PROBLEM-SOLVING STRATEGIES AS A MEASURE OF LONGITUDINAL CURRICULAR EFFECTS ON STUDENT LEARNING

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This study examined the longitudinal effects of a middle school reform mathematics curriculum on students' open-ended problem solving in high school. Using assessment data from a large, longitudinal project, we compared the open-ended problem-solving performance and strategy use of high school students who had used the Connected Mathematics Program (CMP) in middle school with that of students who had used more traditional mathematics curricula. When controlling for sixth-grade state mathematics test performance, high school students who had used CMP in middle school had significantly higher scores on a multipart open-ended problem. In addition, high school students who had used CMP appeared to have greater success algebraically abstracting the relationship in the task.

INTRODUCTION

Problem solving is an integral focus of the school mathematics curriculum. Studies of problem solving in mathematics education have already moved from a focus only on the product (i.e., the actual solution) to a focus on the process (i.e., the set of planning and executing activities that direct the search for solution). Individual differences in solving mathematical problems can sometimes be understood in terms of differences in the uses of different strategies. Proficiency in solving mathematical problems is dependent on the acquisition, selection, and application of both domain-specific strategies and general cognitive strategies (Schoenfeld, 1992; Simon, 1979). Thus, competence in using appropriate problem-solving strategies reflects students' degrees of performance proficiency in mathematics. This implies that assessment tasks should reveal the various strategies that students employ. In addition, students' problem-solving strategies become more effective over time. In fact, researchers have long used the examination of problem-solving strategies to assess and evaluate instructional programs and education systems (Cai, 1995; Fennema et al., 1998). Therefore, both the examination of the strategies that students apply and the success of those applications can provide information regarding the developmental status of students' mathematical thinking and reasoning.

The purpose of this study is to use problem solving strategies to investigate how the use of different types of middle school curricula affects the learning of high school mathematics for a large sample of students from ten high schools in an urban school

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district. This paper reports findings from a large project, *Longitudinal Investigation of the Effect of Curriculum on Algebra Learning* (LieCal).

BACKGROUND AND RATIONALE OF THE STUDY

The LieCal Project began with an investigation of the differential effects of a reform middle school mathematics curriculum called the Connected Mathematics Program (CMP) and more traditional (called non-CMP) curricula on middle school students' learning of algebra. The CMP and non-CMP curricula are very different. In particular, they make use of strikingly different conceptions about algebra – a functional approach in the CMP curriculum and a structural approach in the non-CMP curricula. For example, the CMP curriculum defines a variable as a quantity that changes or varies. The variable idea is needed to describe relationships in the problem situations that the CMP curriculum uses. In contrast, the non-CMP curricula define a variable as a symbol (or letter) used to represent a number. Variables are treated predominantly as placeholders and are used to represent unknowns in expressions and equations. By introducing the concept of variables in this fashion, the non-CMP curricula support a structural approach to algebra. In the non-CMP curricula, similarly, equation solving is introduced symbolically by using the additive and multiplicative properties of equality (equality is maintained if the same quantity is added to, subtracted from, multiplied by, or divided into both sides of an equation). On the other hand, in the CMP curriculum, equation solving is introduced using real-life contexts that are incorporated into contextually based justifications of the equation-solving steps.

In the LieCal Project, we found that on open-ended tasks assessing conceptual understanding and problem solving, the growth rate for CMP students over the three middle school years was significantly greater than that for non-CMP students (Cai et al., 2011). At the same time, CMP and non-CMP students showed similar growth over the three middle school years on the multiple-choice tasks assessing computation and equation-solving skills. These findings suggest that the use of the CMP curriculum is associated with a significantly greater gain in conceptual understanding and problem solving than is associated with the use of the non-CMP curricula. However, those relatively greater conceptual gains do not come at the cost of lower basic skills, as evidenced by the comparable results attained by CMP and non-CMP students on the computation and equation solving tasks.

The LieCal Project has subsequently followed the students into their high school years. All high schools in the district are required to use the same district-adopted mathematics curriculum. CMP and non-CMP students were mixed into each class in each of ten high schools in the same district. Thus, all of the former CMP and non-CMP students used the same curriculum and were taught by the same teachers in their high schools. We have been examining whether the superior problem-solving abilities gained by the CMP students in middle school result in better performance on a delayed assessment of mathematical problem solving in high school.

In a previous study, we used problem posing as a measure of middle school curricular effect on students' learning in high school (Cai et al., 2013). Using problem posing as a measure, we found that in high school, students who had used the CMP curriculum in middle school performed equally well or better than students who had used more traditional curricula. The findings from this previous study not only showed evidence of the strengths one might expect of students who used the CMP curriculum, but also demonstrated the usefulness of employing a qualitative rubric to assess different characteristics of students' responses to the posing tasks. In the same vein, the present study uses open-ended problem-solving strategies as a measure to examine longitudinal curricular effect on students' learning.

