REVEALING STUDENTS' CREATIVE MATHEMATICAL ABILITIES THROUGH MODEL-ELICITING ACTIVITIES OF "REAL-LIFE" SITUATIONS

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The study described herein is part of a larger, inclusive research study exploring the effects of model-eliciting activities (MEAs) of "real-life" situations on the development of students' mathematical creativity. This part aim at revealing students' cognitive abilities that are involved in the creative modeling processes using a qualitative analytical method. The participants were mathematically talented students, members of the "Kidumatica" math club. The data include videotapes, classroom observation and modeling products. Three core categories—appropriateness, 'mathematical resourcefulness' and inventiveness—of students' cognitive creative abilities are identified, defined and illustrated. These findings may give a better understanding of the larger concept of mathematical creativity.

INTRODUCTION

The knowledge revolution and the impressive technological innovations that characterize today's world require the facilitation and development of "the innovators of tomorrow who can lead the way forward" (National Science Board, 2010, p. 7). In line with this, educators and researchers are still investigating how the educational system can identify, promote and develop students' innovative and creative potential (Sriraman, 2009; National Science Board, 2010; OECD, 2013).

According to the new report of PISA's mathematics framework (OECD, 2013), formulating "real-life" situations mathematically is a fundamental ability that invokes creativity since "outside the mathematics classroom, a challenge or situation that arises is usually not accompanied by a set of rules and prescriptions…Rather it typically requires some creative thought in seeing the possibilities…" (p. 31).

Model-eliciting activities (MEAs) involving "real-life" situations outside the classroom not only provide students with the opportunity to apply their creative skills, but also encourage the development and improvement of those skills (Lesh & Doerr, 2003; Lesh & Caylor, 2007; Amit & Gilat, 2013). This development of creativity goes "hand-in-hand" (National Science Board, 2010, p. 20) with its identification, which predefined the goal of the present study to identify and reveal the cognitive abilities applied and activated by students when modeling creative processes for "real-life" situations.

CREATIVITY AND MATHEMATICAL MODELING

The following review is organized around the creative process, abilities and production or product (Guilford, 1950, 1967; Sternberg & Lubart, 1999; Sriraman, 2009).

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Guilford (1967) described the creative process as a sequence of thoughts and actions resulting in a novel production, and defined creativity as divergent thinking with its four mental abilities: fluency, flexibility, originality, and elaboration. According to Kruteskii (1976), mathematical creativity appears as flexible mathematical thinking which involves “switching from one mental operation to another qualitatively different one” (p. 282), and depends on openness to free thinking and exploration of diverse approaches to a problem. Sriraman (2009) revealed the common characteristics of mathematical creativity through the Gestalt model of the creative process, defining mathematical creativity as the ability to produce a novel or original solution to a non-routine problem. Sternberg and Lubart's (1999) widely accepted definition asserts that creativity is "the ability to produce work that is both novel and appropriate" (p. 3).

Mathematical MEAs provide the student with opportunities to deal with non-routine "real-life" challenges. These activities are designed according to six principles: reality, model construction, self-evaluation, documentation, sharability and reusability, and an effective prototype (Lesh & Caylor, 2007). This thoughtful design not only engages students in multiple cycles of modeling development in which they are given the opportunity to construct powerful and creative mathematical ideas relating to complex and structured data (Lesh & Caylor, 2007; Gilat & Amit, 2012; Amit & Gilat, 2013). It also allows following students’ thinking and pattern of reasoning and requires students to represent a general way of thinking instead of a specific solution for a specific context. Therefore, the current study was designed to identify and conceptualize students’ cognitive abilities that are involved in, promote and contribute to the development of the creative modeling process and its significant outcomes.

METHODOLOGY

This study made use of deep qualitative analyses based on an intervention program of model-eliciting activities (MEAs) to answer the above-defined questions. This study is part of more inclusive research aimed at developing creativity through MEAs of "real-life" situations. The study was conducted with 71 "high-ability" and mathematically gifted students in 5^{th} to 7^{th} grades who are members of the "Kidumatica" math club (Amit, 2012), for an entire academic year, applied in weekly 75-minute meetings. The intervention program included four workshops based on different MEAs reflecting “real-life” situations, which were worked on by small groups of 3–4 students. Each MEA workshop had three parts: a warm-up activity, a modeling activity and a poster-presentation session. The modeling task asked students to solve a mathematically complex “real-life” problem for a hypothetical client.

