MATHEMATICAL ACTIVITIES IN A SOCIAL LEARNING FRAMEWORK: HOW MULTIMODALITY WORKS IN A COMMUNITY OF PRACTICE

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In this paper, we explore an approach to understanding how multimodality works in a community of practice. Using a social learning framework, we show how a community of practice, involving a pair of high school students, engaged in perceptual, bodily, and imaginary experiences while discussing about calculus concepts in a dynamic geometry environment. Our findings suggest that learners' multimodal experiences emerge in both visible and invisible uses of the artefact and are situated in the mathematical activities. This study enriches our understanding about how students participate in the mathematical activities with dynamic geometry environments.

INTRODUCTION

This paper brings multimodality in the lens of social learning, in particular, of community of practice. While many studies using Lakoff and Núñez's (2000) ideas have provided insights into the embodied and multimodal nature of mathematical cognition, this line of work tends to focus on thinking in the individual sense rather than with respect to the social nature of learning. Adopting the non-dualistic view that mathematical *thinking* is part and parcel of *doing* mathematics, we see here compatibility with conceptualising learning as participating in mathematical activities in a community of practice.

Our study seeks to apply the idea of multimodality—seen as an interplay of perceptual, bodily and imaginary experiences situated in the resources at play (Ferrara, 2013)—in social dimensions of learning (Lave & Wenger, 1991; Wenger, 1998). Toward a greater purpose, we hope that the results of our study will provide a better understanding of how multimodality "works" in a community of practice and in social learning contexts involving artefacts. Moreover, to pursue our interest in multimodality with respect to the use of dynamic geometry environments (DGEs), we anticipate that the notion of *transparency* under this framework can be meaningfully extended to the kinds of multimodal experience upon mathematical activities using DGEs. Within this perspective, we investigate:

- What repertoire of resources do learners use as they participate in mathematical activities using a DGE?
- What kinds of *visible* and *invisible* mathematical (multimodal) "talk" do learners develop as they participate in mathematical activities using a DGE?

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THEORETICAL PERSPECTIVES

Social dimensions of learning: Community of practice and transparency

A community of practice is a unit of social interaction situated in practice; it is part of a broader framework for conceptualising learning in its social dimensions (Wenger, 1998). This perspective suggests that learning is located, not in the heads or outside of the individual, but in the relationship between a social person and a social world. Meaning-making in social contexts requires a dual process of participating in-action and reifying actions into artefacts:

> On the one hand, we engage directly in activities, conversations, reflections, and other forms of personal participation in social life. On the other hand, we produce physical and conceptual artifacts—words, tools, concepts, methods, stories, documents, links to resources, and other forms of reification—that reflect our shared experience and around which we organize our participation. (Wenger, 2010, p. 180).

The interplay between participation and reification is dynamic: the person and the world intertwine to shape meaning both individually and collectively. Over time, this creates a social history of learning and a dynamic social structure that define a community of practice. Participants use a set of criteria and expectations to recognise membership in a community of practice, which include: an understanding of what the enterprise of the community is (*domain*), mutual engagement in the activity (*community*), and appropriate use of the repertoire of resources that the community has accumulated through its history (*practice*).

Lave and Wenger (1991) also posit that, when learners work within communities of practice, a dual visibility—*visibility* and *invisibility*—develops in the use of artefacts with respect to their *transparency* for the communicating subjects.

> Invisibility of mediating technologies is necessary for allowing focus on, thus supporting visibility of, the subject matter. Conversely, visibility of the significance of the technology is necessary for allowing its unproblematic—invisible—use. This interplay of conflict and synergy is central to all aspects of learning in practice: It makes the design of supportive artifacts a matter of providing a good balance between these two interacting requirements. (Lave & Wenger, 1991, p. 102).

