HOW SHOULD STUDENTS REFLECT UPON THEIR OWN ERRORS WITH RESPECT TO FRACTION PROBLEMS?

Tim Heemsoth, Aiso Heinze

Leibniz Institute for Science and Mathematics Education, Kiel, Germany

Educational research assumes that error reflections are efficient if they include the rationale behind the own error instead of just correcting the error. However, thus far there is a lack of empirical evidence regarding this aspect. Thus, we conducted a field experiment with pre-post-follow-up design and with 7th and 8th grade students (N = 174). The study was conducted during standard mathematics lessons. We compared two different error-handling strategies. Our findings indicate that students who reflected the rationales behind their errors enhanced their procedural knowledge more than students who reflected on the corresponding correct solution only. Regarding conceptual knowledge we found this effect only at the follow-up-test. The implications for theory and school instructions are discussed.

INTRODUCTION

Educational researchers assume error reflections to comprise a high learning potential for the students' learning (e.g. Siegler, 2002; VanLehn, 1999). Yet, most of the previous studies investigated learning from errors committed by someone else (e.g. Große & Renkl, 2007). To our knowledge studies investigating learning from reflections on one's own errors are very rare. Moreover, thus far it is unclear what error-handling strategy supports the students' learning from own errors most efficiently. A core assumption is that students develop more comprehensive cognitive models if the error-handling strategy includes the rationale behind one's own error (Ben-Zeev, 1998). The main objective of the study presented in this contribution is to address these desiderata and to investigate the question whether 7th and 8th grade students learn fractions better by reflecting on the rationales behind their own errors or by only reflecting on the corresponding correct solution only.

Student reflections upon errors

If errors occur during the learning process, they have the potential to trigger the reconstruction of the students' concepts and strategies (Duit & Treagust, 2003; Siegler, 2002). Thus far, these concepts and strategies might have been absolutely sufficient to solve previous problem solving situation. However, in the error situation these concepts and strategies need to be reconsidered in order to solve new problem solving situations. Educational research assumes that the corresponding error reflections comprise elaborate learning: It is easy to explain why a correct answer is correct just by citing the given answer. However, explaining why an incorrect answer is incorrect forces the learner to reflect on both the correct solution and its scope of application (Siegler & Chen, 2008). In this study, we assume that an error-handling strategy

2014. In Oesterle, S., Liljedahl, P., Nicol, C., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 3, pp. 265-272. Vancouver, Canada: PME.

supports these elaborate learner's reflections if the strategy builds on the *rational*e behind the error. A rational error occurs if a learner applies a strategy that has worked successfully in a previous problem-solving situation to a new and similar problem that would require another strategy (Ben-Zeev, 1998). For example, students erroneously *overgeneralize* a specific strategy: From addition exercises with two fractions having the same denominator they internalized the rule "numerator plus numerator and denominators remain the same". Some students overgeneralize this rule to multiplication exercises and calculate 3/9 x 4/9 = 12/9 (see Padberg, 2009). Such rational errors indicate a principle misunderstanding. Reflections on these rationales behind the errors can enable the learner to access and adjust his/her insufficient cognitive models (cf. Ben-Zeev, 1998).

