MATHEMATICAL COMPETENCIES IN A ROLE-PLAY ACTIVITY

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This paper concerns a competencies-based analysis of the outcomes of a role-play activity aimed to foster conceptual understanding of mathematics for first year engineering students. The teacher role has been considered in order to investigate the competencies addressed by the questions created by the students and their matching with the activity's educational goal. The analysis shows that the quality of the posed questions made by the students highlights the moving from the instrumental approach, the students are used to, towards a relational one.

INTRODUCTION

In this paper we focus on the analysis of the outcomes of a role-play activity aimed to foster *conceptual* understanding of mathematics for first year engineering students. The design of the activity was suggested from the fact that, during some interviews, some students ascribed their poor performance to *strange* and *unexpected* questions. This suggested the idea to support the students by on-line, time restricted activities based on role-play, which actively engage them and induce them to face learning topics in a more critical way. Students are expected to play the role of a teacher in order to force them *to ask questions*.

In the following we are going to investigate the outcomes produced by the students and to discuss the findings with respect the goal of the activity. We have used the Niss competencies framework (2003), also referred by the European Society for Engineering Education – SEFI (2011), to analyse the questions created by the students assuming the teacher role. Our research questions were:

a. what competencies are addressed by the questions posed by the students?
b. does the posed questions address relational knowledge/conceptual understanding rather than instrumental ones?

Finally, we try to draw some ideas for further work concerning the other roles played by the students.

THEORETICAL BACKGROUND

Competence in mathematics is something complex, hard to define which requires the students not only knowledge and skills, but at least some measurable abilities, which Niss names *competencies* (Niss, 2003). He has distinguished eight characteristic cognitive mathematical competencies. The following table lists them in two clusters:

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The ability to ask and answer questions in and with mathematics	***The ability to deal with mathematical language and tools***
[1] Mathematical thinking competency	[5] Representation competency
[2] Problem handling competency	[6] Symbols and formalism competency
[3] Modelling competency	[7] Communication competency
[4] Reasoning competency	[8] Tools and aids competency

Table 1: Cluster related to cognitive mathematical competencies

Mathematical thinking competency includes understanding and handling of scope and limitations of a given concept; posing questions that are characteristic of mathematics and knowing the kinds of answers that mathematics may offer; extending the scope of a concept by abstracting and generalizing results; distinguishing between different kinds of mathematical statements (theorems, conjectures, definitions, conditional and quantified statements).

Problem handling competency includes possessing an ability to solve problems in different ways; delimitating, formulating and specifying mathematical problems.

Modelling competency includes analysing the foundations and properties of existing models, and assessing their range and validity; decoding existing models; performing active modelling in given contexts.

Reasoning competency includes understanding the logic of a proof or of a counter-example; uncovering the main ideas in a proof, following and assessing other's mathematical reasoning; devising and carrying out informal and formal arguments.

Representation competency includes understanding and utilising different kinds of representations of mathematical entities; understanding the relations between different representation of the same object; choosing, making use of and switching between different representations.

Symbols and formalism competency includes decoding symbolic and formal language; understanding the nature of forma mathematical systems; translating back and forth between symbolic language and verbal language; handling and manipulating statements and expressions containing symbols and formulas .

Communication competency includes understanding other's mathematical texts in different linguistic registers; expressing oneself at different levels of theoretical and technical precision.

Tools and aids competency includes knowing and reflectively using different tools and aids for mathematical activity.

EXPERIMENT SETTING AND METHODOLOGY

The setting is a University with a 3-year BSc degree in Electronic Engineering and first year students taking part in a two trimester intensive modules in mathematics. Our research focus on the second module, which concerns topics from linear algebra and calculus. The module has ten hours per week in face-to-face traditional lectures/exercises sessions, supported by an e-learning platform which provides the students with various learning resources and communication tools. The experiment has been performed with voluntary students, who were liked to be involved in a massive and more interactive use of the e-learning platform.

