RELATING STUDENT OUTCOMES TO TEACHER DEVELOPMENT OF STUDENT-ADAPTIVE PEDAGOGY

Nicola Hodkowski[1], Ron Tzur[1], Heather Lynn Johnson[1], Evan McClintock[2]

[1]CU Denver, [2]NYU

We examine a 4th grade teacher's development of a constructivist-based, adaptive pedagogy (AP) approach—and its contribution to student multiplicative reasoning and outcomes, mixing qualitative analysis of segments from her interviews with quantitative analysis of her student outcomes on the state-mandated test. Her reflections indicate a shift to this pedagogical approach, which tailors the intended mathematics and classroom activities to students' available conceptions. The data reflect how, via professional development, her new understanding of students' learning to reason multiplicatively promoted learning opportunities for them and thus—their outcomes. We discuss how linking teacher development to student conceptions—adaptive pedagogy—can contribute to improving their outcomes.

INTRODUCTION

In an era of growing emphasis on teachers' accountability for their student outcomes in mathematics, this case study with a 4th grade teacher (Nora, pseudonym) examined possible links between a teacher's development of a constructivist-based, student-adaptive pedagogical (AP) approach and student outcomes. The study was conducted within our team's efforts to promote and study K-5 teachers' development of pedagogical perspectives and practices that revolve around and adapt to students' available conceptions. This paper focuses on how changes detected in Nora's understanding of and capitalizing on student thinking contributed to their improved outcomes on the Transitional Colorado Assessment Program (TCAP)—the state, annually mandated test in mathematics. Specifically, the study addressed the questions: (a) What shifts can be detected in a teacher's pedagogical understandings and practices to incorporate research findings about students' thinking and (b) how might these shifts contribute to student learning and outcomes? Nora chose to focus on teaching multiplicative reasoning because it constitutes a conceptual milestone for her fourth graders. In this domain, the Common Core State Standards (CCSS) (National Governors Association Center for Best Practices, 2010) emphasized students' learning to reason about and solve multiplicative, realistic (word) problems along with using algorithms to calculate 1-digit x 4-digit numbers as well as 2-digit x 2-digit numbers. Linking conceptual and procedural understandings in all children is vital not only for multiplicative reasoning but also as foundations for fractional, proportional and algebraic reasoning (Thompson & Saldanha, 2003).

2014. In Oesterle, S., Liljedahl, P., Nicol, C., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 3, pp. 321-328. Vancouver, Canada: PME.

CONCEPTUAL FRAMEWORK

We consider teaching mathematics to be a goal-directed activity (Ernest, 1989) that involves teachers in promoting students' progress to ever more advanced ideas (Schifter, 1998). Thus, teacher perspectives of mathematical knowing and learning drive the goals for and ways they implement their activities in practice (Thompson, 1992). To account for teacher development, we use a 4-perspective framework (Table 1) that explicates a continuum of stances in teachers' thinking about math knowing, learning, and teaching (Jin & Tzur, 2011; Simon et al., 2000).

Perspectives	View of knowing	View of learning	View of teaching
Traditional (**TP**)	Independent of knower, out there	Learning is passive reception	Transmission; lecturing; *instructor*
Perception-based (**PBP**)	Independent of knower, out there	Learning is discovery via active perception	Teacher as *explainer* ('points out')
Progressive Incorporation (**PIP**)	Dialectically independent and dependent on knower	Learning is active (mental); known required as start; incorporate new into old	Teacher as *guide and engineer* of learning-conducive conditions
Conception-based (**CBP**)	Dynamic; depend on one's prior knowledge (assimilatory schemes)	Active construction of the new as transformation in the known (via reflection)	Engage in problem solving; Orient reflection; *Facilitator*