METHOD

Participants

In the LieCal Project, we followed more than 1,300 students (650 using CMP and 650 using non-CMP curricula) from a school district in the United States for three years as they progressed through grades 6-8. In the 2008-2009 school year, most of these 1,300 CMP and non-CMP students from the middle school study entered high schools as freshmen. We then followed the students enrolled in the 10 high schools that have the largest numbers of the original 1,300 CMP and non-CMP students.

Assessment Tasks and Analyses

As part of the LieCal Project, we developed and used 13 open-ended tasks to assess students' learning in high school, specifically the 11th and 12th grades. Students' responses were analyzed in two ways. The first was to quantitatively score each student response using a prior-developed holistic scoring rubric. The second was to qualitatively analyze students' responses with a focus on their solution strategies. In this paper, we mainly draw on results from an analysis of solution strategies to a pattern problem called the doorbell problem (see Appendix). This five-part task assesses students' ability to find regularities of a pattern and make generalizations. We chose to report the results from this task as it is a representative task that assesses students' generalization skills.

Data Collection and Coding

As part of the larger longitudinal study, we assessed students in the fall of 11th grade (Fall, 2010), spring of 11th grade (Spring 2011), and spring of 12th grade (Spring 2012). The data for the analyses of students' strategies came mainly from the 12th grade spring assessment. In a small number of cases, if a student did not participate in the Spring 2012 assessment but did participate in the Spring 2011 assessment, we used the data from the Spring 2011 assessment. If a student did not participate in either the Spring 2012 or Spring 2011 assessments, but had participated in the Fall 2010 assessment, we used the data from the Fall 2010 assessment. This allowed us to look at the students' most recent attempt at each task.

As noted above, students' responses to the doorbell problem were first scored using a holistic scoring rubric that took into account the students' numerical answers and their explanations of their strategies. The responses were then also qualitatively coded for the types of strategies used. We coded students' solution strategies for parts A, B, C, and E as an abstract strategy, a concrete strategy, an unidentifiable strategy, or no strategy. Students who used an abstract strategy were able to recognize that the number of guests entering for each ring was equal to either two times the ring number minus one (i.e., $y = 2n - 1$) or the ring number plus the ring number minus one (i.e., $y = n + (n - 1)$). Students who used a concrete strategy were able to identify that the number of guests who enter increases by two for each doorbell ring and then sequentially adding two until they reached the desired number of rings, but did not abstract an algebraic formula. An unidentified solution strategy was a strategy that did not particularly make sense for the problem (e.g., $y = [r(100) + 2] - 1$). Lastly, a student was said to have used no strategy if the student did not show work for his or her answer, or if he or she did not attempt to answer the question at all.

Students' strategies for part D were coded in one of five ways. First, the student could have completely abstracted the algebraic formulas $2n - 1$ or $n + (n - 1)$. Secondly, they could have completely abstracted the pattern in a verbal description (e.g. "The number of guests who entered on a particular ring of the doorbell equalled two times that ring number minus one."). Third was an incomplete abstraction that only captured a recursive relationship, such as, "When the bell rings, two more people come." Fourth was an unidentified strategy, which either represented the strategies for students who incorrectly answered the question or had a provided a strategy that did not make sense. Finally, a strategy was coded as "no strategy" if no attempt was made to solve the problem.

RESULTS

Overall Performance on the Doorbell Problem

We first conducted two ANCOVA analyses based on the quantitative scoring to student responses to the doorbell problem. The ANCOVA analyses indicated significant curriculum effects under two covariates for the doorbell problem. When controlling for overall state math test exam scores for 6th grade, CMP students scored significantly higher than non-CMP students on the doorbell problem ($t = 2.09$, $p = 0.0371$). When controlling for scores on the algebra subtest on the overall state math test for 6th grade, CMP students still scored significantly higher than non-CMP students ($t = 2.47$, $p = 0.0141$).

Performance on Individual Parts of the Doorbell Problem

Chi-squared tests were performed to look for relationships between curriculum and correctness of answers on each part of the doorbell problem. For part A, there was a significant relationship between curriculum and correct answers ($\chi^2 = 6.5363$, $p < 0.040$). That is, a significantly larger percentage of the CMP students had correct answers than the non-CMP students. For parts B, C, D, and E, there were no significant relationships between curriculum and correct answers. For each of the five parts of the problem, Table 1 provides the percentage of students with correct answers in that part. Note that Table 1 shows a considerable decreasing trend in the number of students who found a correct solution from part A to part E.

	Doorbell Problem Part				
Curriculum	A	B	C	D	E
CMP (n = 321)	80.1	38.6	27.7	18.1	7.5
Non-CMP (n = 212)	74.5	35.4	27.4	16.0	5.2

Table 1: Percentages of CMP and non-CMP students who correctly solved each part of the Doorbell Problem

Concrete and Abstract Solution Strategies

Focusing specifically on the solution strategies of those students who provided correct solutions for parts of the Doorbell problem, some differences in strategy use arose between the two groups. For part B (see Table 2), 73.4% of CMP students (n=124) and 60% of non-CMP students (n=75) abstracted the problem to an algebraic formula to find the correct solution, whereas 17.7% of CMP students and 24.0% of non-CMP students used a concrete strategy. A significantly greater proportion of CMP students used the abstract strategy than did the non-CMP students (z = 1.97, p < 0.050), but there was no significant difference in proportion between CMP and non-CMP students for the concrete strategy.