In the case of using a mediating technology like a DGE, transparency means that the DGE fades into the background and becomes a means by which participants achieve something else. On the other hand, if the DGE remains to be the focus, there is little room for learning about its affordances—it will be a black box that is in control. This invisible and visible character of the DGE allows for considering its relevance in communities of practice and its relationship to learning about particular domains.

Multimodality in mathematical activities

In the special issue on gesture and multimodality in mathematical thinking, Radford et al. (2009) point out that in our acts of knowing, different sensorial modalities—tactile, perceptual and kinaesthetic—become integral parts of our cognitive processes. Other studies discuss gestures in mathematics teaching and learning, with respect to teacher's gestures in relation to students' meaning making (Arzarello *et al*., 2009), the cultural dimension of gestures (Radford, 2009), and the role of gestures in mathematical imagination (Nemirovsky & Ferrara, 2009).

Further contributing to the discussion on multimodal mathematical cognition, Ferrara (2013) describes how multimodality manifests, that is, "as a constitutive expression of thinking, which encompasses complex networks of perceptual, sensory–motor and imaginary experiences" (p. 19). In particular, it is proposed that the contemporary and entangled emergence of such experiences shapes mathematical thinking on the one hand, and, on the other hand, is shaped by the resources at play.

It is this idea of multimodality that we think works suitably in the lens of social learning and of community of practice, where the resources at play are relevant both for the community and for the practice at hand, and at the same time strictly contextual in terms of the domain of interest.

METHODOLOGY

Participants, task and data collection

The participants of the study were two pairs of 12th grade students (age 17) enrolled in an AP Calculus class in a culturally diverse high school in Western Canada. In the class of 26 students who all volunteered to participate in the study, the participants, R, G, J, S, were selected. They were selected randomly as a group of four because they had been seated in the same row in their regular calculus classroom and were regular partners during assigned group and pair-work activities. Their teacher (second author) describes them as motivated and comfortable working with each other. The study took place at the end of the first trimester of the school year in the participants' regular calculus classroom, outside of school hours. At the time, the participants have just finished learning about key concepts in differential calculus using an iPad-based DGE called *Sketchpad Explorer*. So, students have experienced with exploring and discussing, in pairs, calculus concepts such as derivative, derivative functions and related rates through geometrical, dynamic sketches.

The participants were divided into two pairs, and each pair was asked to discuss two different sketches presented in *Sketchpad Explorer*. For the purpose of comparing patterns of communications, they were given one sketch that they had seen before in class and another sketch related to a topic that was new to them. For example, the pair R and G were given a sketch related to the definition of derivative which they have seen before, and then a sketch related to area-accumulating functions (Figure 1a and b) which was new to them (they had not learned the topic of area-accumulating functions in class). The researcher gave the instructions, turned on the videotaping function of the camera facing the student-pairs, and then left the room, until the students finished talking about all the diagrams. Each student-pair took around 25 minutes on completing the task for each session.

