ALGEBRA FOR ALL: THE HIDDEN COST

Tracy Carolan

University of Wisconsin, Madison

PISA 2012 results indicate that school systems that group students based on ability levels tend to have lower performance than those that do not divide students by ability. One way some in the United States have sought to increase equity of opportunity is to mandate enrollment of students in college-preparatory mathematics, i.e., Algebra 1 in eighth or ninth grade. This paper is based on a study conducted on one such curricular change. It uses a multiple linear regression model to compare two graduating class cohorts—one from before the initiative and one after—on test scores, courses completed, grades, and drop-out rates. There were positive gains for select groups of males and negative results for most females with the highest losses found for White females, especially those qualifying for special education services.

BACKGROUND

Countries around the world vary in their approach to mathematics education. Some have a highly stratified system, sorting students from an early age, while others delay sorting students until their last two years of schooling, if at all. PISA 2012 results indicate that countries with systems that group students according to their ability tend to have lower performance than those that do not. Across countries, students in schools that do not use ability grouping on average outperform students in schools that do. Furthermore, ability grouping or "tracking" has a disproportionate impact on students of lower socio-economic status (SES), and that impact is greater the earlier the age at which students are divided according to ability (OECD, 2013).

In the United States, ability grouping in mathematics is often firmly established by grade eight, around age 13, with results similar to those found in PISA 2012: lower levels of mathematical achievement—particularly for students from historically marginalized or economically disadvantaged groups. Many scholars in the U.S. have called for increased access to college-preparatory mathematics curricula at grade eight or nine (age 13 or 14), especially for historically marginalized or disadvantaged groups, as a way to increase equity of opportunity (e.g. Pelavin & Kane, 1990; Silva, Moses, Rivers, & Johnson, 1990; Smith, 1996; U.S. Department of Education, 1997). Initiatives aimed at this grade level are often called *Algebra for All* initiatives because Algebra 1 is the course students generally enroll in at age 13 or 14 if they are to complete a college-preparatory mathematics course sequence by age 18.

This study examines one such initiative in an economically and ethnically diverse school district in the Midwestern United States: the Madison (WI) Metropolitan School District's (MMSD) *Algebra for Everyone* initiative. Analogous to results seen internationally, students of color were under-represented in the district's higher-level

2014. In Nicol, C., Liljedahl, P., Oesterle, S., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 2, pp. 241-248. Vancouver, Canada: PME.

mathematics classes and over-represented in basic and vocational mathematics classes, thus denying many students of color the opportunity to apply to and attend college due to inadequate high school mathematics courses. The MMSD identified institutional and systemic racism as a large contributor to this situation and decided to discontinue using staff recommendations for students' mathematics class placements and instead place all students in a college-preparatory mathematics track.

In 2003 non-college-preparatory mathematics classes at the high schools such as Pre-Algebra and Consumer Math were discontinued district-wide and all students were required to enroll in an Algebra 1 or higher-level mathematics class by grade nine. The only students who had an option for enrollment in non-college-preparatory mathematics were Special Education students who planned to apply for an exception based on their diagnosed disabilities (graduating via Individualized Education Plan (IEP)).

By 2004 the *Algebra for Everyone* initiative was in full swing and a disturbing new trend was appearing in the Algebra 1 classes: higher and higher failure rates were observed across all sections and for all teachers. By 2007, failure rates in Algebra 1 had skyrocketed to 40% (from an average of just 10% in 2000) with some Algebra 1 classes having 65% of students failing.

Anecdotally, students of color seemed to be more likely to fail than White students, Special Education students (those with diagnosed cognitive and/or emotional disabilities) seemed to be more likely to fail than those not qualifying for Special Education, and the students who struggled the most in Algebra 1 seemed to have very low middle-school mathematics achievement. It was extremely disheartening for classroom teachers to both literally and figuratively fail so many students.

When examining results for the school district as a whole, the policy of largely eliminating ability grouping in ninth grade seemed to be a success. More students were completing a college-preparatory mathematics course sequence than had ever before, and yet there was this seemingly contradictory anecdotal evidence that the policy was actually *lowering* achievement for many students. Was it just that these students were struggling at first but were able to recover and catch up, or was it that the positive effects on some students masked the negative effects on others when outcomes were aggregated? This study was conceived in order to explore the effect of eliminating ability grouping via the *Algebra for Everyone* initiative on students in the MMSD and whether that effect differed at all depending on a student's demographic group.

