LEARNING NEGATIVE INTEGER CONCEPTS: BENEFITS OF PLAYING LINEAR BOARD GAMES

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Linear board games have shown great promise as tools to teach whole number concepts (Ramani & Siegler, 2008), but little is known about their utility for supporting negative integer concepts. This study sought to extend the use of linear board games to teach integer concepts. Forty-eight first graders (ages 6-7) counted along an integer board game with negative numbers or did control activities with integers. Students who played the board game made significant gains on several measures related to identifying and ordering integers. Findings suggest that even young children can benefit from games with negative integers, and we provide implications for instruction.

Young children are often exposed to negative numbers in contexts, such as negative temperatures, negative points in golf or video games, or when selecting floors below zero in elevators in some countries, before they are formally introduced in school. An understanding of negative numbers can help students better understand that zero is not the smallest number, a conception they have difficulty overcoming (Bofferding, 2014). Further, having experiences counting backward through zero can help them understand that expressions such as 3-5 are meaningful (Bofferding, 2011), making it unnecessary to teach the misleading rule that you cannot subtract a larger number from a smaller one. However, there is little data on how children's informal experiences with negative numbers might influence how they make sense of these numbers. This study provides initial data on the benefits that playing linear board games can have on first graders' (6-7 years-old) understanding of negative numbers.

THEORETICAL FRAMEWORK

According to Case's (1996) Central Conceptual Structures for Number Theory, by the age of about 6, children have coordinated their understanding of number concepts involving symbols, number order, number values, and the relations among them. Therefore, they can say the counting sequence, know that numbers further in the counting sequence correspond to larger values, and know that the larger values correspond to larger sets of objects, all of which can be represented with numerals. Further, they understand that saying the next number in the sequence corresponds to getting 1 more (or adding 1) and that saying the previous number in the sequence corresponds to getting 1 less (or subtracting 1). Researchers describe this understanding as using a mental number line. These concepts form students' initial mental models of number that they must change to accommodate new numbers like negative integers (Vosniadou, Vamvakoussi, & Skopeliti, 2008).

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In order to extend their mental number line to include negative integers, they must extend the number sequence to the left of zero (or less than zero), with numerals symmetric to the positive ones, but marked with negative signs. They also must extend the idea that numbers further to the left on their mental number lines correspond to smaller values, e.g., -5 is less than -3. The purpose of this research is to determine the extent to which informal experiences with negative integers could help students extend their mental number lines in this way.

RELATED LITERATURE

Drawing on Case's (1996) theory, Ramani and Siegler (2008) posited that young children could learn whole number concepts if they played linear board games which supported the development of concepts important to establishing a mental number line. Through a series of experimental studies, they found that pre-schoolers (3 to 5 year-olds) who "counted on" from 1 to 10 while playing linear board games made significant gains on a variety of number concepts compared to children who did numerical, control activities. The children who played the game improved in counting to 10, determining which of two numbers is larger, correctly identifying numbers 1-10, and estimating the positions of numbers 1-10 on an empty number line (Ramani & Siegler, 2008).

The literature on young student's understanding of negative integers is fairly sparse but suggests that they are capable of learning about them if given purposeful experiences with them. Before learning about negative numbers, first graders order negative integers next to their positive counterparts (Peled, Mukhopadhyay, & Resnick, 1989) or treat them as numbers that have been taken away and order them before or after zero (Bofferding, 2014; Schwarz, Kohn, & Resnick, 1993). Similarly, they treat negative numbers as positive and consider -5 > 0 and -5 > -3 (Bofferding, 2014; Peled et al., 1989). However, after instruction on the order and value of negative integers and working with these concepts through a series of card games utilizing number line contexts, first graders at the end of the school year (7 and 8 year-olds) improved significantly on such tasks compared to students who had not received this instruction (Bofferding, 2014). Further, students who know the order of the negative numbers can begin to reason about integer addition and subtraction problems and successfully solve some of these problems (Bofferding, 2010; Bishop, Lamb, Philipp, Whitacre, & Schappelle, 2013). Because integer concepts rely on the same order and value relations underlying the whole number mental number line, it is likely that the board game experiences that help students develop their whole number mental number line could also help students develop an integer mental number line.