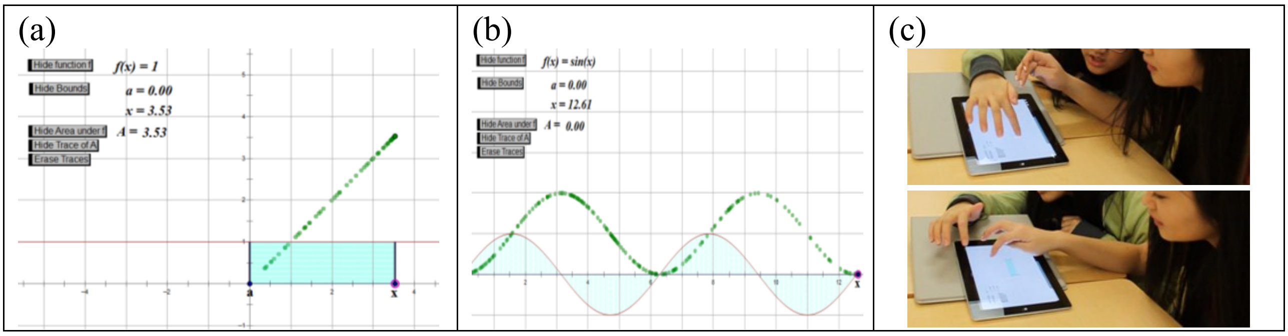

Figure 1(a) & (b): A dynamic sketch used in the study (with all *Hide/Show* buttons, "*Show function*", "*Show bounds*", "*Show Area under f*", and "*Show Trace of A*" activated). The bounds "a" and "x" are draggable; the green traces represent function $A(x) = \int_a^x f(t)dt$. Figure 1(c): Snapshots of R and G interacting with the sketch.

DATA ANALYSIS

In this section, we extend the notion of transparency to analyse the use of the DGE and the kinds of multimodal experiences that one of the student-pairs, R and G, developed in the task. The analysis is divided into three parts each containing a condensed transcript, for the purpose of identifying themes in each part.

Visible talk: Exploring *Hide/Show* buttons and dragging

The following is a transcript of R and G's initial interaction with the first page of the sketch (see Figure 1a). At the start, all buttons were in the "hide" position, except for the "show function" button, which showed a constant function on the page.

1 2	G:	So this is like a straight line. What is show bounds? So there is an interval, so it's like a domain. *<G presses the "show bounds" button>*
3	R:	Can we change this one? Can we change "a"? *<G drags "a">*
4	G:	No, no, no, make it zero. Show area under "f". *<G presses the "show area under f" button>*
5	R:	What's this point? Can we move it? *<R taps on the green trace trying to move it>*
6	G:	Show, show, what's trace of "A"? *<G presses the "show trace of A" button>*
7	R:	What are you doing?
8	G:	I don't know, I'm just, oh, when you are moving it, it's graphing like the
9		area, or, no, ya, ya, it's just graphing the area. *<G drags the entire rectangle horizontally>*
10	R:	Oh, interesting, and it goes up and down. *<R drags point "a" horizontally>*
11	G:	Well yeah, 'cause you're making the area.
12	R:	Can I move "x" still? Oh, interesting… *<R drags point "x" horizontally>*
13	G:	So if you just move "x", area is a positive slope. What if you just move "a"?

14 R: It's the up and down thing.

The 3-minute transcript highlighted the way R and G entered visible "talk" about the dynamic sketch. It showed that the student-pair was trying to learn *how* to use the "black box" sketch by exploring the functions of the *Hide/Show* buttons and dragging the points "a" and "x" respectively. As the students had yet to fully grasp the many buttons in the sketch, they posed questions three times in lines 1, 4, 6 to inquire the functions of each button, "*Show bounds*", "*Show area under f*", and "*Show trace of A*" as each button was pressed. Since their perceptual and bodily experiences were focussed on the use of the *Hide/Show buttons*, it can be said that at this point, they attended to the buttons *visibly* rather than *invisibly*. The students also seemed unsure about what to do with the two draggable points "a" and "x" initially and therefore tried to use dragging as a resource to investigate the behaviour of the points. This was evident in lines 3 and 13, where R asked G if they could "change" "a" and then "move" "x". The word use of "can" in both questions suggests that R did not know if the points were draggable, and therefore, proposed to drag "a" and "x" for the purpose of learning about the sketch. R and G's use of the resources at hand, particularly the draggable points (that are used as parameters) gave relevant feedback about the enclosed area, allowing the two students to begin the process of *imagining* how the area would change and behave as a function.

Invisible talk: Gesturing, dragging, and using the Trace tool

After about 5 minutes of interacting with the first page, the students moved onto a new page of the sketch, which initially showed the sine curve (see Figure 1b).

15 R: Oh, sine. It's gonna be complicated, it's gonna be crazy.

16 G: Oh, is it "cos"? No, it's not. <*G drags "x" horizontally continuously*>

17 R: It's like it's been shifted, transformed.

18 G: So this is the area right now, so when "x" equals to 3, the area is like 1.2.

19 R: So "a" is always gonna be there, and "x" is the one that's always gonna

20 like, where it corresponds.

