THE IMPORTANCE OF ACHIEVING PROFICIENCY WITH SIMPLE ADDITION

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In this study we investigated how 200 students in seventh grade (mean age = 12.38 years) solved simple addition problems and if the way they performed simple addition was related to their achievement in mathematics. Four performance groups were identified: proficient, almost proficient, inaccurate min counting and accurate min counting. More than half the participants did not display proficient or close to proficient performance despite expectations that proficiency is achieved around third grade. Findings unique to this study were that accurate min counting was associated with lower math achievement and that girls were more likely to display this pattern of performance than boys. The findings corroborate a growing awareness that many students are not achieving proficiency and that this is a concern requiring attention.

INTRODUCTION

Simple addition involves adding together single digit numbers. Curriculum documents (or standards) indicate that by around second or third grade (eight years of age), children solve simple addition problems using retrieval (e.g., Australian Curriculum, Assessment and Reporting Authority, 2010; Department for Education, 2013; National Council of Teachers of Mathematics, 2000). Contrary to curriculum expectations, a large study conducted in the UK found that not just a few but *many* children in second and third grade were not retrieving solutions to most simple addition problems (Cowan, Donlan, Shepherd, Cole-Fletcher, Saxton, & Hurry, 2011).

It is not clear if educators need to be concerned about this finding. Cowan et al. (2011) postulated that a lack of retrieval with simple addition may not be the barrier to achievement that it is often predicted to be given children in their study also showed typical achievement. In this study we investigated how students who were well beyond the stage when retrieval is expected to dominate performance solved simple addition problems and if an association between simple addition performance and math achievement was evident among students who were expected to learn higher-order mathematical procedures and concepts.

BACKGROUND

The issue regarding the importance of retrieval is somewhat clouded by different views of what it means to achieve *proficiency with simple addition*. Curriculum documents tend to view proficiency as the accurate and exclusive use of retrieval to solve simple addition problems. Retrieval refers to the direct retrieval of an answer from a store of facts held in long term memory (Ashcraft, 1995). Proficiency can also be viewed as

2014. In Oesterle, S., Liljedahl, P., Nicol, C., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 3, pp. 337-344. Vancouver, Canada: PME.

encompassing the correct use of other efficient strategies, not just retrieval (Baroody, 2006). These include *decomposition strategies*, strategies that involve applying number principles, deriving answers from related facts or decomposing numbers to solve problems (Cowan et al., 2011). Proficiency with simple addition is defined in this paper to be performance that is accurate and is dominated by the use of retrieval and decomposition strategies.

Before proficiency is achieved, children's simple addition performance is characterized by the use of counting strategies. As children develop in their mathematical thinking they generally progress from using a *counting-all strategy* where the count is started at one, to using more sophisticated counting strategies such as a *counting-on from first strategy*, where the second addend is counted on the first addend, and a *counting-on from larger strategy*, where the smaller addend is counted on the larger addend (Carpenter & Moser, 1984). The counting-on from larger strategy requires the minimum number of counts and is also referred to as *min counting* (Fuchs, Powell, Seethaler, Cirino, Fletcher, Fuchs, et al., 2010; Geary, Hamson, & Hoard, 2000). With continued correct practice (Shrager & Siegler, 1998) and growth in an understanding of number principles and rules (Baroody & Tiilikainen, 2003), children generally progress from using less efficient counting strategies to using min counting, retrieval and decomposition strategies (Hopkins & Lawson, 2002).

While min counting is the most efficient counting strategy, its frequent use is not considered appropriate beyond third grade in most curriculum documents. The issue regarding the importance of achieving proficiency with simple addition it is not about whether children's performance is dominated by retrieval, or retrieval and decomposition strategies, the issue is that children's performance is accurate and is no longer dominated by counting strategies including min counting.

This research is concerned with investigating the importance of achieving proficiency with simple addition. The first aim of this research was to document how students who were at a stage well beyond when proficiency is expected, performed simple addition. Students in Year 7 were chosen as this is the final year before secondary education in the state. The second aim was to investigate if the way students performed simple addition was related to their achievement in mathematics. The third aim was to explore possible gender differences in how simple addition was performed.

METHOD

The study cohort comprised 200 students in Year 7 with a mean age of 12.38 years (*SD*=0.43 years) from 13 government primary schools located in the Perth metropolitan area in Western Australia (WA). Participants included 116 females (58%). Mathematics achievement scores for each participant were based on numeracy results from the Western Australian Literacy and Numeracy Assessment (WALNA) administered by the government as part of the national testing regime. The numeracy section of the WALNA assesses math outcomes associated with Space, Measurement, Chance and Data, Number, Pre-algebra and Working Mathematically and takes approximately 45 minutes to complete. Assessment results for the WALNA are calibrated on a common logit scale based on the Rasch measurement model and scores range from 0 to 800. The numeracy assessment used showed excellent reliability (Pearson Separation Index = 0.879). Numeracy scores for the study cohort did not differ significantly from the state's population mean score, $t(196)=.219$, $p=.827$.