RESEARCH AIMS

The primary aim of this research is to test whether students who play linear, numerical board games develop a deeper understanding of negative integer concepts than peers who participate in integer-related control activities. We hypothesized that the board

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game group would make significant gains on negative integer tasks compared to students in the control group because the board game provides students with the opportunity to experience an extension of the positive number line into negatives.

METHODS

Setting and Participants

The study was conducted at an elementary school located in a low-income area where approximately half of the students were Hispanic and a third spoke a language other than English at home. The study was conducted during the first three months of the school year. Across six classrooms, students were randomly selected for participation until we had a sample size of 50 (26 female; 24 male).

General Design

To test our hypothesis, we employed an experimental design involving a pre-test, random assignment to control or treatment group, intervention, and post-test. The design extended the methods and measures used by Siegler and Ramani (2009) to include negative integers. For the intervention, each participant worked with a researcher for three, 15-minute sessions. However, two children in the treatment group withdrew from the school before the post-test measure was given, resulting in data for 23 children who played the integer board game and 25 children who did the control activities. A professor and graduate research assistant collected data in corners of the classrooms, replacing a portion of the students' whole-class mathematics instruction. Most participants worked with both researchers during the course of the study.

Pre-test and Post-test Measures

The pre-test and post-test were identical so that we could measure gains in the participants' mathematical knowledge. We conducted both tests as individual interviews with the students, and we did not provide specific feedback on their performance, just general encouragement.

The first section of the tests involved students *counting* forward to ten and backward from ten as far as they could go. If they stopped at 0, we asked if they could keep counting down any further. The second section involved verbal *integer identification* of the numerals from -10 to 10, presented on isolated pages in random order. The third section concerned *integer order*. Students were asked questions such as, "What number comes two numbers after 7?" They also ordered a set of eleven cards labelled -5 to 5 and indicated which were the least and greatest.

The fourth section on *integer values* asked students to identify which of two integers was closer to 10 (n=10) and further from 10 (n=10). The two numbers were a mix of positive integers, negative integers, and zero. For each question, children were shown an equilateral triangle with 10 inside the peak and the pair in question equidistance away, placed in the left and right corners. Further, each integer was named by the researcher. If students were confused on the use of the words "closer" and "further,"

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the researcher would make a statement such as, “If 10 is the largest, which of these [pointing] -9 or 5 is less or further away from 10?”

The fifth section dealt with *operations*. Students were first given additive expressions involving strictly positive integers, and then negatives were introduced into the expressions. Second, the researchers presented expressions with subtraction, initially with positive differences, and then negative differences. The section ended with a contextualized addition problem and two, two-digit arithmetic problems.

The final section of the tests involved students placing integers on *number lines*. Students completed a packet involving positive integers followed by one involving negative integers. Each page of the packet contained an empty number line 25.5cm long with two integers marked. On the first page of both packets, students were asked to put a pen mark where 0 would go, given the locations of -5 and 5. For the positive packet, the remaining pages contained empty number lines marked with 0 and 10. The placement of zero in the middle, i.e., leaving space for the negative numbers to the left, was an important feature. Students were asked to make a mark where a given integer should go a total of 18 times (1 through 9 in random order, twice). The researchers gave instructions such as, “If here is 0 [point to the middle] and here is 10 [point to the right], then make a mark anywhere on this line [motions to whole 25.5 cm line] where 6 should go.” The negative number packet worked similarly only with -10 marked on the left and 0 marked in the middle. Students were told to place the negative integers -1 through -9 on the respective pages (see Figure 1 for examples).

