WHAT DID THE STUDENTS LEARN FROM MATHEMATICS TEXTBOOKS? THE CASE OF L. F. MAGNITSKII'S *ARITHMETIC*

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In this paper, we investigate the Arithmetic authored by L.F. Magnitskii (1669-1739) especially focusing on the methods of teaching and learning represented in this arithmetical manual that remained highly influential among the Russian educators for more than a century after its publication in 1703. We suggest that Magnitskii, even though drawing upon arithmetical manuals published in Western Europe in the 17th century, introduced a number of new elements that can be properly interpreted only if one takes into consideration his didactical agenda.

HISTORICAL AND SOCIOCULTURAL PERSPECTIVE

When conducting research on the history of mathematics education, the historians usually work with the extant documents they have at their disposal, especially the mathematical textbooks or other written materials used for instruction. However, as Michael Polanyi (1891 – 1976) argued in his seminal work of 1958, in the process of transmission of scientific knowledge its considerable part is not verbalized; a substantial part of knowledge is transmitted via direct interaction between the individuals involved. If we adopt his hypothesis, the materials found in modern mathematics textbooks cannot suffice to reconstruct the actual interaction between teachers and learners who use these textbooks nowadays, and additional methods (e.g., classroom observation, interviews, etc) are needed to discern and analyse the processes of learning. However, when dealing with the history of mathematics education, the classroom observations and interviews, for obvious reasons, are impossible to conduct, and alternative research methodologies have to be designed to reconstruct, at least partly, the processes of instruction on the basis of the extant written materials.

This paper is focused on the tradition of mathematics education in Russia, in particular, on the first printed school mathematics manual, the *Arithmetic, or Science of Numbers* (Арифметика, сиречь наука числительная) published by L.F. Magnitskii's (Л. Ф. Магницкий, 1669-1739) in 1703. The conventional descriptions of mathematics education in Russia have always been based upon the extant textbooks of which the first ones were compiled during the period antedating the publication of Magnitskii's manual (see, for example, Yushkevich 1968), yet no historiography of Russian mathematics education that would take into consideration the role of tacit knowledge in educational practices, to the best of our knowledge, has ever been published.

It appears plausible to distinguish two types of "tacit knowledge". The first type is directly related to the subject matter of mathematics instruction, in other words, it

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comprises conceptions (and sometimes misconceptions) concerning numbers, figures, and other mathematical objects, as well as operations with them. The second type is related to didactical aspects of instruction, in particular, to the style of interaction between the teachers and learners. This type is determined by a more general framework which, in turn, is related to the traditions of teaching and learning specific for the respective social group and for the embracing cultural tradition. In our case, we are dealing with the traditions of teaching and learning which existed in Russia some time before the publication of Magnitskii's textbook and which, arguably, continued to exist after it. These two types of tacit knowledge can be identified as ideas, concepts, and representations concerning, on the one hand, the contents of the respective discipline (in our case, elementary mathematics), and on the other, the processes of its transmission which were not verbalized or at least were not described explicitly in the extant materials.

We therefore assume that the mathematical knowledge represented in the *Arithmetic*, even though based upon Western textbooks originating from a different educational tradition, was adjusted by Magnitskii to fit into the classroom activities different from those taking place in the Western classroom. In other words, we can interpret the modifications of Western teaching materials (mathematical problems, definitions of mathematical objects) made by Magnitskii in his textbook as resulting from requirements (tacitly) imposed by the Russian didactical tradition that differed from those of Western Europe. The sources of information used for our reconstruction of the Russian didactical practices are: (1) the elements found in Magnitskii's *Arithmetic* and other Russian textbooks which distinguished them from their (hypothetical) Western prototypes and which cannot be explained as caused by purely mathematical or linguistic reasons, and (2) the practices adopted in Russian/Soviet schooling tradition in the 19th and 20th centuries that have been, at least partly, documented.

Paradoxically, we will begin our study of the Magnitskii's textbook with a discussion of the case of "Asian/Confucian learners". Recently a considerable number of publications were devoted to the phenomenon of Chinese mathematics education; the difference with the learners from other countries (in particular, from the USA) was perceived, but not always clearly stated or identified. The path-breaking monograph of Ma Liping (1999) was followed by a large number of studies of Chinese mathematics classroom, and these studies, including the book of Ma, contain an amount of data concerning the second kind of abovementioned tacit knowledge, such as detailed descriptions of Chinese methods of learning, teaching etc. From North-American perspective some of the Chinese didactical approaches may be seen as somewhat exotic and/or inapplicable in American/Western classroom (for example, memorization of multiplication table), yet for the Chinese educators and students such practices, especially memorization, look highly relevant.

