EFFECTS OF A HOLISTIC VERSUS AN ATOMISTIC MODELLING APPROACH ON STUDENTS' MATHEMATICAL MODELLING COMPETENCIES

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The paper deals with the question of the practicability and the effectiveness of different approaches to foster students' mathematical modelling competencies. Within the modelling project ERMO (Acquirement of modelling competencies) a holistic and an atomistic approach of mathematical modelling were compared in order to find out which approach is more effective in fostering the students' modelling competencies. The results of modelling tests with three measurement points show that both approaches foster students' modelling competencies, but both approaches have strengths and weaknesses. The data indicates that the holistic approach is more effectively for students with weaker performance in mathematics.

INTRODUCTION

For several years, there was an intense national and international didactic discussion and research in mathematical modelling (see Blum et al., 2007; Kaiser et al., 2011; Stillman et al., 2013). Furthermore, the development of students' mathematical modelling competencies is a central goal of German mathematics lessons, since the competency of mathematical modelling has been described as one of the central competencies in German educational standards in mathematics. Projects to foster students' mathematical modelling competencies can each be assigned to one of two approaches: either a holistic or an atomistic approach (Blomhøj & Jensen, 2003). The main goal of the presented study is a comparison of the effectiveness of these two approaches in terms of the development of modelling competencies of students.

In the first part of the paper the theoretical framework will be documented. Then, the design of the modelling project will be presented as well as the methods of data collection and evaluation. Finally, selected results of the study will be described.

THEORETICAL FRAMEWORK

In recent years, mathematical modelling was an internationally highly discussed topic of didactics of mathematics. From the discussion resulted various perspectives of mathematical modelling that include different representation of the modelling process as a cycle as well as goals and modelling competencies. An overview is given for instance in Kaiser and Sriraman (2006).

However, the various definitions have in common that mathematical modelling is described as a process of solving real world problems by using mathematical methods (Niss, Blum, & Galbraith, 2007). In addition, an ideal-typical process of mathematical

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modelling is usually illustrated in the form of a cycle (Kaiser, Blomhøj, & Sriraman 2006), while in reality such processes are characterized by frequent switching between the various stages of modelling cycles (Borromeo Ferri, 2011; Martinez & Brizuela, 2009). Corresponding to the different perspectives of mathematical modelling there are various modelling cycles, which are either more useful for application in mathematics lessons or in science (Borromeo Ferri, & Kaiser, 2008). The project ERMO refers to a didactical modelling cycle developed amongst others by Kaiser and Stender (2013; see Figure 1).

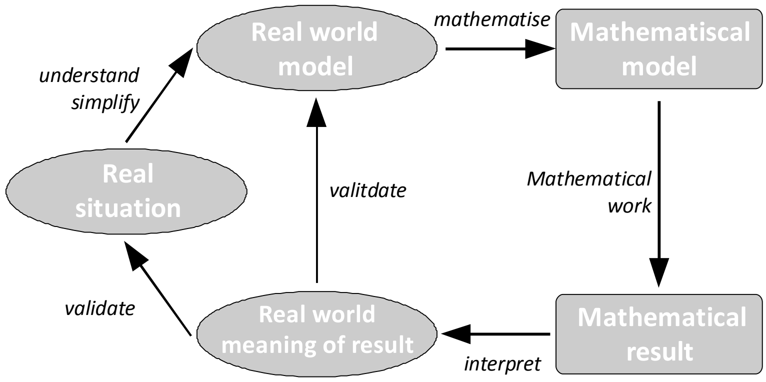

Figure 1: Modelling cycle (Kaiser & Stender, 2013)

The specific definition of modelling competencies depends on the particular underlying concept of mathematical modelling (Zöttl, Ufer, & Reiss, 2010). Widely accepted is that modelling competencies include abilities and a willingness to solve real-world problems by using mathematical modelling (Maaß, 2006; Blomhøj & Jensen, 2003). The concept of mathematical modelling competencies contains different components, namely sub-competencies of mathematical modelling, metacognitive modelling competencies, competencies of structuring given problems appropriately and goal-oriented, competencies of argumentation and documentation and competencies of realising the possibilities of mathematics as well as positively valuing these (see for example Maaß, 2006).

