included a cognitive component, mathematics competences, and an affective component, attitudes toward mathematics and learning mathematics. The choice of items in the components was based on both a literature review and the results of a qualitative pilot study (see the section of instrument).

### **Mathematics competence**

The two major types of mathematics competence (MC) are content-oriented and thought-oriented mathematical competence (Hsieh, Lin, & Wang, 2012). Content-oriented mathematical competence is related to specific mathematics topics, for example, the possession of factual knowledge (Niss, 2003). Thought-oriented mathematical competence arises from the characteristics of mathematical thought rather than relates to specific knowledge of particular mathematical topics (Krutetskii, 1976), for example, exploring in mathematics problems, applying mathematics to solve problems arising in daily life (CCSSO & the NGA Center, 2010), manipulating statements and expressions containing symbols, and understanding others' written texts about mathematics in a variety of linguistic registers (Niss, 2003). This study emphasized thought-oriented, rather than content-oriented, mathematical competence.

### **Mathematics and mathematics learning attitude**

Many studies have discussed the structure of mathematics attitudes (MAs), and have developed measures to investigate students' mathematics attitudes (Lim & Chapman, 2013; Perry, 2011). In these studies, the value of mathematics and the usefulness of mathematics are considered critical. These studies have investigated students' perceptions regarding the usefulness of mathematics in daily life and other subjects, the power of mathematics to develop people's thinking, and etc.

Researchers have specified many positive mathematics learning attitudes (MLAs). Kim and Kim (2010) considered the intention to grasp the core mathematical concepts indicative of a positive learning attitude. In another study, being willing and perseverant to do mathematics was considered indicative of a positive learning attitude (Yang & Tsai, 2010). Studies have also regarded the employment of appropriate learning methods to be indicative of a positive learning attitude (Kim & Kim, 2010; Yang & Tsai, 2010).