The phenomenon of "Chinese mathematics education" became famous mainly due to the success of Chinese students in various kinds of competition and comparative studies, but in reality the transfer of Western mathematical knowledge and, to some

extent, Western teaching practices to China happened relatively late, in the late 19th century, even though some attempts were made in the 17th and early 18th centuries, but without particular success. Technically, "Westernized" Russian mathematical tradition that started some time prior to the publication of Magnitskii's textbook had a much longer history and was much more developed, but due to the fall of the USSR and economical success of China, the attention of the researchers turned to China, even though a number of recent publications were devoted to the phenomenon of Russian and Soviet mathematics education (see, for instance, Karp and Vogeli 2010; 2011).

A number of attempts have been made to explain the success of Chinese mathematics learners; for example, a number of authors suggested that it resulted from a particular "Confucian" cultural tradition of teaching and learning, while some other authors expressed their doubts concerning this thesis (see, for example, Leung 2001; Fan et al. 2004); see also the analysis of the philosophical foundations of Chinese and American systems of mathematics education by Xie and Carspecken (2008) and a comparison of European and Chinese "cognitive styles" and their impact on teaching mathematics (Spagnolo and Di Paola 2010). Conversely to the case of Chinese mathematics education, the case of USSR/Russia remains largely underexplored. The innovations introduced by Russian educators were not duly documented, and the remaining documents often do not provide information necessary for reconstruction of educational activities. The study of Magnitskii's textbook was not an exception: a number of historians of mathematics and mathematics education, when dealing with the *Arithmetic*, did not pay enough attention to the didactical techniques found in this book.

DIDACTICAL PERSPECTIVE

We will open this section with a short presentation of the studies devoted to the *Arithmetic*. To identify the "tacit" didactical elements in Magnitskii's textbook, we will compare its contents with those of its hypothetical Western prototypes in assuming that the found differences resulted from the didactical agenda of Magnitskii.

According to A. Vucinich, "In the seventeenth century – the century of logarithms, analytical geometry, and calculus – Russia's mathematical knowledge did not exceed the most elementary principles of arithmetic contained in the translations of Western European (mainly German) texts written during the fifteenth and sixteenth centuries". (Vucinich 1963, p. 33) Therefore, according to the latter author, "the *Arithmetic* was important not only in bringing up-to-date elementary mathematical knowledge to Russia but also in showing the wide range of practical problems – particularly of a military and commercial nature – that could be solved mathematically" (ibid., p. 54). Moreover, based on Peter the Great's dedication to the program of strengthening the nation that required from the emerging new Russian ruling class excellent command of several foreign languages, knowledge of rhetoric as well as of the arts of philosophy, medicine and theology, Magnitskii claimed in the preface of his book that "not only is arithmetic essential to education in the liberal arts, but the practical skills of measuring

