COLLECTIVE PROBLEM POSING AS AN EMERGENT PHENOMENON IN MIDDLE SCHOOL MATHEMATICS GROUP DISCOURSE

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This naturalistic case study investigates the problem posing patterns that emerge as four small groups of 12 year old students in Western Canada work collectively on a structured mathematics task. A method of data analysis is introduced that blurs the data to create transcript "tapestries" providing visual evidence of collective patterns of posed problems that emerge over time. Results in progress suggest that groups vary widely in terms of the problems posed, and in terms of the patterns in which these problems emerge in their discourse. The reposing of problems helps to structure each group's discussion, with the role that each problem plays in the conversation evolving as it reemerges in the discourse.

INTRODUCTION

Problem posing has been defined as "the creation of questions in a mathematical context and… the reformulation, for solution, of ill structured existing problems" (Pirie, 2002). Working from this definition, one might argue that there are two kinds of problem posing, depending on the purpose of the problem being posed (Silver, 1994), and where it occurs in relation to the problem solving process. In the first half of the definition, a new problem is generated from a situation, a problem, or an experience. In the second half of the definition – the "How can I (re)formulate this problem so that it can be solved?" type – a related problem is generated in response to the original problem, as a way of making that original problem more accessible. This study focuses on this second kind of problem posing, describing the behavior of small groups in a mathematics classroom who pose their own problems in the process of solving an assigned problem task. My research question is: What problem posing patterns emerge as small groups of students work collectively on a mathematics task?

THEORETICAL FRAMEWORK

The current National Council of Teachers of Mathematics' Standards document (2000) notes that problem posing is an important component of problem solving, recognizing it as an indication of a "mathematical disposition." Students can be supported as they move from a novice level to an expert level through various forms of instructor intervention ranging from introductory activities to specific problem posing strategies (Bonotto, 2013; Singer, 2009; Singer & Mascovici, 2008) to participating in problem posing (and solving) programs (Brown & Walter, 2005; Crespo, 2003; Crespo & Sinclair, 2008; English, 1997, 1998; Leung, 1993; Pirie, 2002).

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Many studies of problem posing rely on their subjects' written work, a static product, as the focus of analysis. While this has the advantage of allowing researchers the ability to draw on a large pool of subjects, it also has the effect of (appropriately enough) triggering yet more questions about the research itself. In an excellent discussion of the results of one such study (Silver & Cai, 1996), the researchers wondered if middle school students only recorded problems they knew they could solve; perhaps they were able to generate more complex questions, but hesitated to write them down because they were not able to solve them. The researchers also questioned the trend of simpler questions being posed before the more complex ones were. Perhaps the subjects originally had the more complex question in mind first but decided to record the simpler questions at the beginning of their written responses. All of this points to problem posing being difficult, and perhaps simply inappropriate, to capture with an end product consisting of a written list of problems.

Some argue that group work has the potential to provide a safe structure for building problem posing competence (Kilpatrick, 1987; Silver & Marshall, 1989), and offers the opportunity for students to work together less competitively and more productively (Brown & Walter, 2005). Yet, despite these and similar recommendations (English, 1997; Lester, 1994; Silver, 1994; Silver, Mamona-Downs, Leung, & Kenney, 1996), there is little in the literature about how problem posing works on a collective level.

Little documentation exists about the group itself as a learner, how its understanding unfolds (Martin, Towers, & Pirie, 2006), and how it thinks. Although in casual conversation, a teacher might refer to what a certain group thinks or, for example, describe the personality of the class in period three (Bowers & Nickerson, 2001), it can be difficult for researchers to conceptualize the group as a unit of analysis, even a small group. Thus, studies of small groups have often tended to focus on how working within the group affects the learning of the individuals within the group rather than on the group itself (Stahl, 2006). The concept of group learning is "a difficult, counter-intuitive way of thinking for many people" (Stahl, 2006, p. 16) due to the strong association of cognition with an individual psychological process.