METHODS

Data Source

I used transcript and demographic data from two graduating class cohorts: the last cohort in the MMSD whose students were able to enroll in classes below Algebra 1 and the cohort entering high school soon after the implementation of the *Algebra for Everyone* initiative. The first cohort (Cohort A) entered ninth grade in 2000 and was the last cohort for whom Pre-Algebra was still an option at all four high schools in the district. Cohort B entered high school in 2004, which was the second year in which Algebra 1 was the lowest-level math class offered at all four high schools. I chose the second year of the *Algebra for Everyone* initiative to avoid as much as possible any effects of the adjustment period on student achievement.

The raw data I received contained transcript data for grades 8 through 12 for 4,440 students in the MMSD who were either enrolled in ninth grade in the fall of 2000 or the fall of 2004. Students who were not first-time ninth-graders in either 2000 or 2004 were excluded from the study. After these exclusions, there remained 2,019 students in Cohort A and 2,006 students in Cohort B.

Eighth-Grade (Incoming Ninth-Grade) Achievement

Using independent-samples *T* tests to compare the means of Cohort A and Cohort B, I discovered that there were not significant differences ($p \leq 0.05$) in mean eighth-grade achievement between the two groups either overall, or when divided into each of the eight main demographic subgroups (Asian males and females, Black males and females, Hispanic males and females, White males and females), in terms of the number of eighth-grade mathematics credits earned or the eighth-grade mathematics grade point average (GPA).

I also compared scores from the state-wide standardized test given in eighth grade: the Wisconsin Knowledge and Concepts Examination (WKCE). Unfortunately, I was unable to conclusively compare eighth-grade WKCE scores from Cohort A to Cohort B because of changes to the WKCE test which occurred in 2002 (WI DPI, 2003), but the change in the MMSD's 8^{th} grade scores from Cohort A to Cohort B closely resembles the changes seen across those years state-wide. When this result is paired with the favorable comparison of the measures of 8^{th} grade mathematics GPA and 8th grade mathematics credits earned, it gives confidence that students in the two cohorts entered high school with essentially the same prior achievement.

Criteria on Which Cohorts Were Compared

I then set out to measure whether any of the positive effects desired by proponents of *Algebra for All* initiatives, as well as any possible negative effects, were realized during the implementation in the MMSD. I compared the two cohorts on measures of student achievement chosen to address specific claims found in the literature (see Table 1).

Measure(s) of student achievement	Notes on how data were recorded
Level of initial high school mathematics class enrollment. Level of highest mathematics class taken in high school for which credit was received.	1 = Special Ed or Pre-Algebra (non-college prep mathematics) 2 = Algebra 1 3 = Geometry 4 = Algebra 2 5 = Algebra 3, Pre-Calc, or AP Stats 6 = Calc AB or higher.
GPA in high school mathematics classes and overall high school GPA.	GPA was unweighted and on a four-point scale.
Overall ACT scores and ACT mathematics sub-scores.	If there was more than one score, the highest one was used.
Number of mathematics credits earned in high school.	Programming classes were not included in the total.
Drop-out rate.	

Table 1: Measures of student achievement.

Statistical Methods and Justification

I used a standard multiple linear regression model because it allowed me to better isolate the effects of the *Algebra for Everyone* initiative from other known variables, such as gender or socio-economic status. For example, if a particular subgroup had an increase in drop-out rates from Cohort A to Cohort B, it may be due to the initiative, but it could also be due to an increase in the proportion of students in that group with low socio-economic status. Multiple linear regression calculates the magnitude of change we can expect to see in the dependent variable due to each predictor (independent variable) and create a model which quantifies this change.

Regression models for all variables have coefficients for the following predictors where possible: Cohort, Gender, each of the races/ethnicities except for White, Special Education status, English Language Learner status, and Socio-Economic status. I translated the demographic data into dummy codes of 0 and 1 so as to be able to use them as predictors in linear regression models for each measure in Table 1. Because the drop-out variable took only values of yes (1) or no (0), I used a binary logistic regression model to analyze this change.

The focus of this study was the coefficient for Cohort, which represents the amount of change from Cohort A to Cohort B for a given variable that may be attributed to the *Algebra for Everyone* initiative.