Table 1: Teacher perspectives on mathematics knowing, learning, and teaching

The AP (Steffe, 1990) is based on the conception-based perspective. It stresses a teacher's selection and use of mathematical goals and activities for student learning that are tailored, in every mathematics lesson, to students' resources—conceptions and experiences they have and bring to a learning situation as part of their funds of knowledge (Moll et al., 1992). The rationale is that learning a new mathematical idea entails transformation in conceptions available to the child (von Glasersfeld, 1995). Thus, a teacher needs to continually infer into their current reasoning—ways of operating with/on units—and set goals for changes in these operations/units that build on, challenge, and foster construction of the intended ones. We note that AP differs from the well-known CGI approach (Carpenter et al., 1989). CGI seems to equate a child's thinking with a task-structure an adult recognizes whereas AP distinguishes between the two (Tzur et al., 2013). To illustrate AP, we describe how a teacher may foster students' conceptual leap from additive to multiplicative reasoning (Behr et al., 1994), stressing that *their* operations on units may be effected but are not determined by tasks a teacher uses.

Reasoning multiplicatively requires using number as a composite unit—a "thing" made up of sub-parts—and coordinating distributing operations among such units (Steffe, 1992). Additive operations preserve such units (e.g., 5 *dots* + 5 *dots* + 5 *dots* =

15 *dots*), while multiplicative operations transform them (Schwartz, 1991) via distribution of items of one composite unit over the items of another composite unit to yield a third, different unit (e.g., 5 *dots/page* x *3 pages* = 15 *dots*). This key, content-specific notion of our conceptual framework is organized in a 6-scheme developmental sequence (Tzur et al., 2013) that, between multiplication and division (both quotitive and partitive), distinguishes three ways of operating on composite units. After establishing multiplicative double counting (*mDC*—the aforementioned operation on composite units), a child may advance to the *Same Unit Coordination* (*SUC*) scheme (finding sums/differences of compilations of composite units), then to a *Unit Differentiation and Selection* (*UDS*) scheme (noting differences/similarities among units of two compilations), and to a *mixed-unit coordination* (*MUC*) scheme (coordinating operations on composite units and 1s). The latter, with UDS as its predecessor, enables, for example, thinking about and meaningfully solving the following problem: "Juanita has 4 bags with 10 marbles each, and a box with 56 marbles. If she places the additional marbles in bags of 10, how many bags and how many marbles will she have in all?" Using the MUC scheme, a child may either reason from four 10s to forty 1s and add them to the 56 to yield 96 marbles, or from the 1s to 10s ("bags" as a composite unit) to divide the 56 and find five 10s and remaining six 1s, hence nine 10s + six 1s = 96 (note how the child "supplies" *her* way of operating; it is not task-determined).

METHODOLOGY

We used a mixed-method approach, with quantitative (student scores on the TCAP test in mathematics) and qualitative (interview segments) providing the data to address the research questions. Participants in this study included one teacher (Nora), the students in her classroom who were not pulled into an accelerated math class (N=13), and student aggregates at her school (over 85% ELL, 100% eligible for reduced/free lunch), district, and state levels. Restrictions on (not) presenting disaggregated student data from other 4th grade classrooms precluded comparing changes in Nora's and other teachers' work (and student outcomes) at her school. Thus, student outcomes are examined through comparison to publicly available data to illustrate a trend in student changes due to a teacher's shift toward AP.

Quantitative data and analysis include aggregated reports about proficiency levels achieved by 4th graders on the TCAP. This standards-based, yearly assessment consisted of 69 items: 54 multiple-choice items (accounting for 54% of a student's total score) and 15 constructed response items (44% of the score). Topics sampled by the test items included place-value (base-ten) system, multiples and factors, multiplication and division of 1- or 2-digit whole numbers, interpretation of data presented on a graph, and estimation of costs/change for purchased items (i.e., decimals in money). Our analysis juxtaposes proficiency levels attained by students (March, 2012 and 2013) in the four different groups (Nora, school, district, state) after controlling for comparable populations (ELL, lunch eligibility).

Qualitative data and analysis focus on Nora's rationale for teaching activities used in video-recorded lessons she co-planned and co-taught weekly with [Tzur]. These inquiries were part of reflective, post-lesson sessions. Each session started by asking Nora to explain specific aspects of the lesson, including the *mathematics as she understood it*, reasons for actions she took and changes from plans she made, and what she took as evidence for each student's understanding (sense making) of the intended mathematics. [Tzur] then added his analysis about student learning and understanding, particularly distinguishing units/operations *each individual student* seemed to use. A session then culminated with co-planning the next lesson, linking where different students seemed to be conceptually with curricular goals set for their next learning. Video segments that illustrate shifts in Nora's thinking were selected for the analysis (presented below).