There is a benefit for the researcher who studies groups: the group's discourse may be considered to represent its thinking (Stahl, 2006). However, the discourse cannot "be analyzed by solely considering a sequence of statements that are made''(Yackel, 2002, p. 424). One might even argue that the individual pathways of growth of understanding within the collaboration do not exist at all (Martin et al., 2006). An utterance is linked to the past in that it is a response to another utterance, or utterances. An utterance is also a response to what has been, or what is currently, happening and the utterance is connected to the future, in that it is formed in anticipation of an impending utterance. The "conversation" of a group "is crisscrossed by other places and temporalities, by absent third parties, who may express their voice through the participants' discourse" (Grossen, 2009, p. 266) and also by the uptake and reuptake of individual threads of ideas. One might envision the utterance as a part of a tapestry that comes from the past and stretches into the future, an idea I will connect to in my methodology.

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METHODOLOGY

The research took place at a middle school (ages 10 – 13 years) in a large suburban school district in British Columbia. Sixteen students from each of two classes of 30 12 year old students (i.e. just over half) participated in the study for a total of 32 students. The groups were composed of students who were all working at grade level but who had mixed levels of ability in mathematics. The study occurred in the spring of the school year, with session tapings taking place roughly every two weeks depending on the school schedule, for a total of five sessions for each class, with each session lasting approximately 40 minutes. As I was using a grounded theory approach (Glaser & Strauss, 1967), I selected groups "for their ability to contribute to the developing/emergent theory" (Miles & Huberman, 1994, p. p. 28) – namely those who were working collectively on the tasks. Participating groups were videotaped by stationary cameras and also audiotaped. I took field notes throughout the sessions from a location at the back of the classroom, and compared these notes to the video and audio recordings to clarify events captured in the tapings. Other data sources included the task sheets where group members recorded their work and solutions, and the class whiteboard where some groups chose to write their ideas while presenting their solutions to the rest of the class. I refer to the groups through the acronyms JJKK, REGL, NIJM and DATM.

The task that is the focus of this case study reads as follows:

> **The Bill Nye Fan Club Party**
>
> The Bill Nye Fan Club is having a year-end party, which features wearing lab coats and safety glasses, watching videos and singing loudly, and making things explode. As well, members of the club bring presents to give to the other members of the club. Every club member brings the same number of gifts to the party. If the presents are opened in 5 minute intervals, starting at 1:00 pm, the last gift will be opened starting at 5:35 pm. How many club members are there?

DATA ANALYSIS

As this study involves elaborating upon and building theory about problem posing as a process, I analyzed the data using a constant comparison method (Glaser & Strauss, 1967). The process of determining whether or not a group had posed a problem was necessarily a subjective one. Rather than looking at the actual uttered problem, I was looking more at the conversational fabric around the utterance, both before the utterance occurred (what did the intent of the utterance seem to be?) and afterwards (namely, how did the group respond to the utterance?), indications of surfacing differences that the group appeared to be exploring.

The metaphor that I use to document the patterns of collective problem posing, and reduce the transcript to its "visual essence," is that of the "tapestry." Composed of strands of fabric and color, a tapestry reveals different faces depending on its physical distance from the observer. From afar, which would be the equivalent of summarizing a group conversation and then considering it from both a temporal and contextual

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distance, the tapestry shows a panoramic scene – a whole composed of a number of parts. Closer, the landscape of the tapestry might still be evident, but now the individual strands are more visible. Move closer still, and now the individual strands are the focus and the overall scene is no longer clear – much in the same way in which it may be easy to follow the individual turns of a conversation but difficult to summarize the gist of the discussion as a whole while it is taking place. At this level, an overall pattern is invisible, but individual contributions and ideas stand out. These strands of individual utterances are ones that weave together into a tapestry as the conversation proceeds.

The production of the tapestry involved a data blurring process, which started with the transcript itself. After multiple iterations of reading and comparing transcripts from the four groups' sessions, I identified the posed problem categories I color coded the utterances in the transcripts according to the problem posing category they best fit. The color-coded transcripts were then shrunk in size, using computer screenshots, to the point where the words of the transcript were no longer visible and the lines of color coding appeared as a visual pattern. The resulting tapestry provides an overall image of the problems posed during the course of the group's session.