Previous empirical findings

Educational research has shown that the integration of errors into the learning process can enhance the students learning (e.g. Große & Renkl, 2007; Keith & Frese, 2005; Siegler, 2002). Yet, in previous studies the role of the rationale behind the error was not investigated systematically. Research on *error management training* highlighted that learners who were encouraged to conduct errors during the learning process improved their task performance more than learners who were instructed to avoid errors (e.g. Keith & Frese, 2005). However, learners who were encouraged to conduct errors were not instructed to reflect on the rationale behind their own error. Instead, research on *learning from incorrect examples* used prompts to trigger learners to reflect why answers were incorrect, to explain the reasoning behind a student's wrong answer or to change the problem so that the student's answer is correct (Große & Renkl, 2007; Heemsoth & Heinze, 2013; Siegler, 2002). In some of these studies learners who were confronted with incorrect examples improved their performance more than learners who were only confronted with correct examples. Yet, some findings indicated that there is an interaction effect regarding the learners' prior knowledge: Learners with high prior knowledge benefited more from incorrect examples while students with low prior knowledge benefited more from correct examples. These findings were found both for university students learning statistics (Große & Renkl, 2007) and for secondary school students learning fractions (Heemsoth & Heinze, 2013). However, even though in these studies students were encouraged to reflect on the rationale behind the error committed by someone else, they did not reflect on their own errors. In one of the few studies that tested instructions on own errors students were instructed to reflect on their own incorrect physics statements by (1) indicating, (2) explaining and (3) correcting their statement (Yerushalmi & Polingher, 2006). A similar error-handling strategy suggests one additional step that asked the students (4) to take action in order to avoid the same error in future problem-solving situations (Guldimann & Zutavern, 1999). In sum, there are indications how to implement an error-handling strategy including the rationale behind students' own errors. However, thus far the effectiveness of these strategies has rarely been investigated. Moreover, since most of the findings described in this section were derived from strictly controlled experiments with restricted ecological validity, there is a lack of findings with regard to ecologically valid school settings and relevant curriculum topics.

The learning topic: Fractions

In order to investigate our research question we chose fractions as our learning subject. Knowledge of fractions provides a fundamental basis for later algebraic operations, enhances intellectual development and is essential for handling many real-world situations and problems not only occurring during school but during the whole life through (NMAP, 2008) This might be an explanation for why knowledge of fractions has been shown to be a core requirement for mathematical success in later school years (Siegler et al., 2012). Typical student errors have been extensively investigated and many student errors have been shown to be very persistent for the individual student (e.g. Padberg, 2009). In specific, several types of errors can be traced back to a specific rationale. For example these errors result from adopting concepts of natural numbers to fractions (Vamvakoussi & Vosniadou, 2004) or from an overgeneralization of other fraction arithmetic strategies (Padberg, 2009). Thus, fractions seemed to be an adequate domain for our intervention study.

The present study

We examined whether 7th and 8th grade students improved their knowledge of fractions more if they reflected on the rationale behind their own error (error-centered condition) or if the students were instructed to reflect on a corresponding correct solution only (solution-centered condition). The construction of the error-centered strategy was based on the four metacognitive steps provided by Guldiman and Zutavern (1999). We examined the development of procedural and conceptual knowledge of fractions. We assumed that in the error-centered condition the rationale behind one’s own error is included. Thus, students in this condition better adjusted their incorrect cognitive models than students in the solution-centered condition in which the rationale behind one’s own error was *not* considered. Moreover, according to Siegler and Chen (2008) we assumed that in the error-centered condition learning was more elaborate. Since elaborate learning is a prerequisite for a successful recall of knowledge (Wittrock, 1989), we assumed the predominance of the error-centered condition to remain stable over time compared to the solution-centered condition. In summary, our research was guided by the following hypotheses:

Hypotheses 1: Students in the error-centered condition enhance their procedural knowledge more than students in the solution-centered condition. The effect remains stable after a retention phase.

Hypotheses 2: Students in the error-centered condition enhance their conceptual knowledge more than students in the solution-centered condition. The effect remains stable after a retention phase.

METHOD

Design

All students participated in a pre-post-follow-up design. In each class all students were randomly assigned to one of the two the conditions. Before the intervention started we asked for the students' mathematics grade, their gender, and age. During the first two lessons the error-handling strategies were introduced in both conditions. Hereafter, students reflected on their own errors that they conducted either in the pretest (that was conducted after the introduction phase) or in one of two further intermediate tests. The time for reflections on own errors was 135 minutes in total. After the intervention phase a posttest and six weeks later a follow-up test was administered. All tests measured the students' procedural and conceptual knowledge of fractions.

Participants

The sample consisted of 174 students (12 to 15 years of age) who belonged to five 7th and four 8th grade classes from German secondary schools (Gymnasium or comprehensive school). For all students, the intervention study served as a refresher and opportunity to practice fractions. On the whole, 87 students participated in the error-centered condition and 87 students in the solution-centered condition. There were no group differences regarding mathematics grade, age, gender and number of participants with respect to grade level or school type.