RESULTS

This section first reports on the comparison of aggregated, quantitative data among groups of participating students. This comparison points to the significance of Nora's shift toward AP. We note here that a key reason Nora gave for choosing to focus on multiplicative reasoning was her discontent with student learning and outcomes when, as often happens, teaching-learning processes consist mainly (or solely) of executing algorithms while using heavily-practiced, memorized facts. She noticed that her students might be (partially) successful in solving problems highly similar to those solved in class; but they failed to transfer multiplicative thinking to situations that deviated, even if only slightly, from those they solved previously. This indicated to Nora a lack of fundamental understandings needed to solve such problems by mindfully choosing/executing proper calculations. Student outcomes on the mathematics portion of the TCAP seem to support this focus.

Student Outcomes

Figure 1 presents percentages of students who scored at the combined level of Proficient or Advanced (Pr+Ad), and how these outcomes changed from 2012 (before Nora fully implemented AP) and 2013 (post). At this desired proficiency level, her students improved from 58% to 85% (growth of 46%), as compared to school's increase from 46% to 60% (growth of 30%, figures include Nora's class due to aggregation), district's increase from 56% to 58% (4% growth), and no detectable change in state's averages (72% in both years).

These data indicate three important trends. <u>First</u>, a teacher versed in AP can bring the majority of her class (85%) to the Pr+Ad level. Specifically, Nora promoted three students' shift from PP to Pr and one from Pr to Ad. <u>Second</u>, Nora's students exceeded their comparable counterparts in terms of percentages scoring at Pr+Ad and growth from previous year. (Note: all higher-achieving students, pulled from her class and not included, attained at proficient or advanced levels.) <u>Third</u>, these data indicate closing the achievement gap between students from the typically underachieving sub-groups and their white counterparts. Combined, these results suggest changes in Nora's teaching (examined next) as a possible contributor.

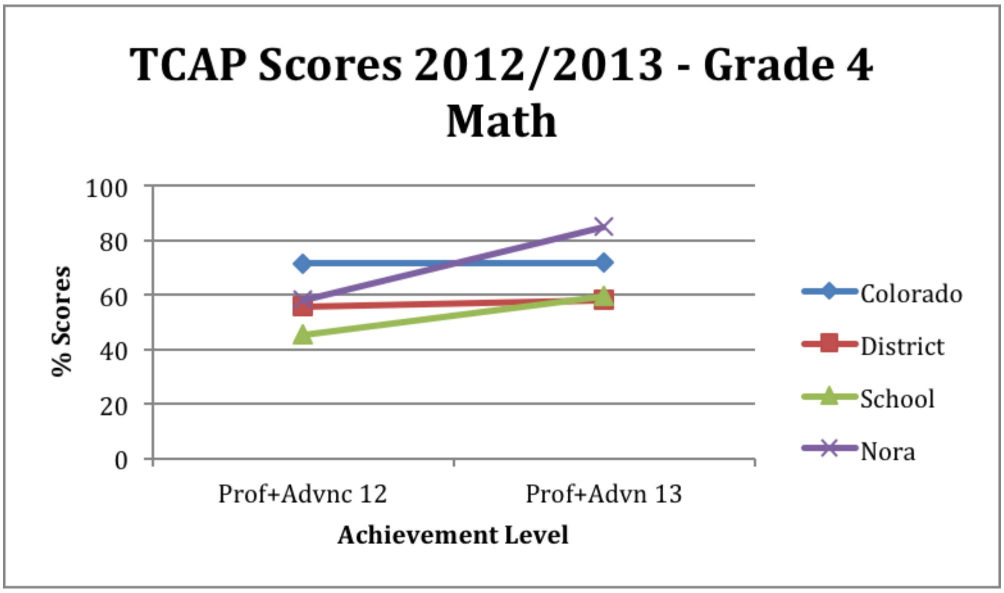

Figure 1: Proficient/Advanced Comparison 2012 to 2013

Shifts in Nora's Teaching

Shifts in Nora's thinking about how her teaching should link to students' learning are illustrated in three excerpts, two from fall 2011 and the third from spring 2012.