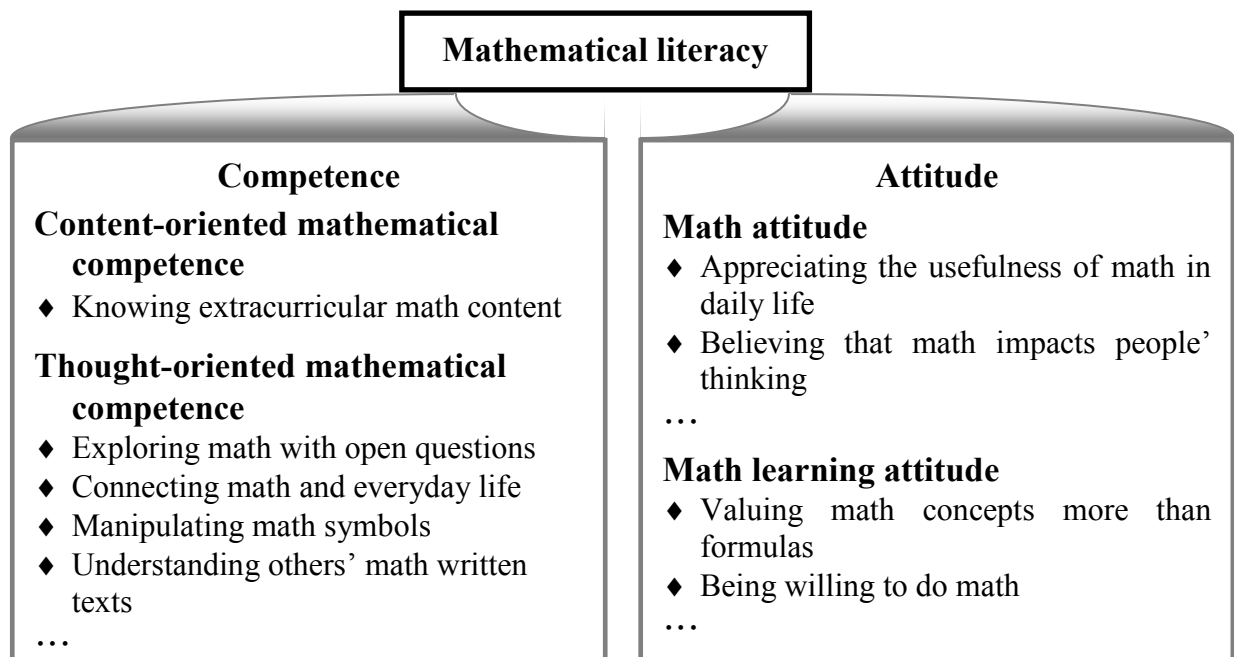


Figure 1: The framework of this study

### Design and Instrument

This study was conducted in two stages. In the first stage, a qualitative pilot study using open-ended questions was conducted, to obtain 238 high school students' opinions regarding what an ideal mathematics teacher would do when conducting a variety of teaching tasks, such as introducing new mathematical concepts. A content analysis of the students' responses was performed to obtain dimensions and items related to mathematical literacy by experts including university mathematics educators and researchers, school-based supervisors of future mathematics teachers, and expert secondary school mathematics teachers. A literature review was conducted to obtain further dimensions and items, which were included in the second stage. In the second stage, a questionnaire with dichotomous items was administered; on the questionnaire, students were asked to state whether or not a good senior high school mathematics teacher would focus on a certain literacy item in a particular teaching context; an example is shown in Figure 2. This study used two questions in the questionnaire, one question measured MC and the other measured MA and MLA; both questions consisted of 11 items. Complete lists of these questions are shown in Figures 3 and 4.

<p>To develop our mathematics competence, a very good senior high school mathematics teacher would...</p> <p><input type="checkbox"/> 1. Cultivate our abilities of expressing mathematics by asking us to explain our own methods to other classmates.</p> <p>...</p> <p><input type="checkbox"/> 4. Teach us how to employ sequencing steps of reasoning according to the information provided by a mathematics problem</p>
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Figure 2: The question related to mathematics competence

## Participants

In the first stage, 238 high school students in 6 classes were surveyed. In the second stage, 1,219 senior high school students from 61 classes and their mathematics teachers (59 in total) were surveyed. These students attended 30 schools in 23 cities out of Taiwan's 25 cities. The sampled schools were randomly selected. In each school, two or three classes were chosen randomly. Students in the 10th, 11th, and 12th grades constituted 24%, 41%, and 35% of the sample, respectively.

## Data analysis

This study employed latent class cluster analysis (LCA), a model-based approach (Muthén, 2001), to analyze the MC and MA&MLA data separately. LCA enabled the interrelationships between observed variables, the responses regarding whether or not an ideal teacher would focus on a literacy item, to be analyzed. Subsequently, latent classes were identified for students' perceptions regarding what aspects of mathematical literacy teachers should focus on. For each literacy item, the conditional probability that a student in a particular class would agree with that item was obtained. The distributions of these conditional probabilities for all literacy items in a certain class were obtained to depict its profiles of students' perceptions of which aspects of mathematical literacy teachers should focus on (hereafter, this conditional probability is referred to as "focusing probability"). For each student, the probabilities of being assigned into each latent class were obtained, which were summed to 1. These probabilities were averaged over the individuals in the same class to obtain the relative size of the class.

Log likelihood (LL) and adjusted Bayesian information criterion (BIC) statistics were employed as goodness-of-fit criteria; smaller values indicated a better fitting model. Entropy was used to measure how well the model classified students. It should be above 0.6 for medium level and above 0.8 for high level (Clark & Muthén, 2009). Relative criteria were also considered. Differences in BIC and LL statistics, and Vuong-Lo-Mendall-Rubin (VLMR) tests, were used to assess the improvement of model parsimony by comparing the model with  $n$  classes to a model with  $n-1$  classes.

Percentages of teachers' checking for each item were calculated to determine teachers' perceptions of which aspects of mathematical literacy a teacher should focus on.

## RESEARCH FINDINGS

### Mathematics competence

Models with more classes were preferred, according to the LL and BIC criteria. However, differences in BIC and LL gradually diminished as the number of classes increased, indicating that improvements in model parsimony shrank. The VLMR tests suggested that the 4-class model did not fit the data better than the 3-class model ( $p = .30$ ), and the 3-class model offered a significantly more adequate fit than the 2-class model ( $p = .01$ ). After further consideration of entropies, the 3-class model was selected because of it offered the optimal and most parsimonious representation.

No. of classes	Entropy	Log likelihood	Adjusted BIC	Diff(LL)	VLMR <i>p</i> -value	Diff(BIC)
1	-	-4636.577	9316.286	-	-	-
2	0.675	-4234.465	8559.117	-402.112	.00	-757.169
3	0.708	-4194.814	8526.868	-39.651	.01	-32.249
4	0.638	-4170.410	8525.114	-24.404	.30	-1.754
5	0.687	-4157.429	8546.205	-12.981	.60	21.091