RESULTS AND DISCUSSION

At first glance, the structured nature of the Bill Nye task would not appear to allow for many creative possibilities for mathematics students. To solve it, one must understand what the range of time is for opening the gifts, determine the number of time intervals that exist within that time frame, and then find the pair of factors of the number such that one factor is one greater than the other (i.e. 8 and 7). Yet, in working through this apparently straightforward task, these four groups take very different paths to eventually arrive at the same correct solution.

Tapestries

A striking aspect of group work that a tapestry helps to illustrate is how posed problems weave in and out of conversations. A color may appear briefly early in a session – for instance, medium blue in NIJM ("What are the factors of x?") – and not appear again until over halfway through when it begins to occur quite frequently. A problem may be posed and seemingly ignored, only to be reposed later in the discussion, while other problems that seem to have been discussed and resolved may also reappear for more discussion. This suggests that the mention of a posed problem early on in a session may help to seed a later discussion. It also seems to highlight the idea of all ideas being part of the tapestry, visible or not – no utterance truly disappears.

The width of the color bands indicates the approximate length of time a problem is being discussed, and how many connections are made with other posed problems. For example, the chunky[1] pattern displayed in the first third of JJKK's tapestry (Figure 1) is quite distinctive from the tapestries of the other three groups. The chunkiness

[1]Thick bands of color in the tapestry

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reflects how a problem is posed, discussed at some length until some kind of agreement is reached, and then disappears, presumably either having been resolved or dropped completely. This pattern also reflects how JJKK poses and reposes far fewer problems than the other groups do (Figure 2).