<u>Excerpt 1: Early Fall 2011</u>

Tzur: (Probes about how she used to teach multiplication.)

Nora: Previously we have had an introduction to multiplication. Which is hard for them because they memorize the facts but they have no idea why they do it. And it's actually a struggle for a group of kids, because they know they are supposed to have memorized it, and they have no idea why. [A bit later, asked for an example.] We actually did study arrays in the first unit of Investigations and that was so hard for them. I had to bring in Cheez-its; I gave each [student] a bag of Cheez-its. So that they could build the factors necessary to get to their number [arrays] and that was [still] very difficult for them.

<u>Excerpt 2: Fall 2011 (two weeks after the lesson discussed in Excerpt 1)</u>

Tzur: (Asks about her assertion on differentiated attainment of the intended math.)

Nora: I think that there are some—that some students that are—I think it is about half and half. Half of the class, maybe a little more than half, are doing [operating on] 1s; the other half is counting in [composite] units.

<u>Excerpt 3: Spring 2012</u>

Tzur: (Asks about tasks *she planned* for students ready for UDS-to-MUC shift.)

Nora: I’ll create a worksheet [of *realistic* word *problem*s] that has to do with the Mixed Unit Coordination in multiplicative reasoning. I will start with 5s and 10s and then I am going to move to 4s and other numbers and then maybe I will do another [task] with 6, [or] 7, [or] 8; and then we can move on.

The three Excerpts indicate a shift in Nora’s focus on and use of students’ thinking. In Excerpt 1 she recognized some students might not have mastered multiplication facts and that everyone, those who did and those who did not, seemed to have no meaning for what is being memorization. She mentioned the use of a real-life manipulative (Cheez-its), which she decided to add when sensing the difficulties her students faced in learning about multiplication as a rectangular array. This is a typical teaching move informed by a Perception-Based Perspective—trying to help students “see” the mathematics she could see. Yet, during the entire post-lesson session (Excerpt 1 included), she did not explain nor link that manipulative to particular ways in which different students were operating to solve the problems.

In Excerpt 2 (two weeks later), she began distinguishing two sub-groups in her class in terms of different units on which they operated when solving problems. We note that later at the interview she also differentiated nature of these units: tangible, figural, or abstract (i.e., numbers). This distinction then played a role in her planning. She purposely designed activities to advance those students who were counting 1s to counting composite units as a necessary conceptual change in their operation via the unit-transforming distribution of items.

In the three months between Excerpt 2 and 3, Nora focused on inferring students’ thinking by proactively using the 6-scheme framework and on using these inferences to guide her practice. Excerpt 3, shows a shift in her awareness of the role that those schemes could play in her teaching. When introducing a new, challenging concept such as MUC, the teacher needs to carefully select composite units (numbers) for the tasks she designs, so students could bring forth their available schemes (mDC, SUC, and UDS—as she mentioned earlier in the interview) and solve the problem while having an opportunity to transform those to the intended, MUC scheme. That is, Nora seemed to develop conscious attention to the link between her analysis of students’ thinking and tasks that may be useful for the next lesson. Her comments suggest purposeful sequencing of tasks, and numbers used, as a means to moving forward while supporting students’ reasoning.

