THE TEACHING ACTIVITY AND THE GENERATION OF MATHEMATICAL LEARNING OPPORTUNITIES

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We present a case study that has been developed to inform about the teaching activity of a secondary mathematics teacher in a whole group discussion and the mathematical learning opportunities generated for the students in this classroom context. For data of one lesson we determine the episodes that shape the whole group discussion. For each episode we then examine the effects of the observed actions on the type of learning that can be encouraged. Our research reveals significant relationships between the teaching activity of the teacher and the creation and potential exploitation of mathematical learning opportunities on the part of the students.

INTRODUCTION

Research in the field has seriously addressed the study of social interaction as an element to build learning in small group work (Sfard & Kieran, 2001), but little is still known about the construction of mathematical knowledge in the course of whole group discussions (Saxe et al., 2009). We assume that these discussions are a crucial resource for mathematics teaching, since they can facilitate the students' learning. Under this assumption in our work we seek to investigate to what extent and how the teaching activity mediates the generation and potential exploitation of mathematical learning opportunities in the mathematics classroom. In this report we summarize a case study (of a teacher and a lesson) whose results have come to inform about the following goals: (a) to characterise the type of teaching activity of a secondary mathematics teacher in a whole group discussion, and (b) to identify mathematical learning opportunities generated for the benefit of the students in this context.

TWO THEORETICAL DIMENSIONS AND A KEY NOTION

We base the analysis of what we call the episodes of a whole group discussion on the articulation of two dimensions: the instrumental dimension, about the artefacts and the way in which these are used in class, and the discursive dimension, about the interactional patterns that help to understand the generic development of the episodes and some of the particular characteristics shared among them. Therefore, two coordinates and the qualitative type that each coordinate takes define an episode.

In the understanding of the instrumental dimension, six types of orchestration are considered: *exploring the artefact, explaining through the artefact, linking artefacts, discussing the artefact, discovering through the artefact and experiencing the instrument*. The first three types are focused on the teacher's actions and the last three on the students' actions. They are all inspired by the initial types constructed by

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Drijvers and his colleagues (2010), but have been generalised in our research for instructional situations in which the design and implementation of whole group discussions do not necessarily contain an intensive use of technological artefacts.

The discursive dimension is also framed in terms of types that are named stages of the discussion of a problem. They are presented as a sequence of activities that illustrate the process of conducting a whole group discussion toward the resolution of a problem. The stages are organised according to an idealized development of the resolution process: *situating the problem, presenting a solution, studying different solutions or explanatory strategies, studying particular or extreme cases, contrasting solutions, connecting with other situations, generalising and conceptualising, and reflecting on mathematical progress.* Later in the report we exemplify an episode with the coordinates *discussing the artefact* and *contrasting solutions*.

More generally, we interpret episodes as systems of actions that have occurred in the course of the discussion. Our interest is on the effects of actions as some of them may foster basic procedural and/or conceptual mathematical learning (Niss & Højgaard, 2011). Differently to how episodes are seen, actions are tied to the subject performing the action, either student or teacher, and their role in the organisation of participation in whole group discussion. To consider the role of the actions performed by the teacher, we draw on the classification by Schoenfeld (2011): *classroom management actions, discussion actions* and *mathematical content actions*, depending on whether they refer to the organisation of the classroom and its participants; to the development of mathematical activities; or to the mathematical content of the activities as well as the teacher's ability to listen to the students, and become aware of their difficulties and of the aspects that they understand better or worse.

Mathematical learning opportunities

The interpretation of whole group discussions in terms of sequences of episodes and actions has to do with our understanding of interaction as a crucial place for the development of mathematical learning. Various authors have researched the broader topic of mathematical learning opportunities for the case of students (Yackel, Cobb, & Wood, 1991). We consider mathematical learning opportunities as relationships between contents of mathematical knowledge, which are liable to be procedural and conceptual, together with actions that potentially contribute to facilitate the students' learning. These opportunities are identifiable through and from actions generated by diverse situations in the interaction processes of the mathematics classroom.

Several distinct actions can be at the origin of the appearance and possible exploitation of learning opportunities. Classroom actions are a combination of multiple interaction processes, in which the students and the teacher as well as the use of artefacts contribute to the creation and development of relevant instructional situations that can in turn foster the students' mathematical learning (Cobb & Whitenack, 1996). Accordingly, the study of learning opportunities requires the prior systematic preparation, examination and assessment of instructional situations.