From the viewpoint of the theory of mathematics education, the online experimental activity, we are going to describe, can be framed within the so-called 'discoursive' approach (Kieran et al., 2001). The activity is based on role-play and has been organized as follows. The course contents have been split into different parts and each part into as many topics as the involved students. For each part a cycle of activities based on role-play has been created. Three topics have been assigned to each student, corresponding to three roles played by the student. Each cycle took nine days, three per role. For the *first topic*, the student acts as a teacher who wants to evaluate the topic's learning so he/she has to prepare some suitable questions – at least six questions. For the *second topic*, the student has to answer to the questions prepared by a colleague. Finally for the *third topic*, the students again acts as a teacher, checking the correctness of the work made by the previous two colleagues. At the end of each cycle, the files produced by the students were revised by the teacher-tutor of the course and the revised files were made available to the students. All the produced worksheets were stored in a shared area of the platform in order to be available to all the students.

A COMPETENCIES-BASED ANALYSIS

In the following we want to analyse students' work concerned the first role using the framework of the above Niss competencies. The methodology used for the analysis has been adapted from Jaworski (2012, 2013).

Let us see some examples (the number in the square brackets refers to the table 1).

In the first role we find questions asking for:

The definition of some concepts involved in the topic at stake:

Q1: "What is an Euclidian space?"

Q2: "Which means "*f* differentiable in *x*"?"

Q3: "Given the basis B = $\{u_1,\ldots,u_n\}$ of V, you can write v = $x_1u_1+\ldots+x_nu_n$ for suitable $(x_1,\ldots,x_n)$ ϵ to..?"

Q4: "Which relation does exist between vector space and Euclidean space?"

We note the different formulation of the first two questions, which refers to different expectations and then different competencies. While in all the cases the expected answers require the student to recognize and the scope and limitation of the mathematical concept [1], in the second case the ability to deal with mathematical language seems to be stressed [5, 6, 7]. In fact, questions such as “what is…?” let the students to answer using for instance only formal language, reproducing a definition in a textbook; questions such as “which means…?” require more deeply understanding which allow the student to use various mathematical representations, including verbal, to understand formal language and to translate it to verbal language and finally to express oneself mathematically in different ways. The third question refers to the span property of a basis. It requires the students to includes to handle symbolic expression [6], recognize the concept/property [1] and knowledge its scope and limitations [1]. The fourth question, instead, concerns the scope and the limitation of the two concepts at stake [1], but it also requires to make connections between them, recognizing for instance if and how one extends the properties of the other class of objects.

The understanding of the steps in a given proof:

Q5: “In which steps of the proof the linearity of the function is used?”

Q6: “Why the Lagrange theorem is applied in the interval $[x,x+h]$?”

Q7: “The equation $y'(x) = g(x)$ for which theorem has solution in $[a, b]$?”

The above questions refer to proof of theorems seen by students during lectures and available in their textbooks (Q5 – differential theorem, Q6 – dimension theorem, Q7 – Cauchy problem for differential equations). All of them require the students first of all the ability to understand already existing chain of logical arguments in order to prove a statement starting from fixed hypotheses [4]. Moreover, questions such as Q5 require the student to make his own chain of arguments in order to justify the application of a given theorem [4] and also to express himself mathematically [7], whilst questions such as Q7 require to make connections with previous knowledge to justify a statement in the proof. Finally, we note that, in order to answer the questions, students need to recognize some mathematical concepts (homomorphism in Q5) and to understand their scope and limitation (Lagrange theorem in Q6) [1].

The recognition of the main ideas in a proof:

Q8: “Which are the main steps in the proof of the differential theorem?”

Q9: “In the proof of the Steinitz lemma, which is the fundamental step allowing to prove the thesis?

Both questions refer to the ability of uncovering the central ideas in given proofs [4]. At the same time the answer requires the student from one hand the ability of express himself mathematically in different ways [7], also using verbal language, and thus it requires the capability to understand symbolic language in formal proof and translate in verbal language [6]. Moreover, the answer to Q9 requires the student to connect the existence of non-trivial solutions of a suitable homogeneous linear system to the existence of non-trivial solutions of the vector equation in the definition of linear dependence of vectors [1].

The construction of their own proofs:

Q10: "In the proof of the Steinitz lemma, why the rank of A is less or equal to *n*?"

Q11: "In the proof of the differential theorem, prove that all the hypotheses needed to apply the Lagrange theorem are verified"

Q12: "Is in R^n (n>1) differentiability equivalent to continuity?"