Table 1: Fit statistics for latent class analysis of MC

As shown in Figure 3, the focusing probabilities of Class 1 were higher than 90% for all MC items except “exploring math with open questions,” which still had a probability of 85%. Students classified in Class 1, which had a relative size of 57%, indicated that teachers should focus on every MC item listed in Figure 3. The profile this class portrayed was thus characterized as “comprehensive.” By contrast, students classified in Class 2 (38%) indicated that teachers should focus more on the first six MC items than on the last five MC items, as shown in Figure 3 (the first and second categories, respectively).<sup>1</sup> The MCs in the second category were considered less crucial to success on senior high school mathematics tests in Taiwan. The profile of Class 2 was thus characterized as “test-oriented.” Compared with students in Classes 1 and 2, the students in Class 3 (5%) indicated that teachers should focus on limited mathematical competencies, and only four competencies had focusing probabilities higher than 50% for this group, three of which were related to mathematics thought in questions, and one of which was related to mathematics language (Niss, 2003). This study thus characterized the profile as “limited thought-focused.” In terms of teachers’ perceptions, teachers indicated that all MC items should be focused on in the classroom, with only two exceptions—“knowing extracurricular math content” and “exploring math with open questions.”

Cultivating students’ abilities to explore mathematics has been a critical issue in mathematics education worldwide (e.g., Hsieh, Horng, & Shy, 2012). In Taiwan, a project presently underway, the “Highlight-base program,” which aims at promoting mathematics teachers’ professional development, chooses developing teachers’ abilities to integrate mathematics exploration into the classroom as one focus. However, “exploring math with open questions” was the MC item with the lowest focusing probability (students’) and the lowest checking percentage (teachers’). Further research is necessary to determine whether this indicates that students and teachers do not perceive exploring as a crucial aspect of MC or that they do not think exploring on open questions are helpful to cultivate students on this MC? Moreover, teachers indicated that teachers should focus on “orally expressing one’s own math problem-solving method.” Oral expression of the problem-solving process allows

<sup>1</sup> Factor analysis was conducted to divide the competences into two categories.



students to reconstruct their mathematical thinking, enabling them to come to a clearer understanding. However, some students did not necessarily share their teachers' perceptions in this regard.

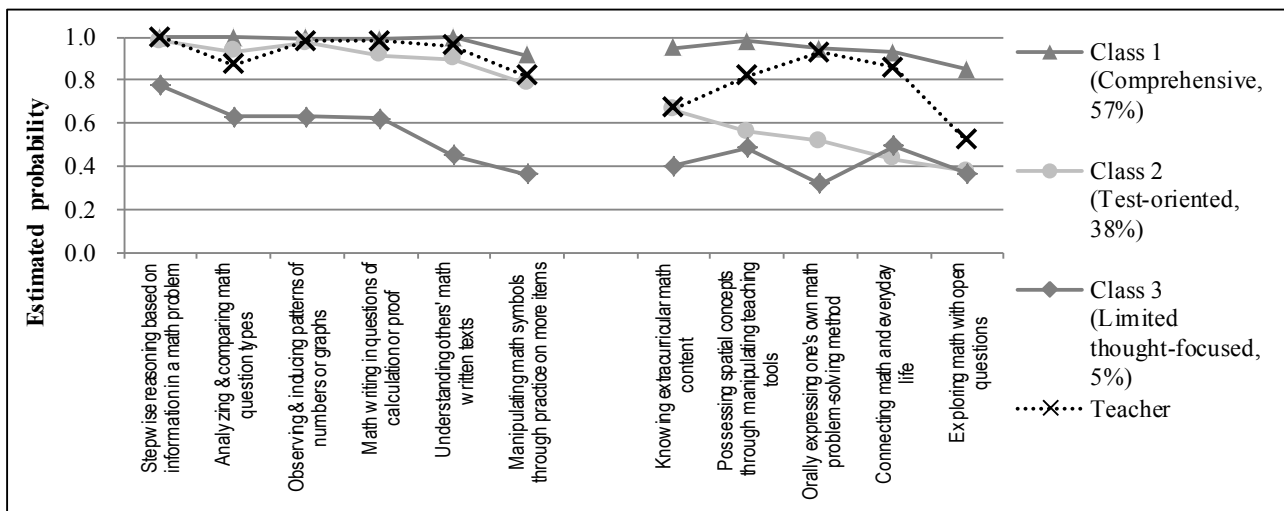


Figure 3: Three profiles of students' perception on the importance of MC

**Mathematics and mathematics learning attitude (MA&MLA)**

Based on the fit statistics shown in Table 2, the 3-class model was selected because it offered the optimal, most parsimonious representation.