DESIGN EXPERIMENT AND DATA

Drawing on the tradition of design experiments in mathematics education research, we designed and implemented an instructional sequence of Geometry with similarity problems. Two teachers conducted it in two classrooms over a total of eight lessons with 8th graders (13 and 14 years of age). In this report we have selected the case of the teacher who at the moment of the experiment had an average teaching experience and was working in an urban school of a medium-high sociocultural area.

Figure 1 shows the first problem in the sequence, whose wording presents an open approach and whose resolution is tied to the activation of high cognitive tasks of proportional thinking. There is more than one solution strategy and connections need to be made with the underlying mathematical concepts (e.g., shape, area, ratio).

Introductory activity 1: Double figures!

Given the following letter from the alphabet, represent another letter which is twice the size. Below, briefly explain how you obtained it and compare it with the original.

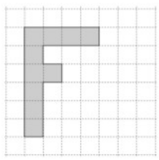

Figure 1: Formulation of the first problem

The work dynamics is collaborative and begins with the paper-and-pencil resolution in pairs. It continues with a 20-minute whole group discussion, and finishes with the students' written individual reflections. Two of the authors were present in the lesson, but did not intervene in the development of the activity. During the whole group discussion three video cameras recorded the interventions made by participants and these were later transcribed for the purpose of the analysis.

The classroom recordings were examined in order to: (a) divide the whole group discussion into episodes, determine the actions that take place in them and, thus, obtain a description of the teacher's activity when managing the lesson; (b) study the effects of the actions on the type of learning that is encouraged and detect and classify the mathematical learning opportunities generated during group discussion. In the next section we illustrate the application of the methods to one episode.

EXAMPLE OF ANALYSIS

First we divided the whole group discussion around the resolution of the problem of Figure 1 into nine episodes. We classified them according to a type of orchestration (instrumental dimension) and a discussion stage (discursive dimension), and we searched for the observed actions of the participants. The nine episodes with their nine corresponding coordinates provide organized information about the teacher's activity when managing the whole group dynamics in the selected lesson.

The fifth episode (*discussing the artefact, contrasting solutions*) of the whole group discussion lasted three minutes. We briefly explain the major phases of the analysis of the episode. In the transcript below, the teacher addresses questions to discuss and obtain a definition of similar polygonal figures. The conversation begins with the projection of solutions onto a screen using dynamic geometry software; this is why we assign the type *discussing the artefact* for the instrumental dimension. As two interpretations of the problem wording are compared, resulting in two different solutions, a figure with twice the perimeter and another with twice the area, we assign the stage *contrasting solutions* for the discursive dimension.

1	Teacher:	[to Student 1] What did you understand? Why did you create this, [figure with twice the perimeter] and not this? [figure with twice the area].
2	Student 1:	Because these two are the same [the original F and the one with twice the perimeter].
3	Teacher:	Okay, then, the definition of similarity... Why do you think they are two similar figures?
4	Student 1:	All the sides multiplied by a number, always the same.
5	Teacher:	Okay, so, how are the sides?
6	Student 1:	Proportional.
7	Teacher:	Proportional, okay. And what else is needed?
8	Student 2:	All the angles need to be equal.
9	Teacher:	Here we won't check the angles because it's an F and it's evident that all the angles are 90º, but we should always check.

The system of actions that have occurred in the episode is also studied and represented by means of a sequence. We distinguish those performed by the students, named participation actions, from those by the teacher, named intervention actions. On the one hand, we mark the description of all actions with italics in the expanded narrative (italics are also used in the description of the mathematical learning opportunities). On the other, the intervention actions are more generally classified and thought of in relation to issues of classroom management, discussion and mathematical content (see the three columns of Figure 2, with the exemplification of the sequence of actions of the third type in the fifth episode).

At the start of the episode the teacher *requests an explanation* [1] from Student 1 as to why he created a figure with twice the perimeter instead of twice the area. The student observes the two representations projected on the screen and reveals that his choice was based on the similarity of the figure whose sides are proportional to the original. We interpret this action as *observation of empirical evidence* [2], since the visual information provided by the artefact helps him to verify a specific mathematical fact, but without this action implying or having the function of justification. Next, the teacher uses the situation to introduce the concept of similar figures and *requests* *another explanation* [3] so that any volunteer defines it. Again, it is Student 1 who uses the representation on the screen to explain that two similar figures must have all the sides multiplied by a number, and that this number must always be the same. However, we interpret this action as *empirical justification* [4], because the student uses the representation projected on the artefact as a complement to his oral explanation and the diagram legitimises his statement. As Hanna (2000, p. 15) states, "the visual representation is used not only as evidence for a mathematical statement, but also in its justification […] since diagrams can convey insight as well as knowledge."