The above first two questions refer to the ability of constructing informal and formal own arguments in order to justify and make clear some steps in a given proof [4]. This require the capability of handling and manipulating symbolic statements and expressions and switch between them and verbal language [6] and the ability of express himself at a certain level of theoretical and technical precision [7]. The difference between the two questions seems to be a greater formality of Q11 with respect to Q10, made evident by the use of the word "prove", highlighting different weights of [6] and [7] for each of them. The last question requires the student to identify the scope of the equivalence between differentiability and continuity – just R [1] – and it is expected that the student is able to prove the true implication and to give a counter-example in the other case [4, 7].

The conversion among various semiotic representations:

Q13: "In the Cauchy's problem which means the expression $y'(x_0) = y_0$ graphically?"

Q14: "Explain in words the Cauchy problem"

Q15: "Write the Cauchy problem (in mathematical language)"

The above questions refer explicitly to the ability of using different kinds of semiotic representation systems of mathematical entities, including verbal language, and the capability of passing from one to another, which is the Duval conversion process (Duval, 2006). Even if we have already noted that such process is implicitly required in other questions, here it is the main focus and it seems us important since Duval states that such capability has to be trained and suggests to make such kind of explicit activities. The pre-requisite of such questions concerns symbol and formalism competency and the answer to Q13 and Q14 requires communication competency.

According to the methodology shown by the above examples, all the questions made by the students has been analysed and for each of them the addressed mathematical competencies have been individuated. The following table resumes the outcomes of this analysis – L* refers to linear algebra topics and C* to calculus ones.

The course setting does not make use of tools, so the related competency has not taken into consideration.

Looking at the outcomes, we can note that the quite predominant competencies addressed by the questions concern the ability to ask and answer questions in and with mathematics, in particular *thinking* and *reasoning mathematically*. Interviews have give evidence that it depends on the teacher role played by the students, which have emulated the way their teachers act with them during exam sessions.

Competency\ Topic	L1	L2	L3	L4	L5	L6	L7	L8	C1	C2	C3	C4	C5
Thinking math.	3	7	6	5	11	9	5	5	3	6	6	3	4
Problem solving	1	2	1	1	5	0	2	1	0	0	3	1	2
Modeling math.	0	1	0	0	0	0	0	0	0	0	0	0	0
Reasoning math.	3	1	4	3	0	5	2	3	2	8	4	1	0
Representation	4	1	0	0	0	5	0	0	2	0	1	4	5
Symbols	6	2	0	0	0	5	2	0	0	2	1	5	3
Communication	6	8	10	6	10	13	8	6	6	7	6	9	9
Aids and tools	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 2: Analysis of mathematical competencies in posing questions.

Moreover, also the *communication* competency is strongly addressed, for the nature itself of the activity which requires the students to express mathematically each other.

Considering the above remarks, we can state that most of the questions address relational knowledge/conceptual understanding rather than instrumental ones, and thus the goal of the activity seems to be achieved from our point of view.

This conclusion has been also supported by:

- students' feedbacks, which reports their appreciation of the teacher role, because it has allowed them to be in the teacher's perspective, so getting able to understand the educational goals which are more conceptual than instrumental;
- students' marks at the next exams, which have obtained a better advancement, due to the fact that this kind of activity has given the students a sort of guidance for the organization of their study, providing time constrictions, topics to revise, indications of the relevant activities.

Moreover, the students report that to *ask questions* have helped them to study in a more critical and deeper way, with greater care, because it is not simple to pose a question due to the fact that there is no method to do that. At the same time, the request of a certain minimum number of questions on a topic requires to range over all the programme, not only concentrating on the specific and restricted topic but also paying attention to all the other linked topics. It is also interesting to note that some students has used this role to make critical points clear (posing as questions exactly their own doubts). Finally we noticed some non-cognitive aspects such as the trend to pose non trivial questions, also for pride reasons, and this has required the mastery of the topics.

CONCLUSIONS AND FUTURE WORK

In this paper we have began to analyze the outcomes of a role-play activity aimed to foster conceptual understanding of mathematics for first year engineering students. The analysis has been performed using the Niss competencies and SEFI framework and has concerned the work of the students in the teacher role.

We plan to continue the analysis of the second role, in particular we are interesting to see what competencies are addressed by answering to the posed questions and its matching with the expected competencies revealed by the questions.

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