# ALGEBRA-RELATED TASKS IN PRIMARY SCHOOL TEXTBOOKS

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*Even though there is growing consensus for engaging primary students with early algebraic ideas, there is limited research knowledge about the relevant learning opportunities designed in textbooks. Textbooks are considered to play an important role in what is happening in classrooms, especially in educational contexts where classroom instruction relies heavily on textbooks. An analytic framework was developed to identify the opportunities designed in textbooks for engaging students with algebra-related tasks and to examine the respective guidance (or lack thereof) in the accompanying teacher guidebooks. The framework was used to analyse a primary textbook series for grades 4-6 and relevant findings are presented. Implications for textbook design, research, and practice are discussed in light of these findings.*

## INTRODUCTION

Algebra is seen as a gateway to higher mathematics, and both researchers and curriculum frameworks recommend that primary students should be offered learning opportunities that can prepare them for formal algebra learning at secondary school (e.g. Carpenter, Franke & Levi, 2003; NCTM, 2000; Stacey, Chick & Kendal, 2004). Even though the issue of which specific algebra-related topics are appropriate for primary students is not settled in the educational community, there is extensive reference to generalised arithmetic (e.g. Kaput, 2008), patterns and functions (e.g. NCTM, 2000), and problem solving and modelling (e.g. Kieran, 2004).

While intervention studies showed that it is possible for primary students to successfully engage with early algebraic ideas (e.g. Carpenter et al., 2003), little is known about what is happening in ordinary primary classrooms in terms of algebra teaching and learning. Textbooks offer a way to investigate this issue, as they can influence what and how mathematics is taught and thus students' learning experiences (Tarr, Chávez, Reys, & Reys, 2006). Indeed, the TIMSS 2007 study showed that, on average, 65% of the fourth grade teachers from the participating countries used textbooks as a primary basis for their lessons while 30% used textbooks as a supplementary resource (Mullis et al., 2008).

Only few studies have investigated how textbooks promote algebra-related topics. A comparative study analysed how algebraic concepts are introduced and developed in five primary curricula: the Chinese, South Korean, and Singaporean curricula, and selected Russian and U.S. curricula (Cai, Lew, Morris, Moyer, Ng & Schmittau, 2005). They found that the main goal in all curricula was to deepen students' understanding of quantitative relationships but the emphasis and approaches in achieving this goal

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differed across curricula. Also, Hodgen, Küchemann and Brown (2010) analysed two textbooks in widespread use in lower secondary schools in England (one currently used and one from the 1970s) focusing on linear relations. They found that none of the textbooks exploited available research knowledge on the teaching and learning of algebra. Both of these studies examined the selected textbooks across some general dimensions such as their goals for algebra-related learning and their content organisation and coverage. Another approach would be to use a systematic unit of analysis such as the textbook task (Stylianides, 2009), and to examine all the textbook tasks in regard to their algebra-related content (or lack thereof). Such an examination could offer a good basis for inferences about the kind of learning opportunities offered to students in classrooms, especially in educational contexts whose teachers tend to follow closely the textbooks.

Primary school teachers tend to recognise algebra-related tasks by the existence of letter symbolism or symbol manipulation (Stephens, 2008). Yet, this conception of algebra does not reflect the breadth of algebra-related topics currently mentioned in the literature. Considering that teachers' use of textbooks depends not only on the opportunities designed in textbooks but also on teachers' interpretations of these opportunities, it is important to explore also whether the accompanying teacher guidebooks offer some support to teachers to understand or appreciate the learning potential of algebra-related tasks in the textbooks. Indeed, the value of curriculum materials that aim to promote teacher learning alongside student learning is well elaborated in the literature (e.g. Ball & Cohen, 1996; Davis & Krajcik, 2005).

This paper presents an analytic framework for investigating algebra-related tasks in primary school textbooks and the respective guidance (or lack thereof) in the accompanying teacher guidebooks. The framework was used to analyse the textbooks for the fourth, fifth and sixth grades in the Cypriot educational context. In this context there is a unique textbook series that is used in all state schools and teachers rely heavily on textbooks to plan and enact their teaching (Kyriakides, 1996). These two characteristics of the Cypriot educational context elevate the importance of a textbook analysis, as such an analysis can offer a good insight into the learning opportunities offered to students in Cypriot primary classrooms.