DISCUSSION

This paper focused on ways in which professional development of teachers of mathematics and their student outcomes may be linked. Particularly, it showed how a shift in a teacher’s perspective changed her practice and student outcomes that seemed to follow from this change. The results of the study suggest two major contributions to the field. First, Nora’s case points to the importance of teacher learning to (a) distinguish students’ ways of thinking and (b) base her teaching practices on research-based learning trajectories, such as the 6-scheme framework (Tzur et al., 2013). The notion of AP used to guide Nora's development entails the need to help a teacher clearly differentiate her own mathematics from the students' ever-changing mathematical schemes, which is consistent with Steffe's (1990) distinction between 1st and 2nd order models, respectively. Nora's case indicates a teacher can surmount the challenge involved in making such a distinction and purposely use accounts of cognitive change to inform her instruction daily. It also raises issues for future studies, including how the intensive nature of co-teaching that promoted Nora's development may be implemented on a larger scale. This study provides a first glimpse into how mathematics educators and classroom teachers can work collaboratively to bring about this desired shift.

Second, the comparison of quantitative data of proficiency levels attained by Nora's students and their counterparts (school, district, state) indicates the potential benefits of a shift toward a student-adaptive pedagogy. Mathematics education literature, particularly since reform pedagogies and materials were introduced (Senk & Thompson, 2003), showed that student outcomes when learning in reform classrooms were not compromised. A typical claim would be that students in those classrooms did not do worse than their counterparts in more traditional classrooms. However, this study, along with data about other classes in Nora's school (and other districts) that we continue collecting and analyzing provide further evidence that a shift to adaptive teaching may promote bona fide improvement in student outcomes. We contend that the improvement in student outcomes presented in this paper were made possible by the teacher's learning to use their ways of thinking as a driving force in creating (and adjusting) lessons conducive to their learning. Simply put, *opportunities* to learn are afforded (or constrained) by what students know. Clearly, further research is needed to more specifically link between the teacher's development of conceptual/practical tools implied by AP and growth in students' learning, reasoning, problem solving, and tested outcomes.

References

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, *26*(4), 499-531.

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching*, *15*, 13-33.

Jin, X., & Tzur, R. (2011). *Progressive incorporation of new into known: A perspective on and practice of mathematics learning and teaching in China*. Paper presented at the Annual Conference of the Association of Mathematics Teacher Educators, Irvine, CA.

Moll, L. C., Amanti, C., Neff, D., & Gonzalez, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Qualitative Issues in Educational Research*, *31*(2), 132-141.

National Governors Association Center for Best Practices. (2010). *Common core state standards initiative.* Washington, DC: NGA & CCSSO.

Schifter, D. (1998). Learning mathematics for teaching: From a teachers' seminar to the classroom. *Journal of Mathematics Teacher Education*, *1*(1), 55-87.

Schwartz, J. L. (1991). Intensive quantity and referent transforming arithmetic operations. In J. Hiebert & M. J. Behr (Eds.), *Number concepts and operations in the middle grades* (3rd ed., Vol. 2, pp. 41-52). Reston, VA: National Council of Teachers of Mathematics.

Senk, S. L., & Thompson, D. R. (2003). *Standards-Based School Mathematics Curricula: What Are They? What Do Students Learn?* Mahwah, NJ: Lawrence Erlbaum.

Simon, M. A., Tzur, R., Heinz, K., Kinzel, M., & Smith, M. S. (2000). Characterizing a perspective underlying the practice of mathematics teachers in transition. *Journal for Research in Mathematics Education*, *31*(5), 579-601.

Steffe, L. P. (1990). Adaptive mathematics teaching. In T. J. Cooney & C. R. Hirsch (Eds.), *Teaching and learning mathematics in the 1990s* (pp. 41-51). Reston, VA: National Council of Teachers of Mathematics.

Steffe, L. P. (1992). Schemes of action and operation involving composite units. *Learning and Individual Differences*, *4*(3), 259-309. doi: 10.1016/1041-6080(92)90005-Y

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.

Thompson, P. W., & Saldanha, L. (2003). Fractions and multiplicative reasoning. In J. Kilpatrick & G. Martin (Eds.), *Research companion to the NCTM Standards* (pp. 95-113). Washington, DC: National Council of Teachers of Mathematics.

Tzur, R., Johnson, H. L., McClintock, E., Kenney, R. H., Xin, Y. P., Si, L., . . . Jin, X. (2013). Distinguishing schemes and tasks in children's development of multiplicative reasoning. *PNA*, *7*(3), 85-101.

von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning*. Washington, DC: Falmer.