Pre-, Post-, Follow-up- and intermediate tests

We used parallel pre-, post- and follow-up tests to measure procedural and conceptual knowledge of fractions. Example items are presented in Table 1. Seven items emphasized procedural knowledge and asked to use fraction arithmetic procedures to compute a fraction problem. Four conceptual knowledge items comprised basic conceptions of fractions (e.g. part-whole interpretation, see example item in Table 1). To achieve parallel tests, procedural knowledge items differed with regard to numbers and the conceptual knowledge items with regard to the context and numbers. Answers were coded with "1" (correct), "0.5" (partial correct) or "0" (incorrect). Performance scores are represented by the percentage of correct items. The scale reliabilitiy for conceptual knowledge at the pretest was low. Thus, findings with regard to conceptual knowledge should be interpreted with caution.

The student reflections were based on the pretest and two more intermediate tests. The intermediate tests were parallel versions of the pre- post- and follow-up-tests. However, due to time restrictions regarding the standard mathematics lessons two shorter versions of the intermediate tests varying in difficulty were administered: Proficiency Level 1 tests only contained two conceptual knowledge items – the two most difficult items were excluded. Proficiency Level 2 tests contained only five procedural knowledge items; the two easiest item types were excluded. Students received Proficiency Level 1 tests if they solved less than 50% of the previous test problems correctly; they received Proficiency Level 2 tests if more than 50% were solved correctly.

Test	Number of items	Cronbach's alpha			Example Item 1
		Pre	Post	Follow-up	
Procedural knowledge	7	.85	.87	.86	Compute (3/4 * 9/4) * 2
Conceptual knowledge	4	.47	.68	.67	Mr. K. pays a monthly rent of 1500 €. This is 3/5 of his salary. What is the salary of Mr. K.?

Table 1: Scale reliability and example item for procedural and conceptual knowledge.

Error-handling strategies

All students reflected on their own errors and used the strategy corresponding to their condition. In the error-centered condition, the students used a worksheet with a table of four rows. The headline of these rows had the following four prompts: (1) Describe your answer and error; (2) Explain, why you thought your answer was correct; (3) Revise your answer; (4) Create a problem in which a similar error could have occurred. Solve this problem correctly. Both the second and the fourth prompt triggered the learners to reflect the rationale behind their errors. In the solution-centered condition, the students worked on examples that corresponded to the exercises that had been solved incorrectly. The examples began with an exercise similar to the exercise the student had solved incorrectly. Below the exercise a correct solution was presented and the students were asked to answer the following three prompts: (1) Describe the student's solution; (2) Explain, why the solution is correct; (3) Revise your answer.

Procedure

In the first two lessons, the error-handling strategies were introduced in both conditions. Therefore, *non-fraction* problems were presented to the students. During three of the following six lessons, the students took a test of procedural and conceptual knowledge of fractions at the beginning of these lessons (the pretest and the two intermediate tests). Having finished these tests after a short 10-minute break, all students received feedback that was directly presented on the test sheet and indicated right or wrong answers. The students reflected on their own errors using the specific error-handling strategy they had learned before. Reflections were continued in the previous lessons after each test. In total, the reflections lasted 45 minutes each. In the error-centered condition, students who struggled to detect the error were allowed to read a correct example of a similar problem. Examples were the same in the solution-centered condition but were not prompted.

RESULTS

Hypothesis 1

To test differences between conditions with respect to procedural knowledge, we used a repeated-measures ANCOVA and entered *condition* as the between-group factor and *time* as the within-group factor. We entered the mathematics grade, gender, age, grade level and school type as covariates to account for possible effects on the students' learning and to estimate results more precisely. There was a significant interaction effect of condition and time on procedural knowledge ($F(2, 334) = 4.97$, $p = .008$, $\eta^2 = .029$). Simple effects analyses showed that there was no significant difference at pretest. Yet, the analysis of the post- and follow-up-tests indicated that students in the error-centered condition showed a higher performance both immediately ($M = 62.32$, $SD = 33.67$) and six weeks after the intervention ($M = 49.18$, $SD = 36.13$) than students in the solution-centered condition (post: $M = 54.51$, $SD = 35.82$, follow-up: $M = 43.43$, $SD = 35.10$), post: $F(1, 167) = 5.99$, $p = .015$, $\eta^2 = .035$, follow-up: $F(1, 167) = 4.12$, $p = .044$, $\eta^2 = .024$ (see Figure 1). Further analyses showed that there were no significant interaction effects between conditions and prior procedural knowledge.

