COGNITION IN THE WORKPLACE: ANALYSES OF HEURISTICS IN ACTION

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Studies on cognition have capitalized on the role of contexts and experience in shaping our cognitive competence. In the past few decades, the mathematical education research field has begun to pay increased attention to the mathematics practices of both adults and children that take place in non-academic settings. As a result, theoretical fields such as ethnomathematics, situated cognition, and workplace mathematics have gained prominence. In this paper, we use Vergnaud's theory of conceptual fields to highlight the workplace mathematical activities of two groups of practitioners-Street vendors in Lebanon, and Bus conductors in India.

INTRODUCTION

In the past two decades, researchers have increasingly emphasized the elicitative role of cultures in impacting mathematical thinking and problem solving. For many researchers, sociocultural settings not only determine how mathematical knowledge is acquired, but also how it is represented, organized and retained (Sanin & Szczerbicki, 2009). Although there are several contexts in which mathematical ideas develop and are discussed, mathematics education has mostly been associated with the institutional context (Mukhopadhyay, Powell, & Frankenstien, 2009). The problem is that usually in the school setting, mathematical knowledge is presented as a "prized body of knowledge" (Millroy, 1992, p. 50), stripped of its rich cultural and historical connotations, and far removed from the " lives and ways of living of the social majorities in the world" (Fasheh, 2000, p. 5). We, alongside prominent mathematics education researchers, take an exception to this view and argue for countering the narrow vision of mathematics that confines it to the school walls.

Investigations that have focused on studying people's use of mathematics outside the classroom is divided into two main groups; namely, those interested in "everyday cognition" where Lave (1988) is a prominent figure; and those interested in "ethnomathematics," where D'Ambrosio (1992) is prominent. Both groups of researchers call for a new conceptualization of mathematics that is rooted in nonacademic practices. The mathematical ideas that are generated and used outside of learning institutions allows people with little or no schooling experience to practice crafts and trades, conduct business transactions and make their livings in a variety of ways.

We, the authors, have been immersed in the field of mathematics education for over two decades in a wide range of settings and in both western and nonwestern countries: from K-12 schools to research universities and graduate schools of education. During

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this tenure, we have encountered many implicit and explicit questions about mathematical competence and its role in defining and shaping the identity of individuals in the classroom, the workplace, and the society as whole (Naresh & Chahine, 2013). We list some of these questions here: *What are the mental processes underlying an act of labor or service? What is the nature of the problem solving behavior of workers while immersed in everyday work practice? How is experienced knowledge represented and employed as part of the daily decision-making manners undertaken by workers?* Researching in the context of the workplace provided us with just the right context for addressing such questions. It also afforded us the opportunity to make connections between what are seemingly two disparate worlds — the world of mathematics learning and the world of mathematics in the workplace. To further pursue this line of inquiry, we devised a study (Chahine & Naresh, in press) to carry out a meta-analysis of the problem-solving behavior of two groups of workers - street vendors in Beirut, Lebanon and bus conductors in Chennai, India. The research reported in this paper is part of this larger project; in particular, we provide a narrative of workers' problem-solving behaviors using Vergnaud's theory (1988, 2000) of conceptual field.

THEORETICAL BACKGROUND

We situate our work in the broader theoretical fields of ethnomathematics and situated cognition. The foundation of ethnomathematics rests in its "openness to acknowledging as mathematical knowledge and mathematical practices elements of people's lives outside the academy" (Mukhopadhyay, Powell, & Frankenstien, 2009, p. 75). There is strong evidence in the literature on situated cognition that supports the hypothesis that by actively engaging in everyday activities, individuals gradually incorporate culturally constructed artefacts into their repertoire of thinking and further develop context- specific problem solving competencies (Wenger, 2000). Such evidence predictably challenges the conventional definition of what counts as mathematics by reinforcing the claim that mathematical activity can be seen as interwoven with everyday practice outside the academic formal settings.