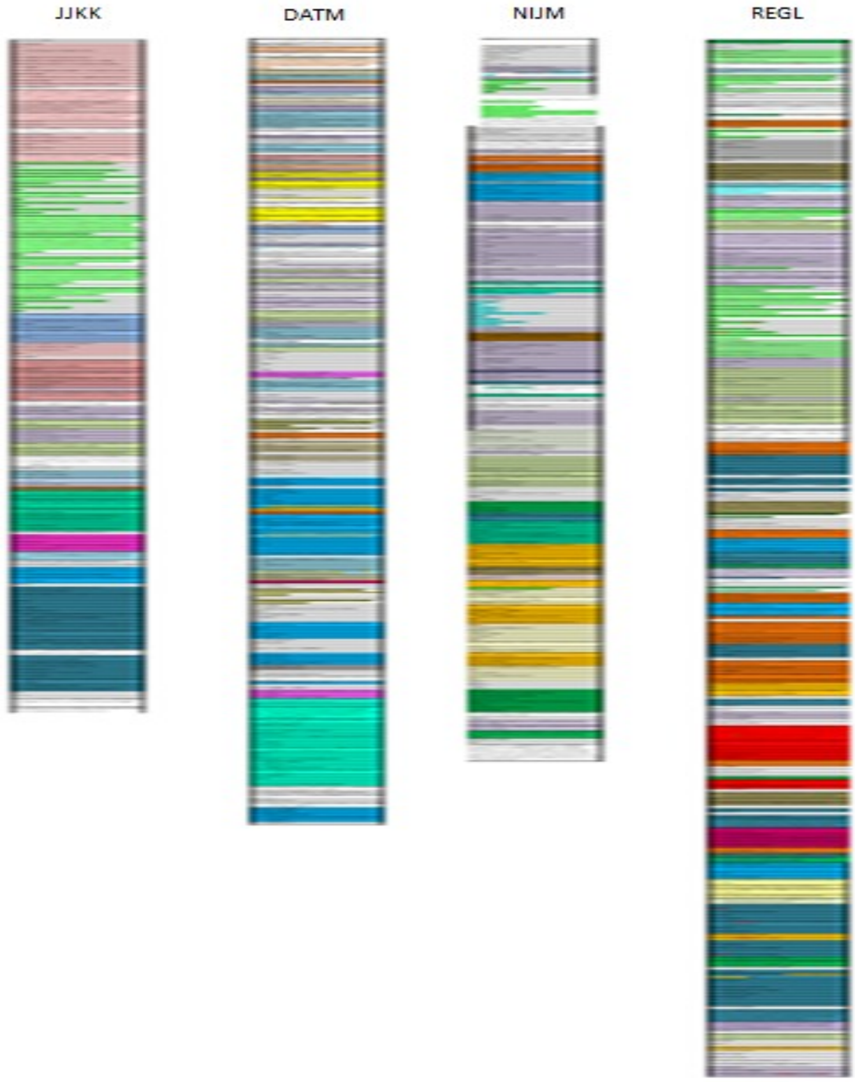

Figure 1: Tapestries

Group	# of different problems posed	Total # of problems posed and reposed
JJKK	13	23
DATM	16	61
NIJM	17	45
REGL	16	66

Figure 2: Comparison of # of problems posed and reposed.

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In comparison, “thready”[2] patterns found in the tapestries of DATM, NIJM and REGL tend to show that a number of different problems are being posed and put “on the table,” so to speak. For all three groups there tends to be a thready pattern of different colors at the beginning of their tapestries when they are first considering the task. Finally, the thready pattern also tends to occur late in the sessions when the three groups have come up with a tentative answer, when earlier problems are reposed as a way of checking their thinking.

While lavender and a few other colors appear in all of the tapestries, there are many other colors which do not. For instance, there is a shade of teal (“Is it a square root?”) that only appears in NIJM and JJKK. And still other colors are unique to certain groups, like the light green (“How can we use the 24 hour clock?”) that occurs at the end of DATM’s tapestry. It might be expected that unique problems might be due to experiences/knowledge that is unique to the group, but this is not necessarily the case. For instance, the topic of square roots was one that the groups were all studying in their regular mathematics class, yet only two of the four groups reference it.

Characteristics of problem posing

A notable trend across the sessions is how the role a posed problem plays in a discussion changes each time it is posed even if, on the surface, the wording of the problem appears to be much the same. On the surface, problems like “Do we use time and divide by 5?” which is featured predominantly in at least three of the group’s discussions, may seem to be a clarification problem. For example, consider it functions during NIJM’s session. Posed and reposed eleven times, this problem functions in order to: propose a method of entry into the task; discuss what method would be easiest; discuss how it might eventually lead to solving the entire task; estimate/predict possible answers; narrate ongoing calculations; check possible answers. Most of the other posed problems in the study also show evidence of their roles evolving as the group discussion develops. The only time that a problem does not appear to evolve is when a group does not repose it.

The number of different individual problems posed (Figure 2) is fairly consistent between the groups but there is a large range in the total number of problems posed. One might posit that the difference is due to each group’s “personality.” For example, REGL, who tends to explore concepts more deeply and connect ideas more frequently than the other groups, poses more problems than JJKK who tends to argue about one problem at a time until a consensus appears to be reached. While some problem posing studies in the literature have focused on the number of problems posed, or the quality of problems posed, my findings suggest that the pattern in which problems are both posed and reposed may ultimately tell us more about students’ mathematical behavior and understanding.

[2] Slim bands of color that alternate with slim bands of other colors.

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. CONCLUSIONS

This study offers a description of problem posing as collective behavior at the level of the group as an agent. It also provides evidence of the groups' ability to problem pose collectively without having been directed to do so, and without having received any formal instructions about how to do so. It is noteworthy that problems do not emerge in the same order for each of the groups. The varied ways in which groups in this study approach the Bill Nye task may suggest that educators need to be careful of presenting problem solving heuristics as lock-step procedures to be followed in a specific order. Even though the four groups have some common experiences with which to work, the fact that certain groups do not necessarily draw on these experiences, or if they do, do not do so in the same way as other groups, suggests that the process of problem posing is more than simply sitting down and "working with what you have." Perhaps the strength of problem posing is not the generation of a list of problems at the end of the task, but the emerging patterns of problems as the discussion continues and how these problems in turn structure pathways to a solution.