The sub-competencies are based on the underlying modelling cycle and include the abilities needed to perform the different steps of the cycle. Based on the modelling cycle from Kaiser and Stender (2013) different sub-competencies of mathematical modelling are distinguishable which can be assigned to three sub-processes of mathematical modelling (referring to Zöttl, Ufer, & Reiss, 2010):

- *Simplifying / Mathematising* (including all competencies needed for the transition between real world and mathematics)
- *Working mathematically* within the mathematical model
- *Interpreting / Validating* (including all competencies needed for the transition between mathematics and real world)

The sub-competencies of mathematical modelling are seen a necessary part of the modelling competencies, as they enable the modeller to perform the different steps of the modelling process adequately. However, the presence of the sub-competencies does not automatically include the existence of the *overall modelling competence* (Zöttl, Ufer, & Reiss, 2010). According to Maaß (2006) or Stillman (2011) the *metacognitive competencies* play a significant role for the modelling competencies. A non-existent or low meta-knowledge about the modelling process as a result may lead to considerable problems while working on modelling tasks, for example at the transitions between the different phases of the modelling process.

According to the survey by Blomhøj and Jensen (2003) projects to foster mathematical modelling competencies can mainly be assigned either to a holistic or an atomistic approach. The holistic approach is based on the assumption that the fostering of modelling competencies will be the most effectively by tackling whole modelling tasks. The complexity and difficulty of the modelling tasks should correspond to the competencies of the students. The atomistic approach is based on the assumption that particularly at the beginning of the work with modelling problems the tackling of whole modelling tasks would be too time-consuming and not be effectively referring to the fostering of sub-competencies of mathematical modelling. Propagated is separated fostering of the sub-competencies by tackling only sub-processes of a whole modelling process (Blomhøj & Jensen, 2003).

DESIGN OF THE STUDY

The central goal of the project ERMO (*Er*werb von *Mo*dellierungskompetenzen: Acquirement of modelling competencies)[1] was to foster the students' modelling competencies. Furthermore, the design of the single modelling activities was oriented towards the promotion of the students' ability to reflect about their own working processes and results.

The modelling project was carried out in 2012 in Hamburg (Germany) and started with a teacher training course conducted in cooperation with Dr. Katrin Vorhölter in February 2012. The participating classes integrated six 90 minutes modelling activities, including the tackling of different authentic modelling problems in co-operative, self-directed learning environments, as well as a modelling test in a pre-, post- and follow-up-design into their mathematics lessons (for an overview see Figure 2). The classes were divided into two groups: The modelling activities of group A were assigned to the holistic approach, while the modelling activities of group B were assigned to the atomistic approach. The students of the holistic group dealt with complete modelling tasks with an increasing complexity, the students of the atomistic group dealt with sub-processes of mathematical modelling separately, especially the transitions real world → mathematics and mathematics → real word. The tasks of the atomistic group contained active parts, i.e. tasks that require for example to develop own real and mathematical models, as well as passive parts, i.e. given models or

[1] The project benefited from experiences of the Hamburg working group on mathematics education with carrying out and evaluating modelling projects (see for example Kaiser, 2007).

solution that were to assess and validate. As hypothesis it was formulated that modelling activities of the holistic approach is more effectively concerning the fostering of the *overall modelling competency* while the atomistic approach might be more effectively regarding the sub-processes of mathematical modelling (*Simplifying / Mathematising, Working mathematically* and *Interpreting / Validating*).