Students were individually assessed as the performed a set of 36 simple addition problems that were presented in random order using a computer. The set included all single-digit addition problems with addends greater than 1 (2+2 to 9+9), written in the form 'm+n=' and presented with the smaller addend first (except for tie problems where m=n) making it possible to distinguish between use of the counting-on from first strategy and min counting. The response time taken to complete each problem was recorded along with the answer given. Strategy use was identified based on a combination of observation and self-report given after each problem was solved. This combined approach of observation and self-report on a problem by problem basis is commonly used to identify the strategies used to solve addition problems (e.g., Canobi, 2009; Geary et al., 2000).

Response times (RT) corroborated the strategies identified using the combined approach. Response times to retrieval trials were generally under three seconds (M=1.78s, SD=0.84) - a time limit often used to infer direct retrieval on forced retrieval tasks (Cowan et al., 2011). Mean response times to trials where decomposition strategies were identified (M=3.26s, SD=1.8) were longer than RTs to retrieval trials but shorter than RTs to min counting trials (M=3.76, SD=2.25). Furthermore, RTs to min counting trials increased in a strong linear fashion as the minimum addend increased (representing the number of counts made). An increase of around half a second was recorded for each count.

Simple addition performance was classified into groups using cluster analysis based on three criteria similar to those used by Siegler (1988): the percentage use of direct retrieval, the percent correct on direct retrieval trials and the percent correct on min counting trials. An ANOVA was used to test group differences in terms of math achievement and a chi-square statistic was applied to test the significance of differences in the number of female and male students in each group.

RESULTS

The three criteria significantly differentiated three performance groups: percentage use of direct retrieval, $F(2,169)=176.50$, $p<.001$, $\eta^2=.67$, percent correct on min counting trials, $F(2,169)=161.61$, $p<.001$, $\eta^2=.66$, and percent correct on direct retrieval trials $F(2,169)=6.03$, $p<0.01$, $\eta^2=.07$. Twenty-eight students could not be classified using the cluster analysis as they exclusively applied retrieval and decomposition strategies and therefore no data were available for the criterion relating to min counting trials. These students formed a fourth group.

The four performance groups appeared readily interpretable and were labelled the proficient, almost proficient, inaccurate min counting, and accurate min counting groups. The characteristics of students' simple addition performance for each of the four groups are detailed in Table 1, including the percentage mix of strategies used to solve the problem set and the percentage accuracy of each strategy.

	Proficient (n=28)	Almost proficient (n=65)	Inaccurate min counting (n=37)	Accurate min counting (n=70)
Strategy		Mean strategy use (*SD*)		
Direct retrieval	88.9% (11.5)	76.7% (10.6)	61.8% (10.7)	39.7% (12.6)
Min-counting	0	16.8% (9.7)	28.3% (13.7)	47.3% (18.9)
Decomposition	11.0% (11.5)	6.2% (8.7)	8.8% (9.8)	12.9% (14.3)
Strategy		Mean accuracy (*SD*)		
Direct retrieval	98.8% (2.3)	98.6% (2.5)	96.3% (4.7)	98.5% (3.6)
Min-counting	-	99.3% (3.0)	74.7% (10.5)	95.7% (7.1)
Decomposition	97.3% (5.19)	91.9% (24.1)	94.9% (12.7)	96.5% (9.7)

Table 1: Characteristics of each performance group

In summary, the simple addition performances of students characterized as proficient comprised the exclusive use of retrieval and decomposition strategies and mostly correct answers. Performance characterised as almost proficient was dominated by the accurate use of retrieval and decomposition strategies, but some accurate use of min counting was evident. Performance characterized as inaccurate min counting encompassed a relatively moderate use of direct retrieval and decomposition strategies, and the frequent use of min counting that resulted in incorrect answers (errors were made on 25% of min counting trials). Students in this performance group also displayed the lowest accuracy for retrieval. This finding is consistent with the strategy choice model (Shrager & Siegler, 1989), which predicts that retrieval errors can occur as incorrect associations are formed in memory when a counting strategy is used inaccurately. Performance characterised as being accurate min counting encompassed the dominant use of min counting and generally accurate performance.

The math achievement scores for students in each of the four performance groups were compared. Four students did not have achievement scores and were dropped from the analysis. An ANOVA revealed a significant difference in mean math achievement scores across performance groups, $F(3,192)=15.84$, $p<.001$, $\eta^2=.20$. Approximately 20% of variance in math achievement scores was explained by differences in how students performed simple addition. Post-hoc comparisons with Bonferroni adjustment revealed that students in the proficient group scored significantly higher on the math assessment ($M=529.33$, $SD=72.89$, $n=27$) than students in the accurate min counting group (*M*=451.09, *SD*=71.37, *n*=69), $t(94)=4.80$ $p<.001$, $d=1.09$, and students in the inaccurate min counting group (*M*=430.32, *SD*=65.50, *n*=37), $t(62)=5.69$, $p<.001$, $d=1.44$. No significant difference in mean math scores was found between students in the proficient group and the almost proficient group, $t(88)=1.94$, *p*=ns. Students displaying almost proficient performance scored significantly higher on the math assessment (*M*=498.54, *SD*=67.12, *n*=63) than students who displayed accurate min counting $t(130)=3.93$, $p<.001$, $d=0.68$, and students who displayed inaccurate min counting, $t(98)=4.95$, $p<.001$, $d=1.03$. No significant difference in math achievement was found between students in the accurate min counting and inaccurate min counting groups, $t(104)=1.47$, *p*=ns. The findings indicate that students who were more likely to solve simple addition problems using min counting displayed lower achievement in maths than their peers who frequently use retrieval and decomposition strategies – regardless of whether they use min counting accurately or inaccurately.