## ANALYTIC FRAMEWORK

The development of the framework involved four stages. First, it was decided that the unit of analysis would be the textbook task, which is taken to be the smallest unit identified by a separate marker in a textbook page (Stylianides, 2009). Second, it was decided that algebra-related tasks would not be limited to tasks that involved the use of letters; this is because letter symbolism is considered to be neither a necessary nor a sufficient condition for algebraic thinking (Radford, 2010). Third, three categories of algebra-related tasks were originally identified by synthesising key definitions of algebra from the literature (Bednarz, Kieran & Lee, 1996; Kaput, 2008; Kieran, 2004;

NCTM, 2000) and were then refined after piloting. Finally, two codes were selected that would provide evidence about the respective guidance in teachers' guidebooks.

Algebra-related tasks were grouped into the following three categories according to the relations between numbers and quantities in the tasks: arithmetically-situated relations, rule-based relations and known-unknown relations. *Arithmetically-situated relations* tasks focus on the structure of arithmetic by attending to the behaviour of arithmetic operations and properties as mathematical objects and why they work. Also, these tasks could engage students in generalising these relations. This category of tasks corresponds to what is referred to in the literature as generalised arithmetic (Carpenter et al., 2003; Kaput, 2008). An example is a task that asks students to form a general expression for the commutative property of addition.

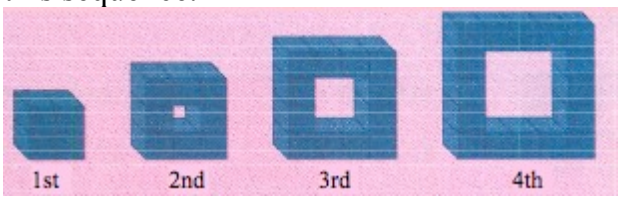
*Rule-based relations* tasks focus on the relations within a dataset or between datasets. These tasks could engage students in forming a rule that applies for all the elements of the datasets, testing plausible rules, extending a rule to nearby and far away cases and generalising a rule. Also, these tasks could provide opportunities for working with equivalent representations of the same rule (e.g. verbal and algebraic expressions). An example is a task that asks students to generalise verbally the functional rule of a growing geometric pattern. This category of tasks relates with the study of patterns, functions, change and variation (Kaput, 2008; NCTM, 2000). The generalisation perspective on the introduction to algebra (Bednarz et al., 1996) includes topics that engage students in generalising activities such as numeric or geometric patterns which would belong to rule-based relations tasks according to this categorisation of tasks, and laws governing numbers which would belong to arithmetically-situated relations tasks.

*Known-unknown relations* tasks focus on the relations between known and unknown quantities and numbers, and treat unknowns as objects (entities that stand on their own) rather than as processes. The nature of the relations range from simple direct relations to complex non-direct relations (i.e., relations for which there is no direct bridge between known and unknown). An example is the following story problem: 'A farm has chickens and rabbits. We counted the heads and we found 27. We counted the feet and we found 78. How many are the chickens and how many are the rabbits?' This category of tasks draws on the description of algebra as a cluster of modelling languages (Kaput, 2008) and the problem solving approach on the introduction to algebra (Bednarz et al., 1996). The potential of forming expressions and equations during engagement with the three categories of algebra-related tasks aligns with the purpose of generational activities as defined by Kieran (2004) (i.e., forming general expressions that arise from patterns and numerical relationships, and equations that represent problem situations).