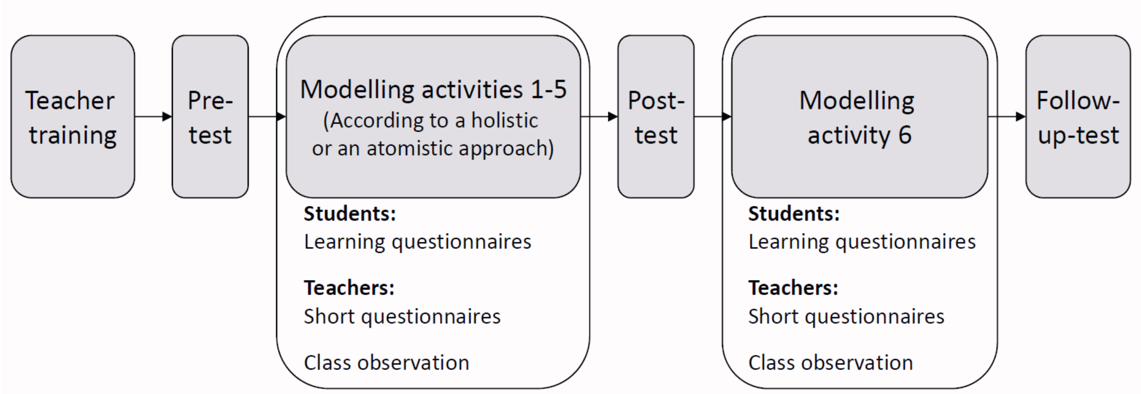

Figure 2: Design of the study

Altogether, N=377 students from 15 classes of 9th grade of four secondary higher-track schools and two comprehensive schools took part in the project, while only 204 students of 13 classes participated in all three measurement points (MP), 132 students of the holistic group and 72 students of the atomistic group (see Table 1). The presented results are based on this panel.

	MP 1	MP 2	MP 3	Panel
Holistic approach	168	164	169	132
Atomistic approach	159	152	97	72
Total	327	316	266	204

Table 1: Sample – number of participating students

The modelling test was designed in a pre-, post- and follow-up-design and conducted to evaluate the students' progress of their modelling competencies. The design of the modelling test refers to work by Haines, Crouch and Davis (2001) and Zöttl, Ufer and Reiss (2011) and others, who developed items that tested different sub-dimensions of the modelling competencies. Because of this structure, it is possible to measure different dimensions of students' modelling competency independently from potential weaknesses in single phases of the modelling process. The developed modelling test covered the three sub-processes of mathematical modelling (*Simplifying / Mathematising, Working mathematically* and *Interpreting / Validating*) as well as an *overall modelling competency* including the competence of carrying out a whole modelling process and matching different parts of a solution of a modelling task to the right phases of the modelling cycle. Per measurement point, the number of used items per dimension of the modelling competency varied between 15 and 24.

The data were scaled by using methods of multidimensional item response theory and with an approach of so-called virtual persons for all items of the three measurement points (Rost, 2004). The various dimensions of the modelling competency are considered as being the latent variables that can be estimated as a multivariate function of the items solved. The scaling was carried out with Conquest (Wu et al., 2007). In a first step, different psychometrical models of the structure of the modelling competency were scaled, a one-dimensional model as well as a four-dimensional between-item model and two multidimensional within-item models. To select the best model for the data, the psychometrical measures Akaike Information Criterion (AIC), Bayes Information Criterion (BIC) and Consistent AIC (CAIC) were used (Rost, 2004). After the model selection, weighted likelihood estimates (WLE) were estimated as individual ability parameters and converted to an average value of M=50 and a standard deviation of SD=10. To analyse the progress of the modelling competencies within the two groups, amongst other evaluations, the average test performances of the students were tested for significance that were corrected by the Bonferroni method. In addition, the effect sizes of the performance differences were calculated.

RESULTS

The comparison of the four psychometrical models points to the four-dimensional between-item model (see Figure 3). Considering the psychometrical measures AIC, BIC and CAIC, which are the lowest for this model, the four-dimensional between-item model describes the collected data the best compared to the others (Rost, 2004). In addition, the reliabilities of the four dimensions are acceptable and vary between 0.767 and 0.821.

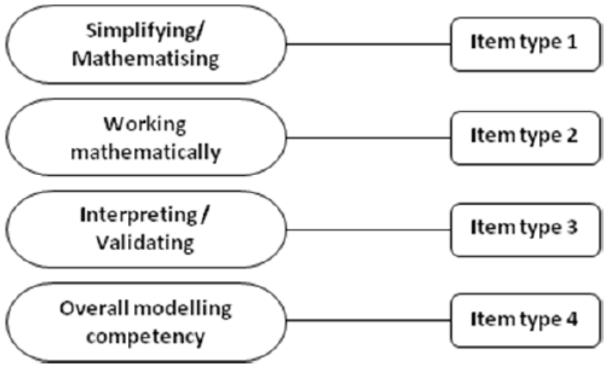

Figure 3: Four-dimensional between-item model

Regarding the development of the four dimensions of modelling competencies the data show for all groups of students highly significant increases between the first and the second as well as between the first and the third measurement points (see Table 2). In the first dimension *simplifying / mathematising* there is a higher effect size in increase of the holistic group between the pre- and the post-test (0.88) compared to the atomistic group (0.72). Between the pre- and the follow-up-test the atomistic group shows a larger effect size (0.68) than the holistic group (0.59). The effect sizes in the dimension of *working mathematically* are larger in the atomistic group between measurement point one and measurement point two (0.57 versus 0.47) as well as between measurement point one and measurement point three (0.46 compared to 0.32). The effect sizes in increase in the dimension of *interpreting / validating* are higher in the holistic group between the pre- and the post-test (0.77 compared to 0.69) as well as between the post- and the follow-up-test (0.65 instead of 0.57). In the fourth dimension, the overall modelling competency, there are larger effect sizes in increase in the holistic group as well (0.90 versus 0.68 between the first two and 0.61 instead of 0.35 between the first and the third measurement point.