This study was conceived primarily out of concern that the *Algebra for Everyone* initiative was having a differing effect on certain demographic subgroups versus others. This interest necessitated that the analysis not stop at simply calculating results for the MMSD as a whole, males vs. females, or even the eight main demographic subgroups. In all, I calculated regression equations for approximately 150 different demographic subgroups: for example, one of the subgroups was the group of White Female Low-SES Special-Ed students. At first glance, this seems like a great deal of unnecessary calculations, but the fine grain size proved to be pivotal in terms of attaining useful results. Many variables did not show significant differences for the larger demographic group but differences became significant as the group was subdivided.

The fine grain size allowed this study to answer, in a way that would not have been possible otherwise, the question of whether the *Algebra for Everyone* initiative had divergent effects on different demographic groups.

RESULTS

Positive Results: Increased Achievement for Select Groups of Males

As hoped, the *Algebra for Everyone* initiative did increase the mathematics achievement in the MMSD of some historically marginalized and/or disadvantaged groups, including Asian and White males of low socio-economic status, Black males who were not of low socio-economic status and were not receiving Special Education services, and Hispanic males who were not classified as English language learners. For these groups, the initiative yielded:

- An increase in the number of credits earned in mathematics classes.
- An increase in the level of the highest mathematics class.
- An increase in the mathematics GPA and the overall GPA.
- Higher college entrance examination scores (measured here by ACT test scores) and more students taking college entrance examinations.
- A decreased or stable drop-out rate.

These are encouraging results because they show that the theory behind an *Algebra for All* initiative is sound: many more students than previously thought are ready for college-preparatory mathematics and when given the opportunity to enroll they will rise to meet the challenge.

Negative Results: Decreased Achievement for Females and Vulnerable Males

Unfortunately, other demographic groups in the MMSD did not fare as well, suffering large losses in academic achievement after implementation. Sadly, these were some of the very groups the initiative was designed to empower, including Black male and female Special Education students, Black males who were of low-socio-economic status, Hispanic females, Hispanic males who were English language learners, White females (especially those eligible for Special Education services), and Special

Education students of all races and genders. For these groups, the consequences of the initiative included:

- Fewer credits earned in mathematics classes.
- A reduction in the level of the highest mathematics class.
- Lower mathematics grade point average (GPA) and lower overall GPA.
- Lower college entrance examination scores (measured here by ACT test scores) scores and fewer students taking the college entrance examination.
- An increased drop-out rate.

Any groups of students not named here showed mixed results, with the exception of Asian females for whom there was inconclusive evidence of either a positive or negative overall effect.

DISCUSSION

These results would show that *Algebra for Everyone* had positive effects on many students in the district, opening up the doors to college to many who would not otherwise have considered it. However, it did this while closing the doors to a traditional high school diploma for many others and leading still others to elect minimal mathematics preparation—the opposite of what was intended.

Teacher Expectations and Student Achievement

When examining the list of students for whom the *Algebra for Everyone* initiative met its goals, the salient feature is the gender they all have in common: male. These results could be an example of what Rosenthal and Jacobsen (1968) termed the "Pygmalion effect" in which teacher expectations of student learning become reality. Males are traditionally viewed as being better at mathematics and, given that their SES may not be readily apparent, for those not receiving Special Education or English Language Learner services there would have been no reason for a teacher or their classmates to expect them not to do well.

Correspondingly, the second list contains students from demographic groups society has historically deemed more likely to struggle or fail in mathematics classes: females, students of color with low socio-economic status, English language learners, and students with diagnosed cognitive or emotional difficulties that qualify them for Special Education services. In these students' cases, disliking mathematics or struggling to do well in it might be seen as common and/or not unexpected and therefore would not be cause for alarm.

Mathematical Identity

Ma's (2003) research on the acceleration of regular students also may apply here. Ma found that when regular students are accelerated (defined as students who score at the 65th percentile or lower, taking Algebra I in seventh or eighth grade), their attitude toward mathematics declines more quickly than their peers who were not accelerated and their anxiety increases at a higher rate than their regular peers who were not accelerated. Ma was unable to find any student-level or school-level factors that could reliably predict this attitude decrease or increase in anxiety level. Using previous research on attitudes and how they relate to learning, Ma came to the conclusion that the negative effects are due to regular students being overwhelmed by the demands of the higher-level class.