Data Sources

Data were derived from: (1) the students' products, i.e. written documents such as mathematical models, poster presentations, letters to the hypothetical client and drafts, (2) video-recordings of the modeling sessions and of students' oral presentations, interviews (performed while students were working on their models in groups and during their model presentation), and (3) classroom observation by the researchers and a trained tutor.

Analytical Methods

Analyses were based on: (1) 'key concepts' (Mostyn, 1985) serving as conceptual ideas for interpreting and coding the data; (2) identification of 'critical events' based on Powell, Francisco, and Maher's (2003) analytical model for analyzing massive videotaped data, and (3) the Way of Thinking Sheets (WTS) (Lesh & Clarke, 2000; Chamberlin, 2004) instrument for organizing and documenting students' massive MEA products.

Phases of Data Analysis

Data analysis was comprised of an exploratory phase (see Figure 1), and three phases that were repeatedly applied to analyse the data and generate the categories:

1. The exploratory phase (research) provides a better understanding of the phenomenon (Gilat & Amit, 2012) and contributes (most) to the refinement and distillation of current theoretical research frameworks and to the determination of preliminary categories (Hsieh & Shannon, 2005).
2. The data-reduction phase involves inclusive data processing of massive video data collected during the course of the MEAs using Powell et al.'s (2003) analytical model of 'critical-event' identification. These identified critical events were transcribed and mapped for further analysis.
3. The data-organization phase allows for a better understanding of the students' work; each group's modeling products were gathered and documented using WTS (Chamberlin, 2004) and mathematically interpreted as shown in Figure 4 further on.
4. The integrated formal phase mainly concerns final assignment of categories to the data obtained from the previous analytical phases utilizing 'key concept' (Mostyn, 1985) as the coding rule for assigning categories to the data.

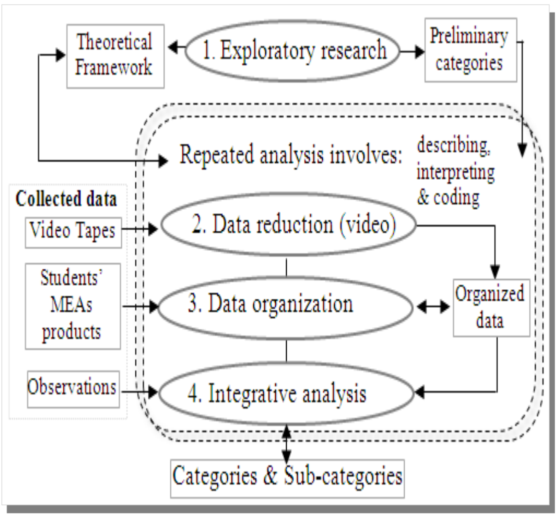

Figure 1: Qualitative analysis flowchart

Throughout the analytical phases (see Figure 1), data are repeatedly described, interpreted and coded for subsequent analysis; each phase strengthens the former phase's interpretations and coding, until a coherent interpretation is obtained. Initial categories were refined and revised until all three main categories and subcategories were generated and defined (based on the theory and the empirical data) and all data were interpreted and coded accordingly. Finally, the categories were ordered hierarchically (see Figure 2) and the relationships between categories and subcategories were identified and conceptualized (Hsieh & Shannon, 2005).

This multi-method triangulation (data-collection methods, analytical methods and analytical phases) provides a richer understanding by uncovering the deeper meaning of the students' cognitive abilities (Lesh & Caylor, 2007), as well as providing us with better validity of data interpretation, enhancing the rigor of the research (Patton, 2002).

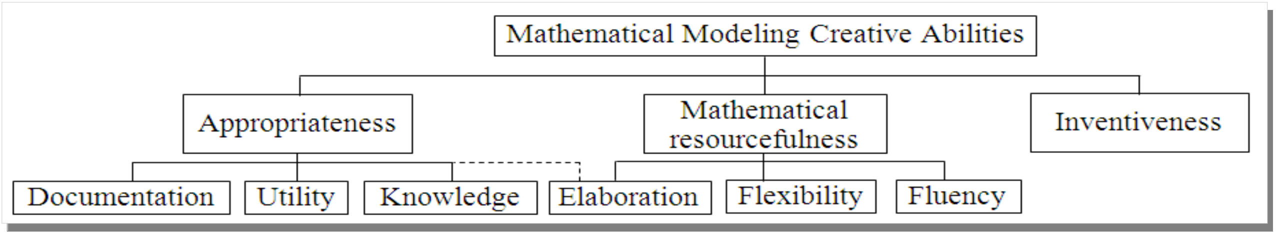

Figure 2: Categorization of students' creative mathematical modeling abilities

Students' Creative Abilities: Categories and Results

The following provide explicit definitions, examples and coding rules utilizing 'key concepts' as conceptual ideas (see Tables 1–3) for each established category and its subcategories. These categories encapsulate the abilities that contributed to, and constituted the creative modeling process and its significant outcomes.