In investigating the guidance provided in the teachers' guidebooks regarding the role of algebra-related tasks, it was examined whether these tasks were explicitly or non-explicitly identified. The code *explicitly identified* algebra-related task was used when there was an explicit reference to the task's relationship with an algebraic idea as signified by the presence of at least one of the following key words in the commentary

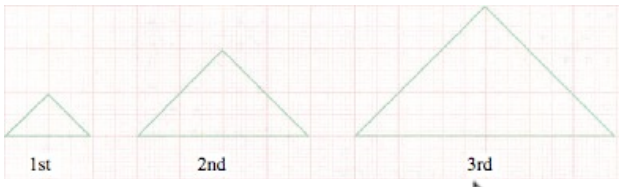
for the task: algebraic symbols/thought/representations/equations, verbal/symbolic/algebraic generalisation, finding the rule/formula, general numbers, investigating relations between numbers/quantities, patterns, functions, arithmetic properties and relations, forming and solving equations, finding the unknown, problem-solving. The code *non-explicitly identified* algebra-related task was used when there was no such relevant key word or commentary in the teachers' guidebook. For example, the two algebra-related tasks below (Figure 1) were both coded as rule-based relations tasks since they can engage students in extending the pattern to a far away case, but only Task 1 was coded as explicitly identified due to the presence of the key word 'identifying pattern' in the commentary for the task in the teachers' guidebook.

1. Observe the figures and find the number of cubes that will compose the 25th figure of this sequence.




(MEC Grade 5 Volume D, 1999, p.43)

2. Draw the fourth figure and complete the table.




| Figure Series | 1st | 2nd | 3rd | 4th | 5th | | 10th | 20th |
|---|---|---|---|---|---|---|---|---|
| Length of base (cm) | | | | | | | | |
| Area (cm $^2$) | | | | | | | | |

(MEC Grade 4 Volume C, 1998, p.80)

Figure 1: Algebra-related tasks in students' textbooks

Inter-rater reliability was tested by comparing the coding of the primary rater (first author) with the codes of a second rater, who coded a subsample of 25% of the tasks in the textbooks of grades 4, 5 and 6. Two reliability values were calculated. The first reliability value concerned the decisions on whether or not a task in the subsample was algebra-related. The inter-rater agreement was kappa=0.82. The second reliability value concerned the decisions on assigning algebra-related tasks to the three categories described earlier. The second inter-rater agreement was kappa=0.84.

## FINDINGS AND DISCUSSION

The framework was applied to the textbook series used in the Cypriot educational context in the three upper grades of state primary schools. It was found that 10.7% of the total number of tasks in the textbooks for the fourth, fifth and sixth grades (N=2814) were algebra-related. The specific percentages for grades 4, 5 and 6 were 10.7%, 9.2%, and 16.7%, respectively.

These findings suggest that algebra-related tasks seem to become more frequent in the sixth grade textbooks. Also, 43.4% of the total identified algebra-related tasks were

found in this grade (see Table 1). This is possibly because grade 6 is the last one before secondary school and students' preparation for algebra gets more priority then than in the previous grades, which may focus more on the development of students' fluency with arithmetic calculations. Another hypothesis is that sixth grade students might be considered more developmentally ready to engage with algebra-related tasks than younger students. However, this hypothesis is inconsistent with the observed decrease of algebra-related tasks from fourth to fifth grade (as shown in Table 1). Of course this inconsistency may be a byproduct of the specific definition used in this study to identify algebra-related tasks, which may differ from the textbook authors' (working) definition of these tasks.

| **Categories of algebra-related tasks** | **Fourth grade (n=90, 29.8%)** | **Fifth grade (n=81, 26.8%)** | **Sixth grade (n=131, 43.4%)** |
|---|---|---|---|
| Arithmetically-situated relations (n=36, 11.9%) | 15.6 | 7.4 | 12.2 |
| Rule-based relations (n=128, 42.4%) | 28.9 | 45.7 | 49.6 |
| Known-unknown relations (n=138, 45.7%) | 55.5 | 46.9 | 38.2 |
| Total | 100 | 100 | 100 |

Table 1: Distribution by percent of the three categories of algebra-related tasks across grades

Opportunities for engaging with rule-based relations (42.4%) and known-unknown relations (45.7%) prevail in this textbook series. This possibly reflects a systemic view on mathematical development that focuses on these two categories of tasks while fewer opportunities seem to be designed for arithmetically-situated relations (11.9%). Table 1 shows further that opportunities for engaging with arithmetically-situated relations are more frequent in the fourth and sixth grades than in the fifth grade. Tasks that involve rule-based relations are increasingly more frequent while those that involve known-unknown relations decrease in frequency from grade four to grade six. This indicates that students have more opportunities to construct first notions of algebra relevant to known-unknown relations while they have more opportunities to develop ideas relevant to rule-based relations in the last two grades of primary school. Opportunities for students to engage with known-unknown relations and rule-based relations are desirable because they are likely to help students recognise the limitations of arithmetic problem-solving approaches and start familiarising themselves with making generalisations.