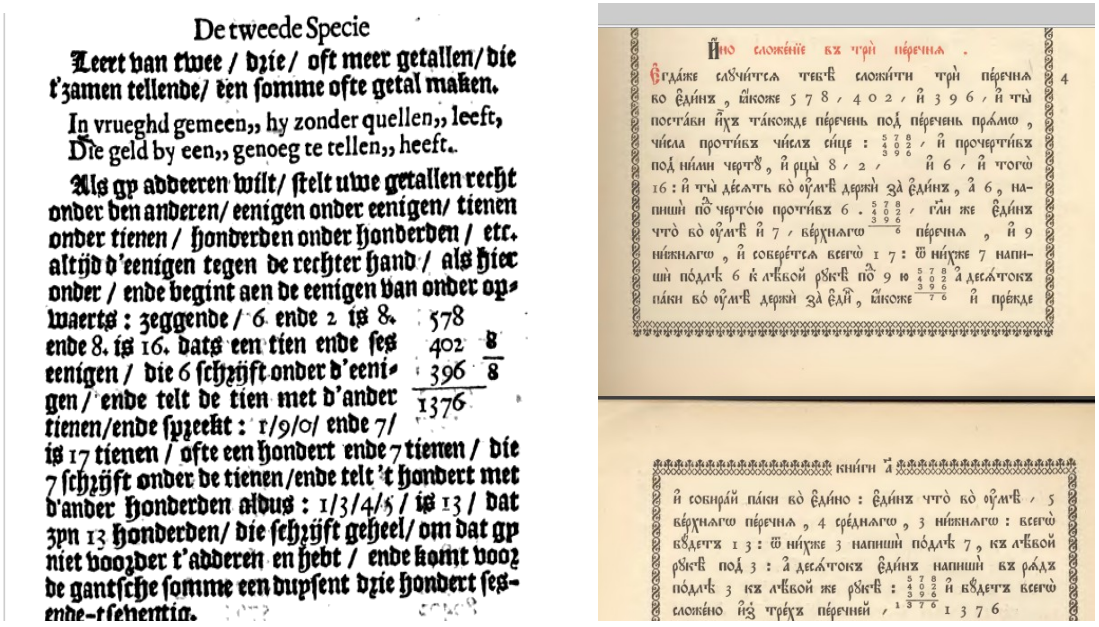
and counting were needed by a dynamic society as well" (Okenfuss 1995, p. 75). When mentioning that Magnitskii wrote his book as a "humanist, concerned above all with the place of mathematics in the mind of an educated man," Okenfuss argues that his work was a "culmination of the impact of the foreign on seventeenth-century Muscovy" (ibid.), thus defining the direction of development of mathematics in Russia for the next half of the century.

The recognition of the didactical value of Magnitskii's work came only a century later when it became one of the central topics in the framework of historical reconstruction of the growth of mathematical knowledge in Russia in the beginning of the 18th century of which Magnitskii's *Arithmetic* was considered an important milestone (Vulcinich, 1963). The significance of the *Arithmetic* for the formation and evolution of mathematical education in Russia was especially emphasized in the 19th century by the historian and educator V. Bobynin (В. В. Бобынин, 1849-1919), who considered it a link between the Russian mathematical texts of the 17th and the 18th centuries, while also serving as an introduction to novel mathematical subjects (e.g., progressions, algebra, etc.) not included in manuscript textbooks that circulated in Russia prior to its publication or were only rarely mentioned in some Russian mathematical manuscripts (such as, for instance, the extraction of roots). Bobynin (1889) claimed that in the Russian mathematical literature it would be hard to find another work of the same historical significance as the *Arithmetic* by Magnitskii. At the same time, he also raised the question of the originality of the book, since Magnitskii himself defined the book as a compilation based upon several Western sources. In the same vein, A. Vucinich (1963) argued that the *Arithmetic* was not a summary of the mathematical knowledge that existed in Russia but rather an encyclopaedia of various relevant items mostly translated from Western sources; still it was not completely unoriginal, and its author showed much "ingenuity in the organization of material, explanatory notes, and selection of examples" (p. 54).

The question of originality of Magnitskii's book remains one of the most frequently discussed by later authors. For example, Ivasheva (2011) mentions that while Magnitskii borrowed much of contents and terminology from the mathematical manuscripts that circulated in Russia prior to the early 18th century, he paid a great deal of attention to general discussions about mathematics in which arithmetic was described as a "honest art, envy-free, readily grasped by all, wholly useful" (p. 39). In turn, Mishchenko (2004) mentions that recent researchers still have no general opinion concerning the sources that Magnitskii used as the basis of his *Arithmetic*. The latter author refers to the analysis of Yushkevich (1968) who believed that Magnitskii used manuscript and printed materials of earlier times, which he carefully selected and substantially modified to compose an original work, taking into account the knowledge and demands of the prospective Russian readers.

In order to provide more insights into the essence of the debates about the didactical value of Magnitskii's work, we briefly summarize a discussion between D. Galanin (1857-1929) (1914) and V. Bobynin (1889) regarding the introduction of addition of

integer numbers in the *Arithmetic* as compared with the same topic in the *Arithmetica oft reken-konst: En een kort onderricht van't italiaens boeckhoudẽ* published in Amsterdam in Dutch by Jacob Van Der Schuere (Schuere, 1643); the latter textbook shared a number of striking similarities with Magnitskii's textbook, briefly discussed in our publication (Freiman and Volkov 2012). While Bobynin called these similarities "borrowing" ('zaimstvovanie'), Galanin referred to them as "inspection/getting familiar with" ('oznakomlenie'), that is, he suggested that Magnitskii knew the textbook of Schuere but introduced the elements of its contents differently, in pursuing his own didactical goals, which brought originality to his work. Schuere's and Magnitskii's introductions of addition are shown in Figure 1:



De tweede Specie

Leert van twee / drie / oft meer getallen / die t'zamen tellende / den somme ofte getal maken.