Analytical Framework: Vergnaud's theory of conceptual field

Vergnaud's (1988) theory of conceptual fields is based on the idea that concepts always involve three facets: invariants, representations, and situations. Invariants refer to the mathematical properties or relations associated with the concept. Vergnaud contends that invariants are expressed through representations and that they are not the only factor affecting performance, but rather the way in which concepts are formed might be important. Also, concepts are always tied to situations which make them meaningful. More importantly, Vergnaud (2000) argues that the existence of these mathematical concepts does not necessarily mean that people are fully aware that they are behaving accordingly, but most often these concepts are only "implicit" in theorems or what he calls "Theorems-in-Action". Vergnaud (1988) has defined theorems-in-action as those "... mathematical relationships that are taken into account by students when they choose an operation or a sequence of operations to solve a problem" (p.144). Vergnaud's theory of conceptual fields brings us to the idea that to understand how mathematical concepts are acquired it is necessary to analyze the situations through which these concepts were made meaningful and useful in the context in which they are invoked. Vergnaud's model provides not only guidelines for coding vendors' and bus conductors' problem solving behaviors, but also an understanding of the underlying properties and relations implicit in these behaviors. Pursuing this model, we conducted comparisons along three major dimensions (representation systems, heuristics-in-action, and situations) to decode and examine the problem solving behaviors of street vendors and bus conductors. In this paper, we will provide an overview of data analyzed along the three dimensions; however, we will present and discuss data specific to one dimension – heuristics-in-action.

METHODOLOGY

The goal of the larger research study (Chahine & Naresh, in press) was to examine cognition at work in order to define and describe practical mathematical knowledge that emerged in the context of work activities. To this end, we chose a methodological approach that was based on an iterative process of data collection, analysis, and hypothesis. Our methods comprised qualitative secondary data analysis (of the ethnographic case studies – case refers to the groups of bus conductors' and street vendors), narrative inquiry of solution schemes, and focused discussions (researchers as participants).

Ethnographic case studies (ECSs) and related data

The overall goals of the ECSs were to unravel, analyze, and describe the mathematical ideas and decisions employed by the participants to solve work-related mathematical tasks. In the street vending context, participants included 10 male vendors randomly selected from two market settings in the southern suburbs of Beirut. Vendors in the sample varied in years of schooling (three to seven years), in age (10 to 16 years), and vending experience (one to eight years). Four of the vendors worked alone while the other six helped their fathers or neighbors. Only three were totally responsible for purchasing the produce at wholesale market and pricing it for selling. In the bus conducting context, five bus conductors selected from two bus depots were included. Four male bus conductors and one female bus conductor participated; the bus conductors varied in their educational qualifications (two had high school diplomas and 3 had Bachelor degrees) and years of experience (9 to 31 years). Data collected from the ECSs on street vending and bus conducting included field observations and notes, transcriptions of interviews, researchers' introspection notes, problem solving narratives, and work sample artifacts.

Data Analysis

Our first approach towards data analysis was to engage in a secondary data analysis (Moore, 2006) using data collected from the ECSs. Examining pre-existing data from the two studies enabled data linkage and afforded powerful insights into the problem-solving behavior of practitioners. Furthermore, revisiting data related to field observations, interviews, and researchers' notes in two workplace contexts allowed transparency within research as we continuously interrogated the quality of qualitative data in terms of coding and completeness. Next we engaged in a narrative inquiry (Coulter & Smith, 2009) to describe the problem-solving heuristics of the street vendors' and bus conductors' workplace activities. Developing a coding system for the problem solving behavior and narratives of vendors and bus conductors involved careful readings of transcriptions taken from practitioners' written solutions as well as interviews, with particular attention to researchers' comments. The first two stages of data analysis required us to engage in focused discussions centered on the secondary data collected through the ECSs. These discussions targeted specific mathematical frames that were captured as the practitioners are immersed in the context of street vending and bus conducting. Such discussions produced nuanced insights that would be less accessible without our intensive face-to-face purposeful interactions. As we listened to each other verbalizing and recollecting our field experiences, memories, ideas, and experiences were stimulated and validated as we discovered a common language to describe our recollections and reveal shared understandings or common views.