An exploratory analysis was also conducted to compare the number of female and male students classified as displaying proficient, almost proficient, inaccurate min counting and accurate min counting. Table 2 shows the number of female and male students in each group.

Performance group	No. of female students	No. of male students
Proficient	12 (10.3%)	16 (19.0%)
Almost proficient	30 (25.9%)	35 (41.7%)
Inaccurate min counting	21 (18.1%)	16 (19.0%)
Accurate min counting	53 (45.7%)	17 (20.2%)

Table 2: Number of female and male students in each performance group

The chi-square statistic indicated that there was a significant difference in the gender composition of the performance groups, χ^2 (3, *N*=200) = 15.421, *p* = .001. More female students displayed accurate min counting performance than male students, and more male students displayed proficient or almost proficient performance than female students. A comparable number of male and female students displayed inaccurate min counting.

DISCUSSION

The findings revealed that less than half the number of Year 7 students who participated in the study were proficient or close to being proficient with simple addition. As students were well beyond the stage when proficiency is expected, this finding highlights the issue that many students are not achieving proficiency with simple addition. The findings also revealed that a close association between proficiency and achievement in math was evident. The implication is that educators need to be concerned about addressing this issue.

We believe it is very important to achieve proficiency with simple addition. Poor proficiency will act as a barrier to developing key conceptual knowledge as emergent understandings of number are not reinforced by efficient procedures and attentional resources are not made available during performance to discern underlying number concepts. The argument is based on the view that mathematical development is influenced by an iterative relationship between procedural and conceptual knowledge, where an increase in one type of knowledge leads to an increase in the other type of knowledge, in turn promoting deeper insight into the first type of knowledge (Canobi, 2009; Schneider & Stern, 2010). The finding of a close association between proficiency and achievement in math supports this view.

The findings are consistent with results from the learning disabilities (LD) field. Research is this field has established that students with a mathematics learning disability (MLD) (often identified by poor achievement) stay reliant on counting to perform simple addition at an age well beyond their typically achieving peers (Geary, 2010; Ostad & Sorenson, 2007, Torbeyns, Verschaffel, & Ghesquière, 2004). Students with a MLD are also more likely to exhibit counting and/or retrieval errors (Geary, et al., 2000) – particularly if they have a combined reading disability (Jordan, Hanich, & Kaplan, 2003). While the findings are consistent, they advance those reported in the literature in three important ways.

The findings of the present study suggest that difficulties achieving proficiency are more common than what is suggested in LD research. Research in the this field has largely relied on a comparative approach where the simple addition performance displayed by a group of students with a MLD is compared to performance displayed by typically achieving students. Students are often identified as having a MLD based on achievement scores that fall below the 30th percentile (Murphy, Mazzocco, Hanich & Early, 2007). Thus by definition, the prevalence of a MLD is at most 30% of the population. It could be assumed that this figure also represents the prevalence of difficulties achieving proficiency. The prevalence of difficulties achieving proficiency appears to be more widespread than this.

The findings from the present study distinguished between the use of accurate counting and inaccurate counting. To our knowledge this distinction has only been made in one other study. Siegler (1988) found that six-year-old children who predominately used counting strategies accurately showed comparable math achievement to their peers who relied on retrieval but children who predominately used counting strategies inaccurately did not. Siegler referred to the first group of children as perfectionists and explained their preference for counting as being influenced by a high confidence threshold for retrieval. The present study is the first examine the effects of adopting a perfectionist-like approach beyond the age of six. Accurate min counting among 12-year-old students *was* associated with lower math achievement. This has important implications for classroom practice. Approaches are needed to address difficulties achieving proficiency for students who display accurate min counting and will need to focus on building confidence with retrieval. These will be different to approaches that address inaccuracy.

A third unique finding of the present study was that girls were more likely to exhibit accurate min counting than boys and boys were more likely to display proficiency. This finding is novel and needs to be corroborated by other research but is important to investigate further. It is generally acknowledged that more males than females are identified as experiencing learning difficulties even though differences in overall math achievement are not strongly evident (e.g., Vogel, 1990). It may be that assessments that contribute to the identification of learning difficulties focus more on accuracy than efficiency and this will need to be rectified in future research.

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