

COMPETENCIES DEVELOPED BY UNIVERSITY STUDENTS IN MICROWORLD-TYPE CORE MATHEMATICS COURSES

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We report on an empirical study grounded in our sustained implementation over ten years of a sequence of three-term undergraduate core mathematics courses centred on microworlds. The survey study investigates students' views on 15 competencies potentially developed as they, individually or in pairs, create 12 Exploratory Objects, i.e., microworld-type environments, on diverse mathematical topics as part of their workload. Results suggest that students develop further the competencies as they repeat designing, programming, and using microworlds to learn and do mathematics, and that original projects in which students start by selecting their own topic, is key to the development of these competencies. No gender differences were found.

INTRODUCTION

Mathematics microworlds have long been acknowledged as providing a rich mathematics learning experience for students (Healy & Kynigos, 2010). There is abundant literature, mainly at the research level, on the topic. Most involve a few student participants (e.g. Wilensky, 1995) or a class for a one-time project (e.g., Jiménez, Gutiérrez, & Sacristán, 2009). There seems to be relatively little sustained classroom implementation of microworlds, probably for the reason that “[t]he ideas behind the microworld culture have not yet been presented in a form readily acceptable not only to school systems, but also to other stakeholders in education” (Healy & Kynigos, 2010, p. 68). In this paper we report on an empirical study on competencies grounded in our sustained implementation over ten years of a sequence of undergraduate core mathematics courses centred on microworld-type activities.

CONTEXT

Mathematics Integrated with Computers and Applications (MICA) program is a unique core undergraduate mathematics program offered at Brock University since 2001 (Ralph, 2001). As a central component of the program are the innovative first-year MICA I and second-year MICA II courses, two core project-based courses for mathematics majors and future mathematics teachers. We can describe these courses by their common activity repeated throughout the two courses (at least four times/term), though each time in a more complex situation and on a different mathematics topic: to design, program, and use an interactive and dynamic computer-based tool, called an *Exploratory Object* (EO), for systematically investigating a mathematics concept, theorem, self-stated conjecture or a real-world situation (Muller, Buteau, Ralph, & Mgombelo, 2009). These microworld-type environments are either assigned to them, i.e. the topic and exploration questions are

provided to students through guidelines, or are original projects in which students start by selecting their own topic. For example in 2011-12, 471 assigned EOs and approximately 98 original EOs were created. Examples of original students' EOs can be found in (MICA, n.d.).

To date we have conducted diverse studies based on our insightful reflections about our MICA students' experiences. For example, we have examined the students' instrumental genesis of programming technology to create their own EOs for their mathematical investigations (Buteau & Muller, 2014). Based on a task analysis (Buteau & Muller, 2010), we have recently conducted a literature review aiming at contextualizing the EO learning activity. As a result, we describe that students engaged in an EO activity "experience, in a context of experimental mathematics, inquiry-based learning and mathematics learning through programming and simulation" (Marshall & Buteau, forthcoming, [p. 17]). The review included literature about microworlds (in the area of 'learning university mathematics through simulation'). The literature study also aimed at theoretically identifying competencies that could be attained through the EO learning activity, resulting in a list of 15 competencies (Marshall, Buteau, & Muller, forthcoming); see Table 1.

	Competencies
a	To self-motivate to learn/do mathematics
b	To engage in divergent thinking
c	To research mathematical topics
d	To develop mathematical intuition
e	To understand mathematical models
f	To closely reflect on problems
g	To program mathematics (simulations, mathematical experimentation etc.)
h	To get a feel for inappropriate answers
i	To work with abstraction
j	To visualize mathematics
k	To connect different representations of concepts
l	To interpret mathematical results
m	To communicate one's mathematical results
n	To engage in the process of mathematics research
o	To learn/do mathematics independently

Table 1: Theoretically identified competencies developed through the EO activity.

These competencies were thereby identified in the literature in areas with common activity elements to the EO learning activity. Many of these competencies were discussed in the context of microworlds (Marshall et al., forthcoming).

Aiming now at empirical research about the students' experiences through the repetitive microworld-type activities implemented in our department, we first conducted a study to gather preliminary empirical evidence of our theoretical results. Our guiding research questions were: what are the students' views on i) the nature of the MICA I & II courses, and ii) the competencies developed in these courses? In the following we report on the results found in relation to the second question.