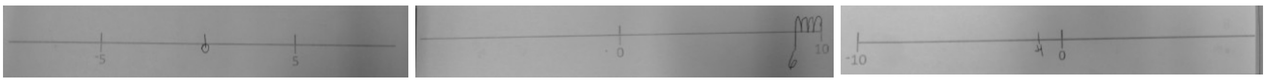

Figure 1: Three number line tasks: mark where zero goes (left), where positive integers 1 to 9 go (middle), and where negative integers -1 to -9 go (right).

Control Group

For their three sessions, the control group students cycled through three types of activities with the researcher. The first activity involved counting a collection of 1-10 poker chips and counting backward as far as they could. They did not receive feedback on correctness.

For the second activity, students put six integer cards in order from least to greatest. The researchers rotated between three sets of cards; for example, one set they ordered included the following integers: 6, -9, -4, 0, 3, and -1. After the students ordered the set, they were asked to point to the least and the greatest. Students were not given any feedback on the ordering or the identification of the cards.

The last activity in the cycle was a game of memory where the goal was to match integers. Students were given corrective feedback if they attempted to collect an incorrect match but were not told the names of the numbers. Both the positive and negative versions of integers appeared, so if students tried to match n and $-n$, the researcher would interject that the cards did not look exactly the same.

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Treatment Group (Game)

During each 15-minute intervention session, the treatment group played a board game against the researcher using a board labelled with the integers -10 to 10 (see Figure 2).

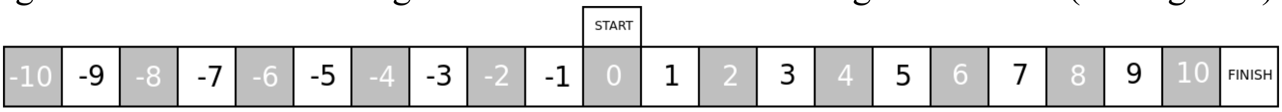

Figure 2: An illustration of the linear, numbered game board.

Players started by placing their tokens at zero, and the first player drew a card from a card deck. In the first version, all but one of the cards was labelled with a 1, 2, or 3. The remaining card contained the text, “All players go back to -10.” When this card was drawn, the student had to count backward while moving their tokens back to -10. The researcher always stacked the deck so that this card would come up in the first few turns of the game, assuring players would proceed from -10 to 10 in each round.

If players drew a 1, 2, or 3, they moved their tokens that number of spaces and named the numbers on the spaces they passed through. For example, if a player on -4 drew a “2”, then she would move her token to -3 and say “negative three”, then move her piece to -2 and say “negative two.” The game ended when a player crossed 10.

During the third session, the card sending players back to -10 was removed and a new stack of cards was introduced. Cards in the new stack were labelled either -2 or -4. Players began the game by drawing from this stack and counting backwards as they moved to -10. Once a player reached -10, on her next turn she would begin drawing with the deck containing positive numbers. From this point, play continued as normal, with the game ending once a player crossed over 10.

The researchers gave enough feedback to ensure legal turns by the students. Sometimes this involved correcting the name of the integer that they landed on, other times it involved correcting the number of spaces the game piece was moved. The game would not proceed until the student had said the correct number names aloud. Students played an average of 4 games in 15 minutes.

ANALYSIS

The focus of the analysis and results will be primarily on the negative integer items. For the *counting* backward to -10 task, students were given 1 point per correct number named below zero until their first error. Therefore, a student who counted “-1, -2, -4,” would receive a score of 2 out of a possible score of 10. Similarly, on the negative *integer identification* task, students received a point per negative integer identified (for a possible total of 10). On the *integer values* task, students received a point per correct problem (for a possible total of 20). Finally, for the *number line* tasks, we calculated students’ percent absolute error (with small numbers being better), comparing where they marked a number on the empty number line to its proper location.

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RESULTS

Based on our preliminary analyses, students in the treatment group made larger gains across item types even after playing the board game for only 45 minutes (see Table 1).