	Mean MP 1 (SD)	Mean MP 2 (SD)	Mean MP 3 (SD)	MP1→MP2 (Cohen's d)	MP1→MP3 (Cohen's d)	MP2→MP3 (Cohen's d)
Simplifying / mathematising						
Holistic group	48.26 (11.29)	57.60 (9.87)	54.90 (11.14)	+9.33*** (0.88)	+6.64*** (0.59)	-2.69* (-0.26)
Atomistic group	51.21 (7.80)	57.62 (9.84)	57.08 (9.39)	+6.41*** (0.72)	+5.87*** (0.68)	-0.54 (-0.06)
Working mathematically						
Holistic group	49.94 (10.18)	54.92 (10.83)	53.10 (9.29)	+4.98*** (0.47)	+3.16*** (0.32)	-1.82* (-0.18)
Atomistic group	48.85 (9.16)	54.76 (11.35)	53.81 (12.33)	+5.91*** (0.57)	+4.95*** (0.46)	-0.96 (-0.08)
Interpreting / validating						
Holistic group	47.93 (9.42)	55.83 (11.07)	54.38 (10.50)	+7.90*** (0.77)	+6.45*** (0.65)	-1.45 (-0.13)
Atomistic group	50.55 (8.86)	56.73 (8.95)	55.79 (9.59)	+6.19*** (0.69)	+5.24*** (0.57)	-0.95 (-0.10)
Overall modelling competency						
Holistic group	49.78 (9.34)	58.08 (9.20)	55.55 (9.62)	+8.30*** (0.90)	+5.78*** (0.61)	-2.53** (-0.27)
Atomistic group	50.75 (9.74)	57.28 (9.46)	54.21 (9.81)	+6.52*** (0.68)	+3.46* (0.35)	-3.07* (-0.32)

***p<0.000, **p<0.01, *p<0.05

Table 2: Means and performance increases of the different dimensions of the modelling competency

To receive detailed information about the differences in the performance increase between different groups of students, two-way ANOVAs with repeated measures were used. The results of the two-way ANOVAs show that there can be seen only significant effects of the modelling approach (in favour of the holistic approach) in the dimension *simplifying / mathematising* and *the overall modelling competency* and only between the first two measurement points. Differentiated between the two school types, the two-way ANOVAs reveal that there is no significant effect of the modelling approach for the students of the secondary higher-track schools while there were significant effects of the modelling approach for all dimensions of the modelling competencies (in favour of the holistic approach) for the students of the two comprehensive schools.

DISCUSSION OF THE RESULTS

The results of the study are on the one hand related to the structure of modelling competencies and on the other hand on the development of the modelling competencies of the students.

The results of the model comparison confirm the possibility to distinguish different facets of modelling competencies. The multidimensionality of the construct modelling competencies was also shown by Zöttl, Ufer, and Reiss (2010), while in their study the *overall modelling competency* was not seen as separated from the sub-processes of mathematical modelling as it could be shown in this study. This fact may be explicable by various aspects, mainly a different definition of the dimension of *overall modelling competency* including meta-cognitive aspects (see above).

The evaluation of the modelling tests shows that the effectiveness of the two approaches towards fostering students' modelling competencies has to be considered in a differentiated way. On the one hand the data showed that, despite the limitations of the reliability of the results particularly in field studies, in the project ERMO both the holistic and atomistic approach fostered the development of the different dimensions of the students' modelling competencies under real teaching conditions successfully. On the other hand, differences between different groups of students and between the four dimensions of modelling competencies became apparent. A general superiority of one approach could not be stated. The results indicate that the approach for high-performance students plays a minor role, since no effect of the approach was found for the dimensions of modelling competencies for students of higher track schools (so-called *Gymnasien*). In contrary, especially for relatively less powerful or for heterogeneous classes the holistic approach seems to be superior to the atomistic approach, because for the students of the comprehensive schools (the so-called *Stadtteilschulen*) there higher performance increases were found in the holistic group.

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