METHODOLOGY

To provide some insight into the research questions a student survey was undertaken. The voluntary and anonymous on-line survey was run, during laboratory sessions, of the MICA I and MICA II courses of the 2012/13 academic year. The questionnaire contained three sections. Section 1 focused on the demography of the respondents; Section 2 inquired about students' views on the nature of the MICA courses; and Section 3 questioned their views on competencies developed in these courses. Questions in the latter two sections were based on our theoretical results (Marshall & Buteau, forthcoming; Marshall et al., forthcoming). In the third section students were asked questions that focused on: each of the 15 competencies listed in Table 1; two competencies related to more traditional mathematics courses, namely y- 'to write mathematics proofs' and z- 'to perform complex calculations by hand'; and an optional open-ended question to comment on any of the competencies. The students' responses were recorded on a five point Likert scale, for example, the question related to competency a) in Table 1 was, "The activities in the [MICA I (and MICA II)] course[s] prompt me to self-motivate to learn and do mathematics" with answer options: "[4]-Very much; [3]-Much; [2]-Some; [1]-Not at all; or No opinion". In this paper, we only report on the survey results related to competencies (third section).

The results from the 17 Likert-scale questions were analysed using simple descriptive statistical methods. Mann Whitney non-parametric tests were used to identify significant statistical differences between groups of participants. Qualitative data from the open-ended question was coded by competencies, followed by a frequency analysis. This data was also used to help interpret the statistical results.

RESULTS AND DISCUSSION

In total 55 MICA students participated in the study (57% participation rate), with 27 MICA I and 28 MICA II students. In terms of gender, 24 female and 31 male students participated. Of the respondents 38% were mathematics majors, 40% future mathematics teachers, and 22% were enrolled in other programs.

Answers to the survey questions provide students' estimations of their acquisition and/or improvement of competencies potentially developed in the MICA courses centred on the EO activity. Although the data is ordinal we decided to calculate and plot means (removing the 'No opinion' responses) in order to provide a visual overview and pointers to possible areas where differences may be found. In the

exploratory graphs created we have joined points to offer more visual distinction between the results by different groups – the lines joining points have no meaning.

Overall the means for each 15 competencies from Table 1 ranged from 2.46 to 3.34 (see Figure 1). It suggests that students view that they've developed, to a certain extent, these competencies through the repetitive EO activity. The two additional competencies (y and z in Figure 1) received the lowest means, i.e., 1.98 and 2.3. In fact, this is in line with the learning objectives of the EO activity, and suggests that the implementation aligns with the activity expectations, namely, to design, program, and use an interactive and dynamic computer based tool to learn and do mathematics. For example, a participant commented that through programming mathematics, “[i]t was interesting to actually visualize the topics studied of mathematical conjectures and projects.” A respondent stressed the self-motivation to learn and do mathematics: “I have been very interested in the exploration of real world applications and creating [EOs] to test real life examples!!!” Another participant also commented that,

[s]ince we code the projects ourselves (mostly) there isn't a nice likes help option to tell us what the results are telling us, so it does require really thinking about what the numbers actually mean.

This aligns with Wilensky's (1995) study involving university mathematics students using microworlds: “It was not until [the student] programmed a simulation of the problem that she began to resolve the paradox" (p. 272).