Examples illustrating the meaning of the categorization are given using research data from one group of 6th-grade students' MEA which was considered as showing the best understanding. This MEA was based on the "Bigfoot" modeling task of a "real-life" situation (Lesh & Doerr, 2003) which required students to develop a conceptual tool that would enable estimating an individual's height. Students received a cardboard with an image of an authentic large footprint's stride (Figure 3) and a measuring tape.

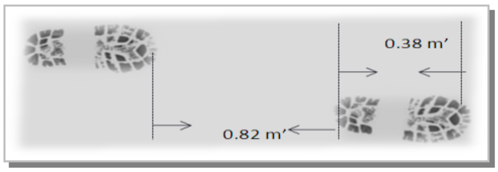

Figure 3: Image of an authentic large footprint's stride

The following is a transcript of the poster presentation given by students A' and S'; Figure 4 shows the students' MEA documentation using WTS (Chamberlin, 2004) and the researcher's (R') mathematical interpretation of their work.

1: A': At the beginning we tried measuring only the length of each of our shoes, and then our height, but we couldn't find any operation that led us to our height.

2: S': We measured the perimeter of our shoes but none of the operations we used led us to a reasonable height.

3: A': Then we measured the width of our shoes.

4: S': We tried width plus length multiplied by a whole number, for instance 5; for me it was right but for him [A'] it wasn't. It was more than his height.

5: A': Then we noticed that my shoe is relatively wider and S's shoe is narrow in comparison to its length.

6: S': So we decided that if the shoe in its narrowest part [pointing to his drawing] is less than 10 cm, we multiply it by 5. Otherwise we multiply by 4.

7: A': We tried it [their formula] on Y' [member of another group] too.

8: R': I can see that you wrote A/S and erased the explanation you wrote in words.

9: S': We didn't have time to complete our solution and find ways to describe the exact ratio so we compared the shoe's width to 10 cm and multiplied it by a fixed number, 4 or 5.

10: A': We wanted to use the proportion between length and width and to find a formula but we didn't have enough time for that so we just wrote A/S.

WTS - Ways of Thinking Sheets

Description of Strategies	Stuents's mathematics and its formal interpretation
Students went through several development phases, at their final phase they utilized both dimensions (shoes' length and width) to calculate the ratio. Strategy: Ratio (depends on different shoe width) multiply by (width÷length) Note: Model is easy to implement, partially accurate, some model testing, revision, or feedback incorporated.	Students took measurements of their shoes' length and width and of their height. Students summed shoe length and width and calculated the ratio between this sum and their height'; Researcher's interpretation of students development phases First phase: $H = R * X$ Second phase: $H = \begin{cases} R_1 * X \text{ if Shoe's width} < 10 \\ R_2 * X \text{ if Shoe's width} > 10 \end{cases}$ Third phase: $H = X * R\left(f\left(\frac{\text{shoe's length}}{\text{shoe's width}}\right)\right)$ $X = (\text{shoes'length} + \text{Shoes'width})$ $H = \text{Height},$ $R = \frac{H}{X}$

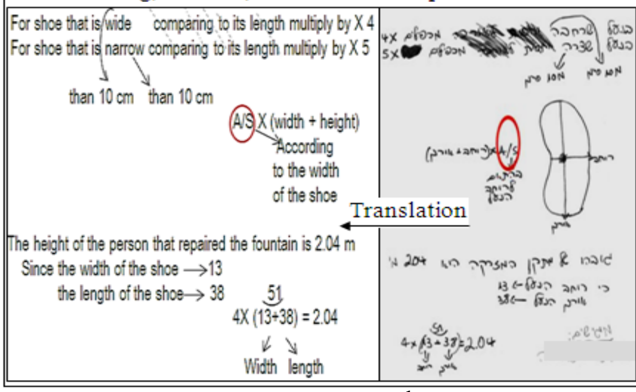

Figure 4: WTS documenting 6th-grade students' "two-dimensional" model

Appropriateness

Main Category & Subcategories	Coding rule: "MEAs' correct response" (as 'key concept') Defined as
Appropriateness	Broader range of mathematical knowledge and abilities to produce a reusable and sharable conceptual tool.
1. Knowledge	Students' ability to utilize their prior and developed mathematical knowledge in various ways to develop an appropriate model.
2. Utility	Deliberate actions or means applied by students to generate useful solutions, not only for the current situation, but for other similar situations as well (reusable).
3. Documentation	Students' ability to apply varied representations to present and share information with others (sharable).