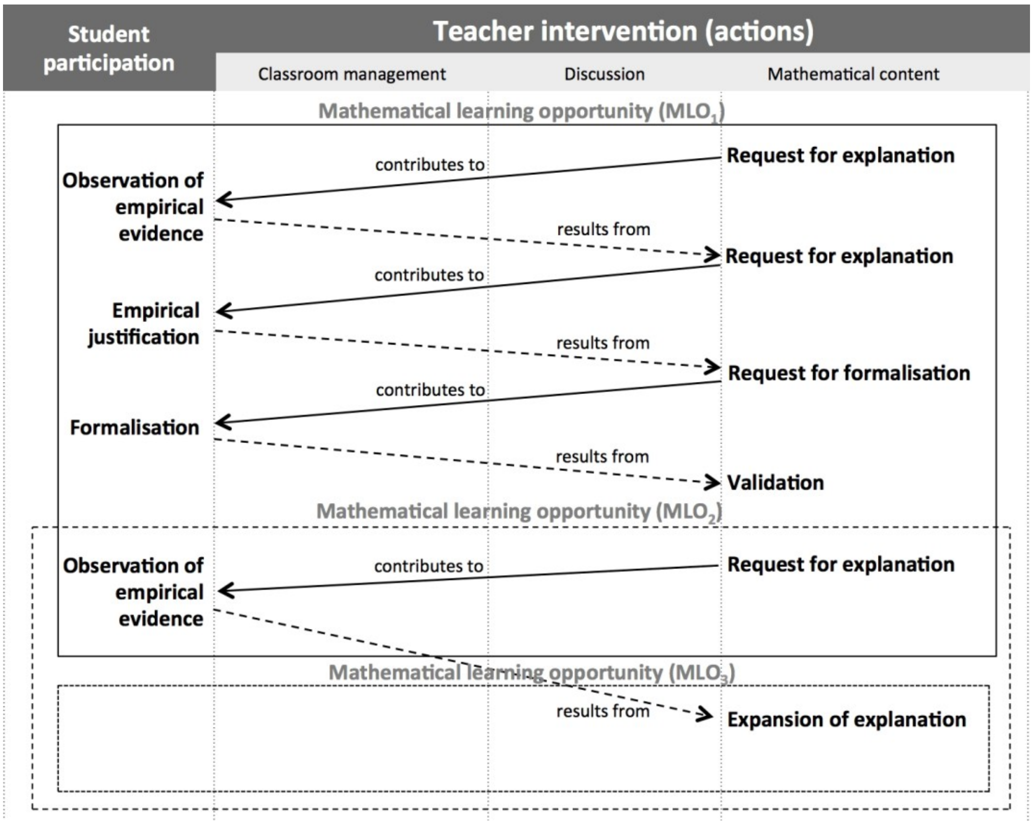

Figure 2: Representation of a sequence of actions in the episode

Later in the episode, the teacher *requests formalisation* [5] to specify particular technical language and to ensure the use of the term 'proportional' [6]. She *validates the reasoning* by Student 1 and *requests further explanation* [7] so that the students complete the definition. Another student uses the construction on the screen *to observe the empirical evidence* [8] that two similar polygonal figures, in addition, must have equal angles. We interpret that Student 2 does not use the artefact to support a mathematical reasoning, but to prove a concrete mathematical fact. Lastly, the teacher *expands the explanation* [9] by this student and states the importance of the equality of angles in the definition of similarity.

In Figure 2 we see that the structure of the sequence of actions presents linked series of interventions between the teacher and the students. This interactional pattern is reproduced in the other eight episodes of whole group discussion in the lesson, since the teacher gives almost no explanations, but manages the discussion with the questions that she asks and mainly elaborates her talk on the students' responses.