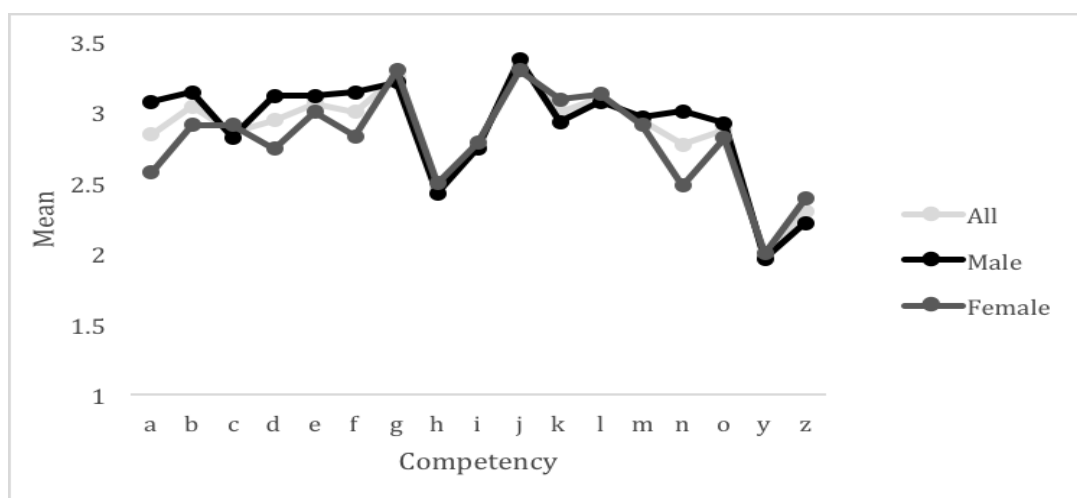


Figure 1: Means of students' views, by gender, on competencies developed through EOs (N=55; with scale: 4-very much; 3-much; 2-some; 1-not at all).

The results by gender in the graph (see Figure 1) seem to be relatively the same. Indeed, when Mann Whitney U tests were performed on data for each competency categorised by gender, no statistical differences ($\alpha=0.05$, two tailed) were found for any of them. This indicates that students, independent of gender, demonstrate a similar awareness that they are acquiring and/or improving the competencies while engaged in the EO activities. This could be contrasted with Barkatsas, Kasimatis, and Gialamas (2009) study which found gender differences about school mathematics students' achievement and views towards the use of technology in mathematics.

A similar comparison was done for responses of MICA I and MICA II students. Figure 2 visually summarizes the results. It is worth noting that for each competency, the mean is greater for MICA II students than the paired mean for MICA I students.

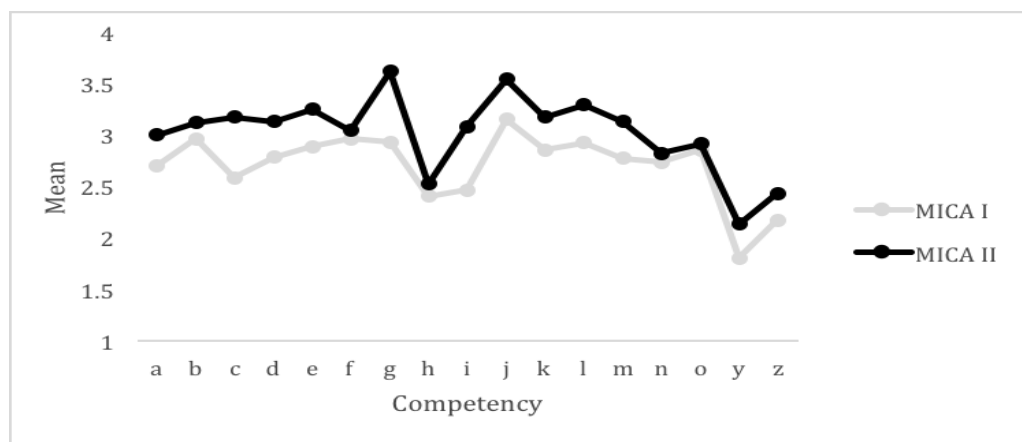


Figure 2: Means of students' views, by course year, on competencies developed through EOs (N=55; with scale: 4-very much; 3-much; 2-some; 1-not at all).

Mann Whitney U tests were performed on the data for each competency categorised by MICA I and MICA II students. Significant differences were found ($\alpha=0.05$, two tailed) for seven of the competencies, namely: c 'to research mathematical topics' ($p=0.032$); e 'to understand mathematical models' ($p=0.029$); g 'to (computer) code mathematics' ($p=0.003$); i 'to work with mathematical abstraction' ($p=0.012$); j 'to visualize mathematical concepts' ($p=0.045$); l 'to interpret mathematical results' ($p=0.021$); and m 'to communicate mathematical results' ($p=0.046$).

The surveys were undertaken close to the end of the academic year, so one would expect that students at the end of their second year of a mathematics program would be more mature, both mathematically and in their ability to work with a microworld-type environment, than their counterparts at the end of their first year. Furthermore students completing MICA II would have realised 11 EOs, including two self-directed Objects and would be working on their third. On the other hand MICA I students would have completed only three EOs and would be engaged in their first major self-directed Object. The differences in Figure 2 point to the possibility that the competencies theoretically identified need repetition and maturity that requires more than one MICA course to become established.