Table 1: Explicit definitions of appropriateness and its subcategories

1. Knowledge: The transcript (lines 1, 2 & 1, 9) demonstrates how students apply their mathematical knowledge to construct (measure, code and synthesize) a relevant mathematical "object" such as their height and their shoe length, and mathematize the relationships between these "objects" to estimate their height (see also researcher's interpretation in the third phase, Figure 4).

2. Utility: In the transcript (lines 6, 7), students explain how they deliberately developed a useful conceptual mathematical tool to estimate the height of students in their group that could also be applicable to other students' data (similar situations).
3. Documentation: The students' poster in Figure 4 shows how students used symbols, "drawing" and written explanations to mathematically communicate "how" they were actively attempting to make sense of the structured problematic "real-life" situation in a way that could be sharable with others.

Mathematical Resourcefulness

Main Category & Subcategories	**Coding rule:** "overcome difficulties" (as 'key concept')
	Defined as
Mathematical Resourcefulness	Students' ability to cope in a coherent and fluent manner and demonstrate flexible thinking involving consideration of different approaches or strategies to construct and elaborate a powerful conceptual tool.
1. Fluency	Students' tendency to consider or evaluate several ideas and perspectives.
2. Flexibility	Students' ease in switching from one mental operation to another, applying redefinition and transformation, and finding new ways to describe both the dataset and its behavior.
3. Elaboration	Students' refinement, generalization and integrating abilities applied to developing a new level of more abstract or formal understanding.

Table 2: Explicit definitions of mathematical resourcefulness and its subcategories

1. Fluency: The transcript (lines 1, 2) shows early stages of the students' modeling process which involved fluent generation of different relevant mathematical objects, including shoe width, shoe length, shoe perimeter and student's height, before an effective solution emerged.
2. Flexibility: In the transcript (lines 5–7), students describe how verifying their early conceptualization of the situation required further refinement that takes into account more "discovered" information and more relationships among the data that better describe their advanced interpretation, leading to the development of a more powerful mathematical model (Figure 4). This example reflects students' ease in switching from one mental operation to another to describe both the dataset and its behavior via different types of representations.
3. Elaboration: The conceptual mathematical instrument demonstrated in Figure 4 and the transcribed explanation (lines 8–10) show how students elaborated (extended, refined and integrated) their ideas to develop a new level of more abstract or formal understanding and create a more generalized conceptual tool, as shown in the researcher's mathematical interpretation in Figure 4.

Inventiveness or Originality

To assign this category to the data, we looked for an appropriate and unique mathematical response in comparison to those developed by other groups (Guilford, 1967).

Main Category	**Coding rule:** "unique responses" (as key concept)
	Defined as
Inventiveness or Originality	Students' ability to break away from routine or bounded thinking to create unique and powerful mathematical ideas that differ from those developed by most other students.

Table 3: Explicit definitions of inventiveness

The conceptual tool in Figure 4 illustrates students' inventiveness. Although there were two other groups (out of 22) that estimated the individual's height based on the ratio between height and the sum of shoe length and width, only this group used a split function to mathematically describe how an individual's height depends on the width and length of his or her shoes.

CLOSING REMARKS

This paper highlights the innovative analytical process and reveals the cognitive abilities that were applied and activated while modeling a creative process by "high-ability" and mathematically gifted students, toward creating and inventing a more significant conceptual tool. Three categories and subcategories were formulated with respect to theoretical framework and empirical data: mathematical appropriateness consisting of three subcategories: knowledge, documentation and utility; mathematical resourcefulness involving fluency, flexibility and elaboration, and inventiveness or originality.

These results have both theoretical and practical implications (Amit, 2012; Amit & Gilat, 2013). In practice, they suggest new directions and alternatives for encouraging and inducing students to draw on those resources and abilities more productively as suggested by Guilford (1950), who argued that creativity can be developed and the "development might be in the nature of actual strengthening of the functions involved or it might mean the better utilization of what resources the individual possesses, or both" (p. 448). Theoretically, viewing students' MEAs through the notions of the three above core types of abilities can provide us with a deeper insight into what is involved in the creative mathematical process of young students engaging in non-routine, "real-life", structured problem-solving (Sriraman, 2009).

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