A qualitative analysis of the problem solving behavior of vendors and bus conductors was established by comparing, contrasting, and synthesizing these properties across work settings namely, vending and bus conducting , and across cultures i.e. Lebanon and India. We conducted three comparisons to decode and examine the problem solving behaviors of street vendors and bus conductors. Comparisons are carried out along three major dimensions: (a) representation systems; (b) heuristics-in-action; and (c) situations. In this section, we present data analyzed along the second dimension -- heuristics-in-action.

RESULTS

Street Vending and Bus Conducting heuristics-in-action

Vergnaud (1988) maintains that all "mathematical behaviors" are tied to certain mathematical concepts and that the existence of these concepts does not necessarily mean that subjects are fully aware that they are behaving accordingly. We call these mathematical behaviors as heuristics-in-action and define them as the ways in which practitioners utilized the mathematical properties or relationships to resolve a problem or complete a task that emerged in their work settings. Two major heuristics-in-action were employed by the participants to reach a satisfactory solution, namely *building-up* and *multiplicative* which in turn led to scalar and functional solutions. These heuristics are virtually based on the properties of linear functions, specifically isomorphic and functional properties (Vergnaud, 2000). To illustrate, consider the following transactions that were extracted from the two contexts:

Transaction 1: Context: Street vending: The following exchange occurred between the researcher posing as a customer and Masri (pseudonym), a 12- year old vendor:

Researcher: I will take 6 kilos of lemon, how much do these cost?

Masri: 1 kilo for 1250 L.L., then if 1 kilo cost 1000 L.L then 6 kilos will cost 6000 L.L and 6 of 250 LL. Will be 1000... then 7500 L.L

Transaction 2: Context: Street vending: This time, we approached Masri selling onions, 750L.L/ 1 kilo:

Researcher: I want 3 kilos of onions, how much do I owe you?

Masri: 2 kilos for 750 L.L plus 750L.L which gives 1500 L.L, and another 1 kilo for 750L.L then 2250L.L".

Transaction 3: Context: Bus conducting: A passenger approached the conductor requesting 4 tickets for destination A and 2 tickets for destination B.

Passenger: I want 4 tickets (tokens of travel) from to Sayani (exit point) and 2 tickets to the Sanitarium (a different exit point)

Conductor: Unit ticket price to Sayani is 3.75 so 4*4 is 16… take away 4 quarters, so the price is 15; unit ticket price to sanitarium is 4.25… so 4*2 is 8 and add 50 to it to get 8.50. The total fare is 15 +8 is 23 … add another 50 to it. Give me 23.50.

Let us consider the second transaction for analysis. We viewed the problem posed as one of multiplication, precisely 750 * 3. However, Masri did not multiply using the standard algorithm; rather, he solved the problem mentally through a *building-up* heuristic involving repeated additions which could be formalized as follows:

$$\begin{array}{lcl} \text{1 kilo} & \longrightarrow & \text{750 L.L} \\ \text{1+1 =2 kilos} & \longrightarrow & \text{750+750=1500 L.L} \\ \text{2+1 = 3 kilos} & \longrightarrow & \text{1500+750= 2250L.L} \end{array}$$

Each variable, i.e., weight and price, remains independent of the other and parallel transformations are carried out on both variables, thereby maintaining their values proportional. When selling something at a price X, the vendors were perfectly aware of the fact that when there is an increase in the number of kilos (k) , there is a proportional increase in the price i.e., as many X's are increased in the price as kilos are increased in the purchase. The solution thus obtained has been termed by Vergnaud (2000) as scalar solution. Representing the above solution formally or explicitly:

Cost(3 k) = Cost(1 k+1 k +1 k) = Cost(1 k) + Cost(1k) + Cost(1 k) = 3 * Cost(1 k).

If we propose a relation between weight and price, more precisely a mapping f: to every weight there corresponds a well-defined price, then the above expression can be formalized as $f(1+1+1) = f(1) + f(1) + f(1)$. More generally, $f(X+Y) = f(X) + f(Y)$, which Vergnaud (1988) describes as the isomorphic property of addition, or more specifically, the linear property of function f.