For the fifteen competencies the views of students in the MICA I and MICA II courses were statistically significantly different for seven of them. Because of space limitations we will comment on only two of the seven cases. We have selected one case in which we anticipated a difference, and another one that was a surprise. MICA I students will have experienced by the end of the semester (i.e., after they filled out the survey questionnaire) their first independent 'research of a mathematical topic'. The MICA II (two semesters) course has important research components arising from both the complex questions generated by the instructor through the EO assignments, that require on line research and computer based experimentation, and also from the

mathematical conjectures or real world problems chosen by the students to be studied in their original final EO projects. It was therefore not surprising to find a significant difference between the views of MICA I and MICA II students in regards to the ‘research of a mathematical topic’ competence.

However we had not expected much of a difference of views for the competence ‘to (computer) code mathematics’. In the MICA I course students learn to code through a well-defined sequence of mathematical problems that require an increasing number of different programming concepts (Buteau & Muller, forthcoming). It is therefore possible that students in MICA I are so focused on acquiring the procedures of programming that is new to them that they lose sight of the mathematics. In contrast students in the MICA II course are sufficiently familiar with programming that they may now become more aware that they are ‘coding mathematics’ in their EOs.

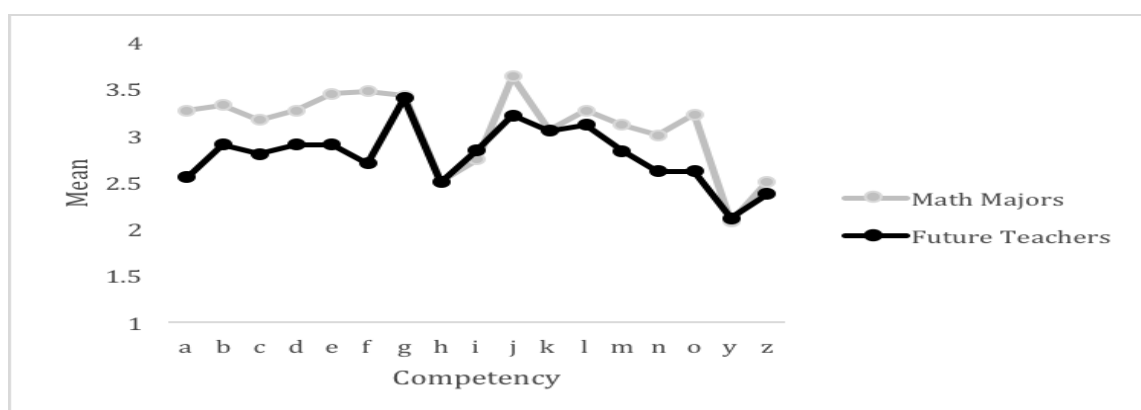


Figure 3: Means of students' views, by program, on competencies developed through EOs (N=55; with scale: 4-very much; 3-much; 2-some; 1-not at all).

Finally we also compared the responses of the students according to their programs, namely the math majors and the future math teachers (Figure 3). We didn't consider students enrolled in other programs. The trends for the responses from the two groups are generally similar, with the math majors mostly providing the greatest agreement (largest means) on their development of the competencies. The Mann-Whitney tests identify significant differences ($\alpha=0.05$, two tailed) between the two groups for only three competencies, namely: e ‘to understand mathematical models’ ($p=0.031$); f ‘to carefully reflect (think over carefully) on mathematical problems’ ($p=0.008$); and o ‘to learn and do mathematics independently’ ($p=0.038$).

Both in MICA I and MICA II courses, there is an over-riding importance placed on the original projects. In both these courses future teachers are allowed to substitute the original EO projects by *Learning Objects*, i.e., interactive, dynamic computer-based environments designed to “engage a learner through a game or activity and that guide him/her in a stepwise development towards an understanding of a mathematical concept” (Muller et al., 2009, p. 64). Thus mathematics majors may experience their original EO project as a means for themselves ‘to learn and do mathematics independently’ (as well as competencies e and f), while the future teacher may experience the Learning Object project as a means for themselves to design a

well-defined sequence of teacher-defined mathematical activities for someone else to learn mathematics. As such, these three competencies (e, f, and o) do not seem to relate to the LO activity, and this could explain why future teachers didn't view developing as much these competencies. We stress that the theoretical list of competencies was generated on the basis of EOs, and for future teachers the list could be modified to include didactic competencies. Overall, this could suggest that original individual EO projects in which students start by selecting a topic of their choice, is key to the development of these three, or many of the 15, competencies.