For part C (see Table 2), 71.9% of CMP students (n=89) and 67.2% of non-CMP students (n=58) abstracted the problem to an algebraic formula, whereas 7.9% of CMP students and 19.0% of non-CMP students used concrete strategies to find a correct solution. A significantly greater proportion of non-CMP students used the concrete strategy than did the CMP students (z = -2.27, p < 0.025), but there was no significant difference in proportion between CMP and non-CMP students for the abstract strategy.

For part A (see Table 2), 67.3% of CMP students (n = 257) and 63.9% of non-CMP students (n = 158) used a concrete strategy to find the correct answer, whereas 26.1% of CMP students and 27.8% of non-CMP students abstracted the problem to an algebraic formula. There were no significant differences in proportion between CMP and non-CMP students for each strategy.

For part D, almost every student who provided a correct solution responded in nearly the same way. All of the 34 non-CMP students and 54 out of 58 CMP students who correctly answered this part generated an algebraic abstraction and provided a mathematical formula. The remaining four CMP students wrote out a verbal description of the mathematical formula, which would still require them to have first abstracted the relationships before translating those relationships into written form.

Problem part	*n*	Type of strategy			
		Abstract	Concrete	Unidentified	None
A					
CMP	257	26.1	67.3	3.5	3.1
Non-CMP	158	27.8	63.9	1.9	6.3
B					
CMP	124	73.4	17.7	3.2	5.6
Non-CMP	75	60.0	24.0	4.0	12.0
C					
CMP	58	71.9	7.9	9.0	11.2
Non-CMP	34	67.2	19.0	5.2	8.6
D					
CMP	58	100.0	0.0	0.0	0.0
Non-CMP	34	100.0	0.0	0.0	0.0
E					
CMP	24	62.5	29.2	4.2	4.2
Non-CMP	11	45.5	36.4	0.0	18.2

Table 2: Percentages of CMP and non-CMP students who used each type of strategy to correctly answer parts of the doorbell problem

Part E seemed to be a challenging question for both the CMP and non-CMP students. Only 24 CMP students and 11 non-CMP students provided a correct solution to this part of the doorbell problem. Given these small sample sizes, although there were noticeable group differences in raw percentages of students using algebraic and concrete strategies, with a greater proportion of CMP students than of non-CMP students using algebraic strategies, these differences were not statistically significant.

DISCUSSION

As part of a larger longitudinal study of curricular effect on mathematics learning, the results we have presented above provide a useful perspective on the potential long-term impacts of reform mathematics curricula on students' mathematical thinking and problem solving. Although we have presented data from only one open-ended task, the results suggest that high school students who used the CMP curriculum in middle school were more successful than their peers who used more traditional middle-school curricula at solving the doorbell problem and explaining their solution strategies. This result accords with those obtained when these students were still in middle school (Cai, et al., 2011). The result is also consistent with our previous findings using problem posing as measure of curricular effect (Cai et al., 2013). Thus, it would appear that the CMP students' problem-solving gains persist well into high school.

The retention of these gains over longer time intervals also parallels the findings from research on the effectiveness of problem-based learning (PBL) in medical education (Hmelo-Silver, 2004). In that context, medical students trained using a PBL approach performed better than non-PBL students on conceptual understanding and problem-solving ability even when assessed at a later time. In a similar fashion, the CMP students in the LieCal project experienced problem-based instruction that focused on developing students' conceptual understanding and problem solving abilities.

In addition, our analysis of the strategies used by the students in this study suggests that the CMP students who correctly solved the parts of the doorbell problem were somewhat more likely to make generalizations. This appears to reflect the emphasis in the CMP curriculum on relationships between quantities (i.e., the functional approach). The ability to abstract algebraic relationships from real-world situations appears to also have persisted in the CMP students.

Note that for this analysis, we focused on the strategies of students who correctly answered one or more parts of the doorbell problem. We did not consider the strategies of students who failed to provide correct answers. Additional analyses that will further probe the strategies of students who provided incorrect answers are in progress at the time of this proposal. Also, we are analysing data from other open-ended problems.

APPENDIX

Sally is having a party.

The first time the doorbell rings, 1 guest enters.

The second time the doorbell rings, 3 guests enter.

The third time the doorbell rings, 5 guests enter.

The fourth time the doorbell rings, 7 guests enter.

Keep going in the same way. On the next ring a group enters that has 2 more persons than the group that entered on the previous ring.

A. How many guests will enter on the 10^{th} ring? Explain or show how you found your answer.

B. How many guests will enter on the 100^{th} ring? Explain or show how you found your answer.

C. 299 guests entered on one of the rings. What ring was it? Explain or show how you found your answer.

D. How many guests will enter on the n^{th} ring? Show or explain how you found your answer.

E. If we count all of the guests who entered on the first 100 rings, how many would we get in total? Show or explain how you found your answer.

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