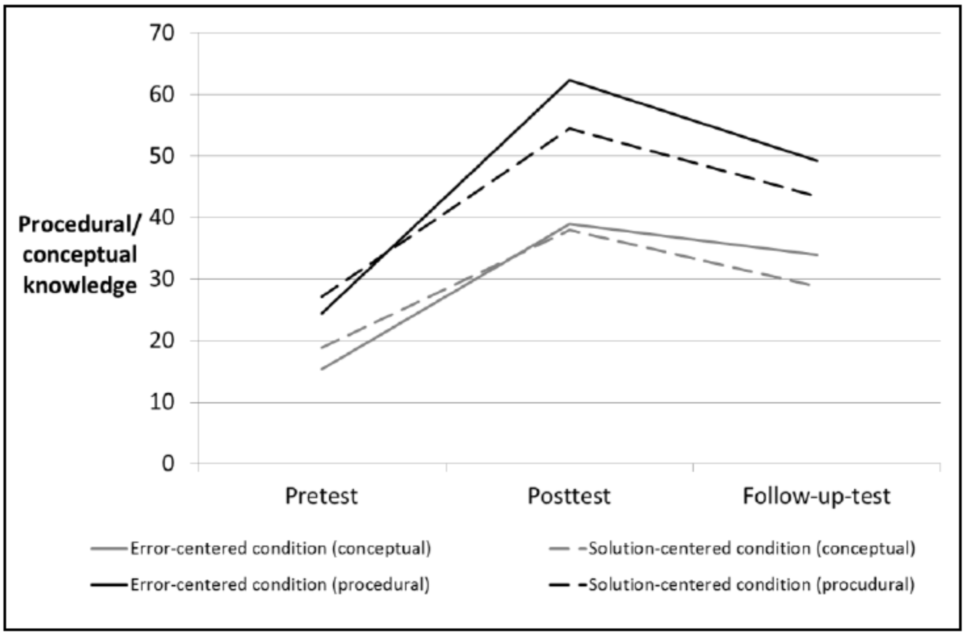

Figure 1: Procedural and conceptual knowledge at pre-, post- and follow-up test, by condition.

Hypothesis 2

We used a repeated-measures ANCOVA to test for differences between the conditions with respect to conceptual knowledge. We could find a significant interaction effect of conditiona and time on conceptual knowledge ($F(2, 334) = 3.26$, $p = .039$, $\eta^2 = .019$). Simple effects analyses showed that the two conditions neither differed at pretest nor at posttest. However, at the follow-up-test the effect was significant ($F(1, 167) = 4.02$, $p = .047$, $\eta^2 = .023$). Students in the error-centered condition had a higher conceptual knowledge ($M = 33.91$, $SD = 30.43$) than students in the solution-centered condition

(M = 28.74, SD = 32.07). Further analyses showed that there were no significant interaction effects between conditions and prior conceptual knowledge.

DISCUSSION

In the current study we examined the role of reflections on the rationale behind own errors. In the error-centered condition the students showed a significantly higher performance with respect to procedural knowledge both at posttest and at the follow-up test compared to the students in the solution-centered condition. Regarding conceptual knowledge we could identify a comparable effect only for the follow-up-test. In total we can state that our results support the explanation that instructions on errors are beneficial if they consider the rationale behind one's own errors (Ben-Zeev, 1998). The effect with respect to procedural knowledge is of particular interest because procedural errors were assumed to be very resistant to instructional interventions in some previous research (e.g. Weinert, 1999). The current study might give indications to cope with procedural errors efficiently. An explanation with respect to the retention effect with regard to both knowledge types relies on the idea of more elaborate learning that is triggered by error reflections and that is essential for a recall of knowledge (Siegler & Chen, 2008; Wittrock, 1989). Yet, we must state that to some extend the support is limited to procedural knowledge. For conceptual knowledge the results need to be replicated with more reliable scales. Beyond, the current study indicates that the teachers' fears that reflecting on errors' might confuse students (Heinze & Reiss, 2007) might be not reasonable. Instead, for both mathematics classes and text books our results encourage considering instructions for reflections on the rationale behind own errors.