The analysis of the teaching activity comes when the analyses of all nine episodes have been finished. In total they reveal an orchestration that is equally focused on the teacher and the students. There are five episodes corresponding to the three first types of orchestration (*exploring the artefact, explaining through the artefact* and *linking artefacts*) and four corresponding to the last ones (*discussing the artefact, discovering through the artefact* and *experiencing the instrument*). The accomplishment of the idealized discussion stages is almost complete and their distribution is sequential from the stages of the initial moments of the discussion (*situating the problem* and *presenting a solution*) to the later ones (*generalising and conceptualising*).

Identification of mathematical learning opportunities

After having represented the sequence of actions for each episode, we are ready to relate the effects of the actions on the type of learning that they can encourage and to identify the mathematical learning opportunities, particularly focusing on the mathematics. For the fifth episode, our analysis suggests that various participation actions by the students can encourage procedural learning, linked to mathematical processes and focused on statements about facts perceived by the students during the debate (e.g., observations of empirical evidence), or on specific clarifications about mathematical aspects (e.g., formalisations). Other actions may encourage conceptual learning, linked to the students' empirical justifications and reasoning, which are centred on the development of mathematical concepts (e.g., notion of shape).

In a similar way, we explore the effects of the intervention actions by the teacher on the type of learning. These are the prioritised actions in the analysis due to our interest in the characterization of the teaching activity. As an example, we pay attention to the effects of mathematical content actions that refer to requests for explanation of mathematical methods or verification as they may encourage procedural learning (e.g., formalisations and validations). Also, this type of actions may encourage conceptual learning in relation to mathematical contents that are specific to the task (e.g., proportion and ratio) through the expansion of the students' explanations. In conjunction with classroom management and discussion actions, three mathematical learning opportunities appear.

The teacher's intervention at the start of the episode, requesting an explanation, initiates a debate that ends with the correct statement of the definition of similarity [1-8]. Therefore the situation generates a conceptual learning opportunity, that of *interiorising the concept of similarity and understanding its definition* (see Figure 2, MLO_1). Although the teacher's questions are crucial to bringing about this situation, the opportunity arises as a result of the participation of students. Student 1 refers to the term 'similar' in his observation of the empirical evidence [2] and Student 2 responds to the teacher and completes the statement introduced by his peer [8].

The request for explanation by the teacher [7], asking about the additional elements that characterise the similarity of two polygonal figures, generates another conceptual learning opportunity, that of *identifying the equality of angles in the definition of similarity* (see Figure 2, MLO_2). Although Student 2 makes an empirical observation stating that the homologous angles of the two figures must be the same, the opportunity arises as a result of the teacher's question, without comments of students directly leading to it.

The teacher's expansion of an explanation [9], emphasising the need to verify the equality of angles in order for the two figures to be similar, generates an interpretative and argumentative mathematical learning opportunity that we see as procedural. This is defined by *realising the importance of being rigorous in the elements constituting a mathematical definition* (see Figure 2, MLO_3). Again it is the teacher with her teaching activity who mainly contributes to its generation.

RESULTS AND FINAL DISCUSSION

We have shown to what extent and how the teaching activity of a teacher in a lesson mediates the creation and potential exploitation of mathematical learning opportunities in whole group discussion. The first goal was to characterise the teaching activity. Our analysis of the instrumental and discursive dimensions suggests that the class is managed with an orchestration that is equally focused on the teacher and the students, and an organised accomplishment of the discussion stages. The distribution of actions in the fifth episode reveals linked sequences of questions and answers in an interactional pattern that alternates teacher and student interventions.

The second goal was to identify the mathematical learning opportunities generated during the lesson. We have shown participation and intervention actions that seem to be at the origin of opportunities. The effects of some of these interrelated actions are likely to generate two major learning types: procedural and conceptual. To identify the opportunities, we have related the mathematical knowledge aspects of the opportunity with the effects of the actions that potentially facilitate their learning. Thus, we have observed that the mathematical learning opportunities occur in multiple discursive and instrumental situations. The data shown in this report only illustrates some of the many opportunities that were generated in all the episodes of the discussion. If looking at all of them, it can be inferred that the teaching activity is a strong mediator of mathematically significant actions whose effects may generate mathematical learning. Our results suggest that the activity by the teacher, which is balanced in orchestration and complete in the accomplishment of the stages, encourages the creation of diverse mathematical learning opportunities, which can be exploited by the students. Further research can be undertaken in this direction to find distinct degrees of exploitation of opportunities during whole group discussions.

Acknowledgements

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