The first transaction, on the other hand, represents another heuristic employed by the same vendor, namely multiplicative heuristic which can be also formalized as follows:

$$
\begin{aligned}
&\text{1 kilo} \longrightarrow && 1250 \text{ L.L} \\
&\text{6 kilos} \longrightarrow && 6(1250) = [6(1000 + 250)] \text{ L.L} \\
& && = [6(1000) + 6(250)] \text{ L.L} \\
& && = [6000 + (4+2)\ (250)] \text{ L.L} \\
& && = [6000 + 4(250) + 2(250)] \text{ L.L} \\
& && = [6000 + 1000 + 500] \text{ L.L} \\
& && = 7500 \text{ L.L.}
\end{aligned}
$$

Here, Masri's solution method can be conceived in terms of a variable f(X), the price, as a function of a variable X, the number of kilos, and hence a relation can be formed through multiplying the value of X by a constant, unit price, in order to find the value of f(X). The solution obtained is called functional for it relates to two different variables, the ratio thus attained is termed "intensive" or "external" ratio (L.L/ kilo). Using the preceding argument:

$$\text{Cost (6 kilos)} = \text{Cost 1 kilo} * \text{6 kilos}$$

$$f(X) = f(1) * X$$

If we assume that the cost of 1 kilo = f (1) = constant a, then the above expression can be replaced by f(X) = a * X, which is the constant function coefficient (Vergnaud, 1988).

In the third transaction, the bus conductor broke the in-situ problem into three smaller problems: Find 3.75 * 4 (b) Find 4.25 * 2 (c) Add the answers from (a) and (b). We can combine the derived facts and related discussions from the first and the second transactions and propose the following (note that the currency denomination is in rupees abbreviated as Rs.):

Cost of four Rs. 3.75 tickets and two Rs. 4.25 tickets

= Cost of four Rs. 3.75 tickets + Cost of two Rs. 4.25 tickets

= Cost of one Rs. 3.75 ticket * 4 + Cost of one Rs. 4.25 ticket* 2

= (3.75*4 + 4.25*2) = (3.75*4) + (4.25 *2) = 3.25 * 4 + 4.25 * 2

This expression can be stated as

g (3.75*4 + 4.25*2) = g (3.75*4) + g (4.25*2) = 3.25 * g (4)+ 4.25 * g (2) or more generally as g (aX + bY) = a g (X) + b g (Y). Here, we can conceive the conductor's solution in terms of g (aX + bY), the price, as a function of the variables X and Y, the number of tickets two different ticket denominations with unit prices a and b respectively. Thus we form a relation by multiplying the values of X and Y by constants a and b.

DISCUSSION

In analyzing the practitioners' problem solving at work, one thing was clear namely, the fact that the participants utilized common heuristics-in-action in their understanding of simple proportional relationships, a model which Vergnaud terms "the isomorphism of measures model of situations". When using *building-up* heuristic, practitioners maintained the proportionality between the values by carrying parallel transformation on the variables without dividing or multiplying values in one variable by values in the other variable. It is worth mentioning here that, the rule-of-three, the algorithm learned in school to solve simple proportional problems differs from the isomorphism schema because it involves the multiplication of values across variables instead of parallel transformation on the variables. Hence, practitioners employed concepts that challenges the rule-of-three algorithm taught in school today clearly preferring the use of multiplicative heuristic due to its strong ties to problem situations, which led to functional solutions.

It seems fair to conclude that street vendors and bus conductors have developed mathematical concepts as a result of immersion experiences in everyday situations. The work setting represented vendors and bus conductors' natural habitat and thus introduced familiar problems. As a result, practitioners systematically and smoothly built up their solutions using intuitive computational strategies and without losing track of the strategy, even if many numbers are involved. In other words, the practitioners kept the meaning of the problem in mind during problem solving. This understanding that the practitioners acquired in the work situation elicited a coherent problem solving behavior that was attained through the following steps: (a) translating the problem from its real life context into an appropriate mathematical calculation problem, (b) performing the mathematical calculations, and (c) translating the result of this calculation back into the context of the problem to see whether it made sense.

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