Students who were enrolled in a grade-level class when they would otherwise have enrolled in a below-grade-level class may have an experience similar to a regular student who was accelerated to an above-grade-level class. Students who had lower prior academic achievement may have been more susceptible to feeling discouraged and overwhelmed, leading to the increased dropout rates and a loss of the lower-achieving students from the group of students taking the ACT.

Another influence on how students experience mathematics classes is how they perceive themselves to perform as compared to their peers. Correll (2001) determined that students' self-assessment of their mathematical ability is done in reference simply to others in their daily classes, not in reference to the entire grade-level or student body. Prior to *Algebra for Everyone*, lower-achieving students would have been placed in a Pre-Algebra or lower class where they could have excelled relative to others in their class. Post *Algebra for Everyone*, these same students were placed in a more difficult Algebra 1 class with students with stronger prior achievement. The lower grades achieved in Algebra 1 vs. Pre-Algebra and their lower performance relative to their classmates may have affected students' views of their mathematical abilities correspondingly.

Individual Agency

A third possible explanation is that the *Algebra for Everyone* initiative inadvertently changed the cost/benefit ratio of pursuing higher mathematics and/or a high school diploma. Correll (2001) found that girls who were strong in both English and mathematics were less likely to elect to enroll in Calculus (the most advanced mathematics course offered at a typical U.S. high school) than girls who were also strong in mathematics but not in English. In a sense, many girls who stayed with mathematics may have done so not because they loved mathematics but because they had no other viable alternatives.

The groups with the greatest negative effects from the *Algebra for Everyone* initiative could perhaps be those for whom another option besides continuing with mathematics was readily available. This may have taken the form of enrolling in more history or English classes or, for those students who also struggle in the other disciplines such as many of the Special Ed students, it could have meant dropping out.

Increasing equity of opportunity without harming vulnerable students

Of course, the theories posited above are simplifications of the complex reality which influences students' choices, but all seem to point to *Algebra for Everyone* not as the cause of the results we see here, but rather as a trigger for amplification of already-existing trends and dynamics.

It would appear that school systems that seek to eliminate ability grouping may unknowingly wield a double-edged sword, and further research is needed to paint a clearer picture of the dynamics involved and the optimal solutions. In principle, a policy designed to increase equity of opportunity, such as an *Algebra for All* initiative, would function only to place underestimated students in classes that were more appropriate, thereby unlocking their heretofore untapped potential. However, this study suggests that this result was achieved for only a fraction of students and that the success of these students was attained only at the cost of their peers' achievement.

This study would suggest that eliminating formal ability grouping is but one factor in increasing student achievement. Another important factor in student achievement is how students incorporate cultural beliefs about mathematics into their identities, and it is one that will be much more challenging for schools to address.

References

Correll, S. J. (2001). Gender and the career choice process: The role of biased self-assessments. *American Journal of Sociology*, *106*(6), 1691-1730.

Ma, X. (2003). Effects of early acceleration of students in mathematics on attitudes toward mathematics and mathematics anxiety. *Teachers College Record*, *105*(3), 438-464.

OECD. (2013). *PISA 2012 results: What makes schools successful? Resources, policies and practices* (Vol. 4). PISA, OECD Publishing. Retrieved from http://dx.doi.org/10.1787/9789264201156-en

Pelavin, S. H., & Kane, M. (1990). *Changing the odds: Factors increasing access to college*. New York: The College Board.

Rosenthal, R., & Jacobson, L. (1968). Pygmalion in the classroom. *The Urban Review*, *3*(1), 16-20.

Silva, C. M., Moses, R. P., Rivers, J., & Johnson, P. (1990). The Algebra Project: Making middle school mathematics count. *The Journal of Negro Education*, *59*(3), 375-391.

Smith, J. B. (1996). Does an extra year make any difference? The impact of early access to algebra on long-term gains in mathematics attainment. *Educational Evaluation and Policy Analysis*, *18*(2), 141-153.

U.S. Department of Education. (1997). *Mathematics equals opportunity: White paper prepared for U.S. Secretary of Education Richard W. Riley*. Washington, D.C.: Author.

Wisconsin Department of Public Instruction. (2003). *Questions and answers regarding the new 2002-03 WKCE proficiency levels.* Retrieved from the Wisconsin Department of Public Instruction website: http://dpi.wi.gov/files/oea/pdf/profnewq_a.pdf