21 G: No you can move "a", when you move "a", it's just a vertical line <*G drags "a"*>

22 R: But it's always gonna stay with "x". The "x" moves at *x* and *y* direction

23 G: And then if you just move "x", it's a vertical line, wait, no it's not. <*G drags "x"*>

24 R: It's still moves it like this. <*R makes "wave" gestures*> It's just when you move

25 "a", then it's like up and down. <*R gestures with one finger moving up and down*>

The transcript opens with G dragging the point "x" horizontally back and forth, therefore tracing a function $A(x)$ that was sinusoidal (line 16). Then, R and G consistently used words like "now", "it", "this" and present continuous verb forms (verb that ends with "-ing") to talk about the state of the sketch. For example, the students took turns to describe what the green traces looked like, referring to the traces as "it" (lines 16 and 17). Then, G started to drag "x" horizontally and said "so *this* is the area right *now*" (line 18). Perceiving the green traces created by G's dragging, R responded that the *x*-coordinate of green traces "always corresponds" (line 19). They moved on to describe the green traces left by dragging "a" which would create a "vertical line" (line 21).

The episode shows that the student-pair moved from questioning about technology to talking *invisibly* about the sketch. The students used the *Hide/Show* buttons and dragged points "a" and "x" purposefully, without struggling with their functions as they did previously. They shifted their focus on the discussion from the act of *dragging* from earlier to the *results* of dragging—and towards *invisible* use of the DGE. During this discussion, R performed two hand gestures using her right index finger to describe the shape of the traces. First, she made "wave" gestures (Figure 2a) to explain that the green traces should be sinusoidal (line 24); then she made "up and down" gestures (Figure 2b) to explain the vertical movement of "a" (line 25), when "x" and "a" were dragged respectively. These gestures provide further evidence that the students were engaging in *invisible*, multimodal "talk" around the DGE.

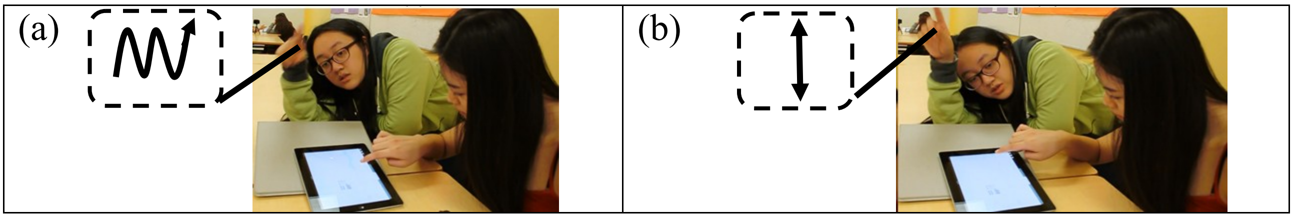

Figure 2(a): R's "wave" gestures, and (b): "up and down" gestures.

Through dragging and gesturing, the students extended their perceptual and bodily experiences. Moreover, the *Trace* tool and shaded area gave feedback about the relationship of the green traces and the area under curve, which enabled the students to *imagine* the possible shape of the corresponding area-accumulating function. It is also worthy of note that the coordination of the two students that reveal their real being of a community of practice: one drags and the other gestures; one moves depending on the movement of the other.