At the end of the survey, participants were invited to comment on some of the competencies. Figure 4 shows a summary created using *Wordle* (Feinberg, n.d.):

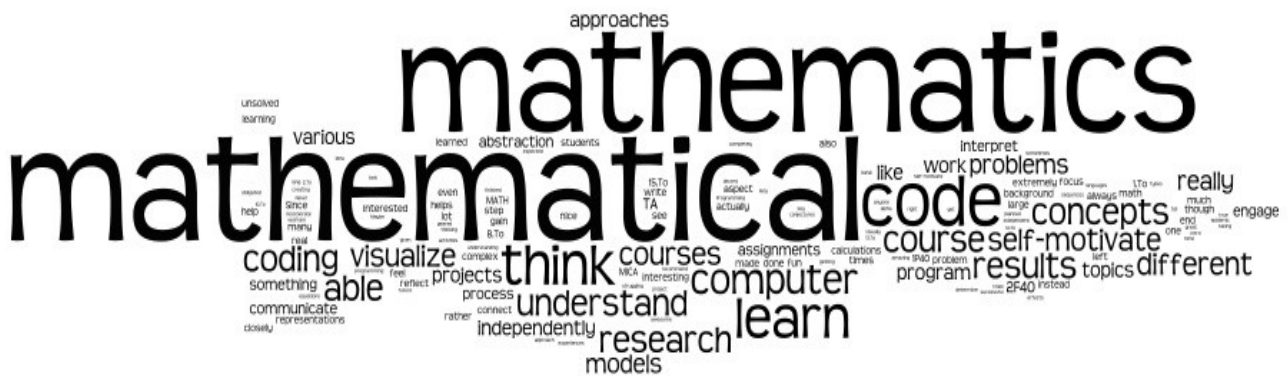


Figure 4: Word cloud of MICA students' comments on competencies.

Clearly the participants identified mathematics as the main focus within the competencies in the MICA courses, followed by coding, thinking, learning, understanding, researching, computer, concepts, and able. When analysing how often each competency appears in the comments (see Figure 5), we find that the most often selected competency for comments was 'to program mathematics', followed by 'to self-motivate to learn/do mathematics', and 'to engage in divergent thinking. Whereas g is the competency, or skill, likely to be most easily identified in relation to the EO MICA courses, the other two (a & b) are deep competencies normally beyond first and second-year university mathematics students.

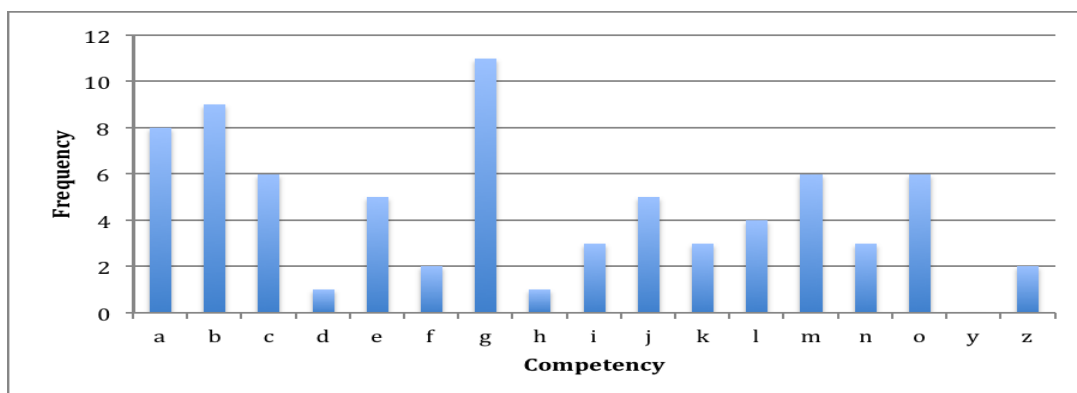


Figure 5: List of competencies selected by students for further comments.

FUTURE RESEARCH

Results of our preliminary empirical study suggest that the 15 theoretically identified competencies (Table 1) may be further developed through a process of *repetitive* microworld-type activities. In addition, the original EO projects, where typically mathematics students independently carry out an investigation of their choice, may be key in developing these competencies. We now aim to conduct a comprehensive empirical study to investigate the evolution of these students' competencies throughout their 12 individual EO activities in the three-term MICA core courses.

Finally, results of the survey study also suggested no gender difference in students' competency development. We postulate this may be linked to the creativity aspect of the EO activity (Buteau & Muller, 2014), which could be a topic of further research.

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