In vrueghd gemeen,, hy zonder quellen,, leeft,
Die geld by een,, genoeg te tellen,, heeft.

Als gy addeeren wilt / stelt uwe getallen recht onder den anderen / eenigen onder eenigen / tienen onder tienen / honderden onder honderden / etc. altijd d'eenigen tegen de rechter hand / als hier onder / ende begint aen de eenigen van onder opwaerts : zeggende / 6. ende 2 is 8. 578
ende 8. is 16. dats een tien ende ses 402 8
eenigen / die 6 schrijft onder d'eenigen / ende telt de tien met d'ander 396 8
tienen / ende spreekt : 1/9/0/ ende 7/ 1376
is 17 tienen / ofte een hondert ende 7 tienen / die 7 schrijft onder de tienen / ende telt 't hondert met d'ander honderden aldus : 1/3/4/5 / is 13 / dat zyn 13 honderden / die schrijft geheel / om dat gy niet voorder t'adderen en hebt / ende komt voor de gantsche somme een duysent drie hondert seseende-tseventig.

Figure 1: Explanations of addition in Schuere (left) and Magnitskii (right).

In his textbook, after mentioning that two, three or more numbers taken together produce a sum, Schuere provides an example of adding 578, 402, and 396 by placing them one under another, aligning the numbers by the position of units and separating them from the sum with a horizontal line. The same example (using the same numbers) can be found in Magnitskii's book, yet explanations of each step are much more detailed. Moreover, before giving this example, Magnitskii introduces another, simpler one, with only two numbers to be added (532 + 46) which he uses to introduce the steps needed to perform addition. This example is missing in Schuere's book. Magnitskii completes his explanation of the procedure with yet two more examples placed under the sub-title "Common rule" (missing in Schuere), and also extends the introductory part of the section, very short in Schuere, beyond the definition of what addition is ("collection or combination of several numbers") by providing a table of basic facts about numbers: in each pair of columns, we see numbers 1-9 on the left, other numbers, from 1 to 10 shown in the middle part can be added to them, and the results are shown on the right side, e.g., "7 + 6 = 13"; see Figure 2:

Figure 2: Magnitskii's 'innovations' – addition table (left) and common rule (right).

In both textbooks of Schuere and Magnitskii, the explanation of addition is followed by several examples that look like exercises or "drills" for the learners. There is an obvious similarity between the two sets of drills:

Figure 3: Exercises from Schuere (left) and Magnitskii (right).

The comparison of the contents of the two books made Bobynin claim that Magnitskii simply translated Schuere's book, while for Galanin, Magnitskii work was a way of enriching sources known to him with original didactical ideas that we still need to grasp. The size limits of this paper does not allow us to provide a deeper analysis of the examples; meanwhile, it is important to stress that Magnitskii's book contains more examples than that of Schuere, and they are of different kind. It is also interesting that both authors introduced in their texts several word problems that prompted application of addition, yet, according to Galanin, problems in Magnitskii's textbook are simpler than those in Schuere's book. Curiously enough, when Bobynin sees a larger number of examples and more detailed explanations, he considers them Magnitskii's didactical weakness, while Galanin in similar cases emphasizes originality and usefulness of this method for the learner.

CONCLUSIONS: SETTING UP A RESEARCH AGENDA

Magnitskii's arithmetical manual was often mentioned in works on the history of mathematics in Russia; however, no special attention, with very few exceptions, was paid to its analysis in didactical perspective. Meanwhile, the work in this direction cannot be accomplished without a detailed exploration of the didactical tradition in Russian mathematics that existed prior to the publication of Magnitskii's manual; moreover, the circumstances of mathematical training obtained by Magnitskii, while being crucial for the present study, remain unknown. However, even the cursory study briefly reported in the present paper strongly suggests that the modifications of

mathematical methods and concepts most likely borrowed by Magnitskii from a number of Western textbooks of the 17th century resulted from the latter's attempts to make those methods and concepts fit into the didactical framework of the early 18th century Russia.

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