	Pre-test		Post-test		Gain	
Item Type	Control	Game	Control	Game	Control	Game
Counting to -10	.36	.43	1.32	4.35	.96	3.91
Negative Integer Identification	1.08	1.61	3.00	9.22	1.92	7.61
Number Values	7.32	6.70	7.5	9.30	.18	2.61
Number Line Percent Abs. Error	41.60	49.81	50.24	39.61	8.64	-10.20

Table 1: Average scores on the pre-test and post-test and average gain for the Control group (n=25) and Treatment or Game group (n=23) across item types.

We examined the multivariate effects of condition (game treatment versus control) across the four tasks using a MANOVA on the average gain scores for the four item types. There was a significant effect of condition on the average gains for all items, $F(4, 43)=5.821$, $p=.001$, partial eta squared= .351. The game group significantly improved across all measures. Separate univariate ANOVAs on the outcome variables revealed significant treatment effects on identifying negative integers, $F(1,46)=17.059$, $p=.000$ and counting to -10, $F(1,46)=6.540$, $p=.014$, with smaller effects on identifying integer values closer to or further from 10, $F(1,46)=4.521$, $p=.039$, and in their percent absolute error when marking numbers on a number line, $F(1,46)=4.097$, $p=.049$.

Counting back to -10

Other than either stopping at 0 or counting back all the way to -10, one student stopped counting at -3 and another student named negative numbers in random order.

Identifying negative numbers

Generally, students in the control group ignored the negative signs and called negative integers by positive integer names. A few students made up new names for the numbers, e.g., "infinity one" for -1 or "equals three" for -3. Only one student in the game (treatment) group was unable to identify any of the negative integers.

Integer Values closer to/further from 10

The smaller gains that students made on these items reflect the difficulty that some of them had in overcoming their desire to treat negative numbers as equivalent to positive numbers in value. Students in the control group learned that -5 could not be matched to 5 in the matching game, but they had no reason to think that the values were different. When choosing whether -7 or -10 would be further from 10, one student in the control grouped explained that -7 is further because -10 is the same as 10. More nuanced, when

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choosing whether -8 or -5 would be closer to 10, a student in the game group said -8, but then justified it by clarifying that -8 is closer to -10. Therefore, she may have interpreted -10 as something different than 10 but with equivalent values. This is true in some sense as both numbers are equal distances away from 0.

Placement of numbers on an empty number line

Across both groups, students displayed a strong tendency to mark 1 toward the left edge of the number line, even when 0 was marked in the middle of the line. They explained that 0 always starts at the beginning of the line and then 1 would come after that. Most students continued to use this logic even after being reminded that zero was already marked for them on the page. By the post-test, on average, students in the game group made significant progress placing integers closer to their actual locations; however, several continued to place positive numbers to the left of zero on some trials.

CONCLUSIONS AND IMPLICATIONS

As demonstrated by the large gain in the game group, once students knew that the negative sign was an important feature for designating a new number, students were quick to learn the negative integer names. Further, a significant number of students in the game group successfully counted back to -10, suggesting that saying the backward number sequence as part of regular instruction can help students extend the whole number sequence to the negative integers. Interestingly, some of these students did not count backward to -10 even though they could do this while playing the board game. Students are generally expected to stop counting once they reach zero, so continuing into the negatives may not have seemed an appropriate response to our prompting. Stopping at zero could also contribute to students' resistance in subtracting a larger number from a smaller one.

Although the integer value result was significant, even students who could count backward to -10 had difficulty determining which of two integers was closer to 10. This is striking because students in the game group often commented about how they did not want to be in the negatives because their goal was to get to positive ten. These results provide further evidence that students rely on the absolute value meaning of integers, using zero as a reference point, and that this reasoning trumps their inclination to consider numbers further to the right on the number line as larger. Students need explicit experiences to help them negotiate the differences between absolute value and ordered value; including language into the game about integer values could provide needed scaffolding. Finally, the number line results suggest that teachers should talk about and use number lines that do not always start at zero to help students from overgeneralizing ideas about zero. Students already had strong conceptions about zero at the beginning of first grade, suggesting targeted instruction about zero and its placement relative to all integers could be beneficial before this point.

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