References

Bonotto, C. (2013). Artifacts as sources for problem-posing activities. *Educational Studies in Mathematics*, *83*, 37-55.

Bowers, J. S., & Nickerson, S. (2001). Identifying cyclic patterns of interaction to study individual and collective learning. *Mathematical Thinking and Learning*, *3*(1), 1-28.

Brown, S. I., & Walter, M. I. (2005). *The art of problem posing*. Mahwah, NJ: Lawrence Erlbaum Associates.

Crespo, S. (2003). Learning to pose mathematical problems: Exploring changes in preservice teachers' practices. *Educational Studies in Mathematics*, *52*, 243-270.

Crespo, S., & Sinclair, N. (2008). What makes a problem mathematically interesting? Inviting prospective teachers to pose better problems. *Journal of Mathematics Teacher Education*, *11*(5), 395-415.

English, L. D. (1997). Promoting a problem-posing classroom. *Teaching Children Mathematics*, *4*(3), 172-179. Reston, VA: NCTM.

English, L. D. (1998). Children's problem posing within formal and informal contexts. *Journal for Research in Mathematics Education*, *29*(1), 83-106.

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research* (1st ed.). Chicago: Aldine Publishing Company.

Grossen, M. (2009). Social interaction, discourse and learning. In K. Kumpulainen, C. E. Hmelo-Silver, & M. César (Eds.), *Investigating classroom interaction: Methodologies in action* (pp. 263-275). Rotterdam: Sense Publishers.

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123-148). Hillsdale, NJ: Lawrence Erlbaum Associates.

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Lester, F. K. (1994). Musings about mathematical problem-solving research: 1970-1994. *Journal for Research in Mathematics Education*, *25*(6), 660-675.

Leung, S. S. (1993). The influence of task formats, mathematics knowledge, and creative thinking. *Proc. 17th Conf. of the Int. Group for the Psychology of Mathematics Education*. Tsukuba, Japan: PME.

Martin, L., Towers, J., & Pirie, S. E. B. (2006). Collective mathematical understanding as improvisation. *Mathematical Thinking and Learning*, *8*(2), 149-183.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook.* Thousand Oaks, CA: SAGE Publication.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Pirie, S. E. B. (2002). Problem posing: What can it tell us about students' mathematical understanding? In D. S. Mewborn, P. Sztajn, D. Y. White, H. G. Wiegel, R. L. Bryant, & K. Nooney (Eds.), *Proc. 24th Annual Meeting of the North American Chapter of the Int. Group for the Psychology of Mathematics Education* (Vol. 2, pp. 927-958). Athens, GA: PMENA.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14(*1), 19-28.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*(5), 521-539.

Silver, E. A., Mamona-Downs, J., Leung, S. S., & Kenney, P. A. (1996). Posing mathematical problems: An exploratory study. *Journal for Research in Mathematics Education*, *27*(3), 293-309.

Silver, E. A., & Marshall, S. P. (1989). Mathematical and scientific problem solving: Findings, issues, and instructional implications. In B. F. Jones & L. Idol (Eds.), *Dimensions of thinking and cognitive instruction* (pp. 265-290). Hillsdale, NJ: Lawrence Erlbaum Associates.

Singer, F. M. (2009). The dynamic infrastructure of mind – a hypothesis and some of its applications. *New Ideas in Psychology*, *27*, 48-74.

Singer, F. M., & Mascovici, H. (2008). Teaching and learning cycles in a constructivists approach to instruction. *Teaching and Teacher Education*, *24*(6), 1613-1634.

Stahl, G. G. (2006). *Group cognition: Computer support for building collaborative knowledge*. Cambridge: MIT Press.

Yackel, E. (2002). What can we learn from analyzing the teacher's role in collective argumentation. *Journal of Mathematical Behavior*, *21*, 423-440.

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