The current study has some methodological and theoretical limitations that give direction for future research: First, we used parallel knowledge tests. We did not investigate whether reflections on the rationale behind own errors are successful if there are more diverse tasks to-be-learned. Second, we did not assess the quality of error reflections. However, effects of reflections might depend on their appropriateness (Wittrock, 1989). Finally, some students may even have struggled to find the rationale behind their own errors. More specific, there might be errors that are more "treatable" or rather "untreatable" for the students in order to identify the rationale behind the error (see Ferris, 1999).

References

Ben-Zeev, T. (1998). Rational errors and the mathematical mind. *Review of General Psychology, 2,* 366-383.

Duit, R., & Treagust, D. F. (2003). Conceptual change: A powerful framework for improving science teaching and learning. *International Journal of Science Education, 25*, 671-688.

Ferris, D. (1999). The case for grammar correction in L2 writing classes: A response to Truscott (1996). *Journal of Second Language Writing, 8*, 1-11.

Große, C. S., & Renkl, A. (2007). Finding and fixing errors in worked examples: Can this foster learning outcomes? *Learning and Instruction, 17*, 612-634.

Guldimann, T., & Zutavern, M. (1999). “Das passiert uns nicht noch einmal!” - Schülerinnen und Schüler lernen gemeinsam den bewußten Umgang mit Fehlern. In W. Althof & F. Oser (Eds.), *Fehlerwelten. Vom Fehlermachen und Lernen aus Fehlern: Beiträge und Nachträge zu einem Interdisziplinären Symposium aus Anlaß des 60. Geburtstags von Fritz Oser* (pp. 233-258). Opladen: Leske & Budrich.

Heemsoth, T., & Heinze, A. (2013). Learning fractions from errors. In A. M. Lindmeier & A. Heinze (Eds.), *Proc. 37th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 3, pp. 25-32). Kiel, Germany: PME.

Heinze, A., & Reiss, K. (2007). Mistake-handling strategies in the mathematics classroom: Effects of an in-servide teacher training on student's performance in geometry. In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), *Proc. 31st Conf. of the Int. Group for the Psychology of Mathematics Education* (pp. 9-16). Seoul, Korea: PME.

Keith, N., & Frese, M. (2005). Self-regulation in error management training: Emotion control and metacognition as mediators of performance effects. *Journal of Applied Psychology*, *90,* 677-691.

NMAP. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington: US Department of Education.

Padberg, F. (2009). *Didaktik der Bruchrechnung* (4., erw., stark überarb). Heidelberg: Spektrum, Akad. Verl.

Siegler, R. S. (2002). Microgenetic studies of self explanations. In N. Granott & J. Parziale (Eds.), *Microdevelopment. Transition processes in development and learning* (pp. 31-58). New York: Cambridge University Press.

Siegler, R. S., & Chen, Z. (2008). Differentiation and integration: Guiding principles for analyzing cognitive change. *Developmental Science*, *11*, 433-448.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., … Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, *23*, 691-697.

Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction, 14*, 453-467.

VanLehn, K. (1999). Rule-learning events in the acquisition of a complex skill: An evaluation of CASCADE. *The Journal of the Learning Science, 8*, 71-125.

Weinert, F. E. (1999). Aus Fehlern lernen und Fehler vermeiden lernen. In W. Althof & F. Oser (Eds.), *Fehlerwelten. Vom Fehlermachen und Lernen aus Fehlern: Beiträge und Nachträge zu einem Interdisziplinären Symposium aus Anlaß des 60. Geburtstags von Fritz Oser* (pp. 101-110). Opladen: Leske & Budrich.

Wittrock, M. C. (1989). Generative processes of comprehension. *Educational Psychologist, 24*, 345-376.

Yerushalmi, E., & Polingher, C. (2006). Guiding students to learn from mistakes. *Physics Education, 41*, 532-538.