No. of classes	Entropy	Log likelihood	Adjusted BIC	Diff(LL)	VLMR p-value	Diff(BIC)
1	-	-4222.155	8487.505	-	-	-
2	0.760	-3695.289	7480.896	-526.866	.00	-1006.609
3	0.782	-3626.110	7389.662	-69.179	.00	-91.234
4	0.770	-3595.817	7376.199	-30.293	.54	-13.463
5	0.801	-3569.918	7371.523	-25.899	.06	-4.676

Table 2: Fit statistics for latent class analysis of MA&MLA

The profiles of the three classes are shown in Figure 4. The first five items in the figure belong to mathematics attitude (MA), and the last six items belong to mathematics learning attitude (MLA). Regarding Class 1, focusing probabilities were higher than 90% for all attitudes except “appreciating contributions of math to human civilization through stories.” Nonetheless, even this attitude reached a probability of 83%. Class 1 was therefore characterized as “broad.” Students in Class 2 also considered MLA items to be critical, but “believing that math impacts people’s thinking” was the only MA item with a focusing probability over 50%. Students in Class 2 did not think that teachers should focus on application outside mathematics (e.g., to other subjects or daily life). The profile portrayed by Class 2 was thus characterized as “math-interior oriented.” The profile of Class 3 is similar to that of Class 2 in terms of the MA part –

only highly considering that math impacts people's thinking as the MA teachers should focus on. However, students in Class 3 valued MLA items that were pertinent to ideas, thoughts, and volition in mind but not items related to actual actions: "being willing to do mathematics" and "being willing to ask math questions." Therefore, this study characterizes Class 3 as "mind-focused." The teachers' perception is shown in Figure 4, which indicates that teachers believed that every MA and MLA item should be focused on by teachers.

In contrast to their teachers, 33% of students (Classes 2 and 3) did not consider that teachers should focus on applying mathematics to other fields (the 2<sup>nd</sup> to the 5<sup>th</sup> items), and 17% of students did not consider the MLAs related to taking actual actions to work on mathematics should be focused on. Whether student consider these attitudes as not important or may not be teachers' responsibilities to foster is worthy of future investigation.

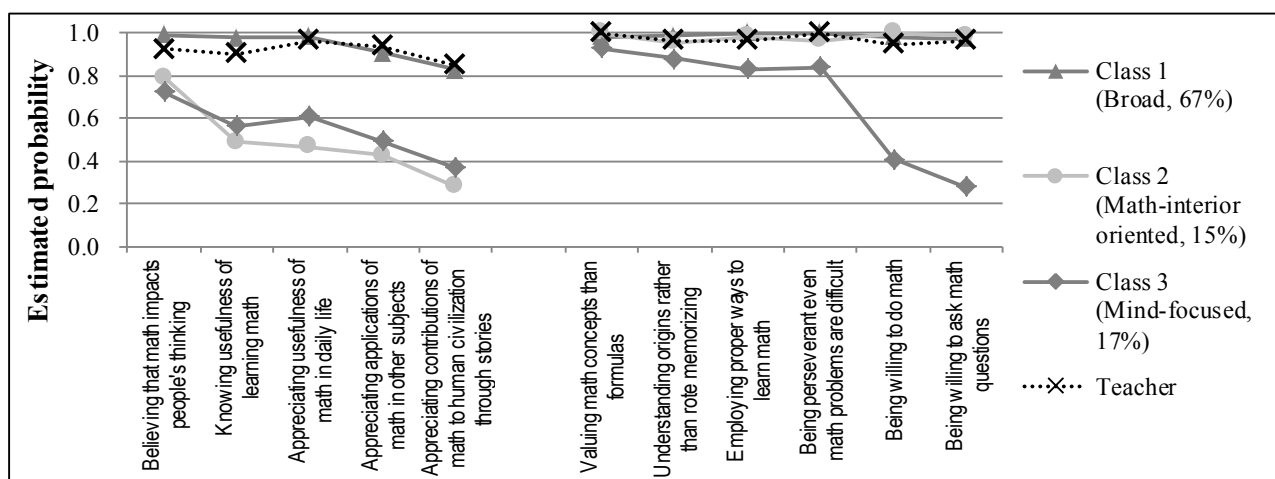


Figure 4: Three profiles of students' perception on the importance of MA&MLA

## CONCLUSION

Using LCA, different profiles of Taiwanese senior high school students' perceptions of which aspects of mathematical literacy teachers should focus on were identified. Regarding mathematics competence, the three profiles were characterized as comprehensive, test-oriented, and limited thought-oriented. In terms of mathematics and mathematics learning attitudes, the three profiles were characterized as broad, math-interior oriented, and mind-focused. Certain students deviated from their teachers' perceptions regarding competencies and attitudes they believed teachers should focus on. Further investigation is required to find out the reason behind, for example, whether students believe the mathematical literacy is unimportant, whether teaching methods are effective at developing the literacy, and whether cultivating the literacy is a teacher's responsibility. The results of this study provide a valuable reference for teachers, to allow them to determine what to emphasize in cultivating mathematical literacy among students.

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