Visible and invisible talk: Confirming conjectures with dragging

R: There's probably some formulas, a generic formula for all of this.
G: So the area gets, it goes like big and small, big and small. *<G drags "x">*
R: Wait, move "a" next to "x", right on top, it goes zero right? *<G drags "a"*
over top of "x"> Yes it's zero. Move one of them somewhere.
G: So "x" increases positively. *<G drags "x" to the right from x=0>*
R: Wait, move it so that it's at the top of the curve, where does it go when
it's at the top? Ok, so this grows from here to here, which represents the
area of the whole. *<G drags "x" towards* π *>*

G: Yea, so it's just like the entire thing, but then it will go back down.
R: Yea it goes negative so it takes away from it. So once we finish this hump, it should be zero, yea it comes back to zero. <*R drags "x" towards 2π*> The area is always, when you graph the point, it's always gonna be at the "x", not the "a".

After about 8 minutes of interacting with the sketch, the transcript shows that R and G began to talk about the significance of the DGE sketch. This was evident in the way the students talked about a generic "formula" (line 34) to relate the green traces with "a" and "x" as well as their use of dragging to confirm their predictions about the sketch. In particular, they made conjectures such as the green trace should reach zero when "a" was dragged towards x=0 (lines 36-37), and that it should go up and back down before arriving at the next zero when "x" was moved towards 2π (lines 39-44). The students' *imaginary* experience was met by perceptual and bodily experiences through perceiving the traces left by dragging. Having confirmed their results, they used high modality words such as "will" (line 42), "should" (line 44) and "always" (line 45) to generalise the ways the green traces should behave. Although they were not able to communicate about the area-accumulating function $A(x)$ clearly, the students grasped the meaning of the green traces, marked by their mutual engagement towards predicting their shape. This suggests that they develop *dual visibility* in the use of the dynamic sketch: unproblematic use and understanding the significance of the DGE.

DISCUSSION

In this section, we direct our discussion with regards to our exploratory approach of applying multimodality in a social learning framework, and the extent to which this approach informed understanding of how multimodality "works" in a community of practice. First, our analysis shows that the student-pair constituted a community of practice. The two students shared a "domain" of interest to advance mathematical knowledge in their activity with the DGE; they were also mutually engaged and used a repertoire of resources in the activity. Some have critiqued the idea that classroom settings do not reflect communities of practice, but we have shown that communities of practice, as units of social interaction situated in practice, may exist in pair-work mathematical activities when students understand and share the goal of the activity.

Secondly, the students used a repertoire of resources in their activities with the DGE, such as the *Hide/Show* buttons, the dragging modality, and the *Trace* tool that they possibly developed through their history of learning during the first trimester of the course. These resources enabled them to initially enter *visible* talk about the DGE, and later talked about the dynamic sketch *invisibly*. After about 8 minutes of interacting with the sketch, they conjectured about the shape of the green traces and used the dragging modality to confirm their predictions. Their unproblematic use of the DGE and ability to talk of the significance of the sketch supports the claim that they had found a "balance" between visible and invisible uses of the DGE.

Thirdly, the students' interactions with the dynamic sketch were analysed both within the lens of transparency and multimodality. Aligned with both the social learning framework and Ferrara (2013) on multimodality, visible and invisible DGE use shaped the students' mathematical thinking on one hand, and on the other, their participation in the activities were constantly shaped by the resources at play. Their participation involved talking, perceiving, dragging, gesturing and imagining, that is, multimodal experiences. In particular, these experiences are situated in the dynamic sketch. The sketch's dynamic essence gave rise to the functional relationship between variables, a, x, and $A(x)$, which the student-pair exploited by dragging and gesturing. These perceptuo-bodily acts, which were also dynamic in nature, led to the students' imagining of the tracing of the green point that was dynamic in nature as well.

In conclusion, our approach did enrich our understanding of how students participate in the mathematical activities with the DGE. In our illustrated episode, we found it helpful to extend the notion of transparency to students' multimodal experiences in mathematical activities. This combined framework informed the interplay between transparency of talk, in a multimodal sense, and the resources at play—the DGE. Because of the scope of the paper, we were not able to examine the evolving discourse between a less experienced user of the DGE (new-comer) and a more experienced user of DGE (old-timer) in mathematical activities. We believe that this process of participation could form the basis for fruitful future research.

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