The low percentage of arithmetically-situated relations tasks indicates that there are relatively limited opportunities for students to attend to the structure of arithmetic and how that relates with algebra. Linchevski and Livneh (1999) mentioned that students' difficulties with the structural properties of the algebraic system originate in a limited understanding of the number system. This difficulty is partly attributed to the lack of attention given to students' awareness of the mathematical structure and of arithmetic operations as general processes during the learning of arithmetic (Booth, 1984).

Table 2 shows that 84.1% of the tasks categorised as 'algebra-related' in this investigation were explicitly identified as such in the teachers' guidebooks while the remaining 15.9% were not. For the non-explicitly identified tasks, there were no relevant key words or commentary in the teachers' guidebooks. It is unknown if these tasks were actually intended by textbook authors to engage students with algebra-related topics, but in any case this lack of clarity leaves space for teachers to interpret in different ways the role of these tasks in the textbooks.

| **Guidance in the teachers' guidebooks for algebra-related tasks** | **Fourth grade** | **Fifth grade** | **Sixth grade** |
|---|---|---|---|
| Explicitly identified (n=254, 84.1%) | 66.7 | 88.9 | 93.1 |
| Non-explicitly identified (n=48, 15.9%) | 33.3 | 11.1 | 6.9 |
| Total | 100 | 100 | 100 |

Table 2: Distribution by percent of explicitly and non-explicitly identified algebra-related tasks across grades

The number of explicitly identified algebra-related tasks increases from fourth to sixth grade. This indicates that the available guidance in teachers' guidebooks differs across grades. Also, the opportunities designed for students seem to become more explicit for the fifth and sixth grade teachers than for the fourth grade teachers. For the non-explicitly identified algebra-related tasks, the lack of key words in the teachers' guidebooks seems to hinder the role that these tasks can serve in the curriculum. Referring back to the two textbook tasks presented above, the tasks seem to promote similar algebra-related learning goals since they can engage students in extending the geometric pattern to far cases. Yet, the algebra-related goal of Task 2 was not explicitly stated in the teachers' guidebook for grade 4 and this may obscure the relationship between this task and other similar tasks in the curriculum, such as Task 1, which students would encounter in grade 5.

## CONCLUDING REMARKS

The distribution of tasks across the three categories of algebra-related tasks raises questions about the possible implications of designing limited opportunities for students to engage with one kind of tasks, in this case the arithmetically-situated

relations tasks. Fewer opportunities are designed for students to understand that the same underlying properties of arithmetic are applied to algebra and more opportunities are designed for them to solve algebraic problems, identify and generalise quantitative relations.

The findings raise also questions about what might be the essential support in teachers' guidebooks. Given the fact that algebra has traditionally been considered a mathematical topic for secondary school, in cases where algebra-related tasks in primary textbooks are non-explicitly identified, there is a danger that the potential of these tasks to engage students with early algebraic ideas will not be fulfilled. Teachers' approaches to tasks are underlain by the different ways they read the textbooks, which in turn are influenced by their beliefs about teaching and their expectations of students' learning (Remillard, 1999). Therefore, by not providing explicit information about the role of these tasks, textbooks allow further space for disparate interpretations among teachers and thus more variability in the opportunities that teachers offer to students to engage with algebra-related topics.

One could argue that it does not matter whether or not these tasks are explicitly identified in the teachers' guidebooks as long as teachers encourage students' engagement with algebraic ideas. However, research suggests that primary teachers have rather narrow conceptions about algebra-related tasks (Stephens, 2008), and this raises concerns about the implementation of non-explicitly identified algebra-related tasks in primary school classrooms. Further research is needed to explore primary teachers' interpretations and enactment of different kinds of algebra-related tasks.

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