PRIMARY SCHOOL TEACHERS LEARN MODELING: HOW DEEP SHOULD THEIR MATHEMATICS KNOWLEDGE BE?

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We taught a group of experienced in-service primary school mathematics teachers the notion of mathematical model, in order to foster the interdisciplinary mathematics teaching in primary school. In particular, we developed an exercise in which they were supposed to construct a mathematical model on the basis of primary school mathematics. We found out that the formal mathematical knowledge needed to perform the exercise was not sufficient to successfully cope with it. The main factor that influences the ability of the teachers to cope with this type of activity is the depth of their mathematical knowledge which we identify with a person's mathematical insight.

THEORETICAL BACKGROUND

The Israeli primary school curriculum explicitly necessitates the linkage between mathematical curriculum and two other components: other disciplines studied simultaneously, and everyday life experiences. This declaration is realized in several paragraphs of the curriculum, such as data organization and analysis and integrative problems. Nevertheless, the mathematics teaching in Israeli primary schools is usually confined to purely mathematical (mostly arithmetic) contents, with no intentional connections made to the world surrounding the pupils (Arcavi & Friedlander, 2007). We regard this to be an essential drawback and seek ways to cope with it.

Numerous studies indicate the insufficient matching between the mathematical knowledge and skills the schoolchildren are expected to acquire at school, and what they need to be able to do with this knowledge outside the school (English, 2009; Gainsburg, 2006; Pollak, 1979; Zawojewzki & McCarthy, 2007). Hence, the mathematical education specialists face the challenge of finding the ways to cope with authentic and related to them interdisciplinary problems, sometimes rather complicated ones. One of the ways to do so is to embed mathematical model construction in mathematics lessons (English, 2009; Gainsburg, 2008; Kaiser & Schwarz, 2006). In order to embed the interdisciplinary teaching into the mathematics class, several components are needed, such as handbooks, time allocation in the mathematics lessons, and the teachers' competence in the issue. This competence is critical for implementation of interdisciplinary teaching at school. Doerr (2007) claims that teachers refrain from dealing with interdisciplinary problems because their knowledge in mathematical modeling is not sufficient. Hence, it is one of the key research issues – to determine what teachers' knowledge is needed in order to implement modeling at school (Garcia & Ruiz-Higueras, 2011).

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In the present research we focus on the process of building-up the teachers' knowledge of the concept of mathematical model which is pivotal for the interdisciplinary approach (Ng, 2010). In particular, we are interested in the connection between this process and the depth of a teacher's knowledge in other issues of the primary school mathematics curriculum.

Mathematical model is a mathematical object – a graph, a sequence, a diagram, an equation etc., reflecting to a certain extent an outer-mathematical phenomenon. The model construction is a kind of a loop-like process which can be schematically represented in the following way:

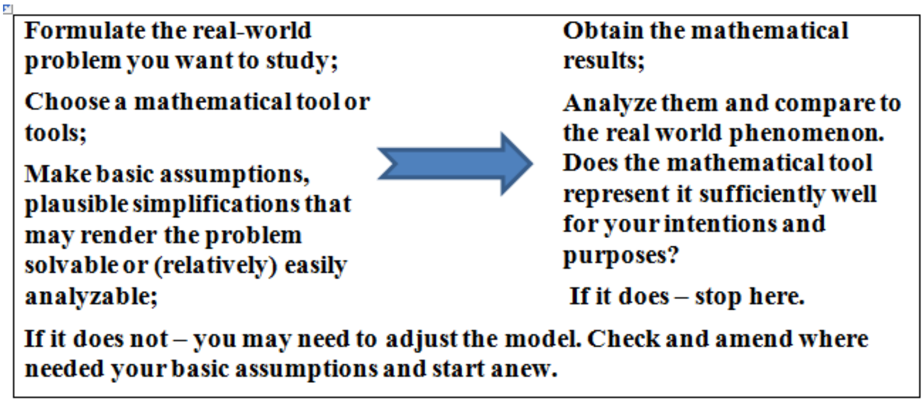

Figure 1: A schematic representation of modeling process

In order for teachers to be able to teach this (as well as any other) approach at school, they must be competent to cope with it at an appropriate level. Speaking of mathematical modeling, we agree with Maab and Gurlitt (2011) who claim that teachers need "modeling competency": the ability to carry out modeling processes independently". Following Cherniak (2007), the empirical research basis in interdisciplinary teaching on which it would be possible to build up practical approaches and curricula, is still lacking, especially in what concerns the teachers' education in these topics. Our research presented here is a part of a bigger research project aimed at the interdisciplinary teaching by expert mathematics teachers in primary school as a part of their professional development.

RESEARCH FRAMEWORK

In this research we follow the process of acquiring the concept of mathematical model by primary school teachers during a one-semester course in mathematical modeling as a part of their M.Ed. program. The notion of a mathematical model was equally new for the audience; nobody has been previously familiar with it.

Our research sought the answer to the following questions:

- Is deep mathematical knowledge of primary school mathematics a necessary basis for the understanding of the concept of mathematical model?
- Is the knowledge of a formal corpus of primary school mathematics a sufficient basis for such understanding?

We studied performance of 14 M.Ed. students who are active and experienced primary school mathematics teachers. In what follows we call them “the teachers”.

Tools and methods

More-or-less in the middle of the course on mathematical modeling the students received an exercise in which they were asked to propose a model for evaluating the paper usage at primary school. The exercise was assessed in two different ways; we looked for possible links between the outcomes of these analyses.

Firstly, we analyzed it by the five parameters included in the assignment formulation. The exercise and the assessment parameters appear in Appendix 1. Each of the parameters was assessed using four-level grading, from the lowest (1) to the highest (4) grade. Table 1 represents the assessment criteria. The abridged notations are explained in Appendix 1.

	DC	BA	MR	MA	EM
1	Non-adequate; data not used in the model	Not formulated	Non-structured tables	Not presented	No evaluation
2	Meticulous sheet-by-sheet data collection on paper usage	Not clearly formulated or not relevant for the model.	Structured tables alone.	Inadequate list; tools not used in the model.	Evaluation in terms of the paper usage outcome.
3	A justified sampling coherent with the model assumptions	Plausibly formulated assumptions but not related to the data collection.	Tables and diagrams of a single type (rod diagrams).	The list of tools is adequate but not complete	Evaluation partly refers to the paper usage outcome
4	A sampling method aimed at the specified model and its purposes.	Coherent, clear and justified assumptions	Adequate variety of tables and diagrams.	A complete and adequate list of tools.	Effective evaluation; clear and motivated amendments.

Table 1: Assessment criteria for the exercise.

Secondly, we assessed it from the viewpoint of the depth of teachers’ understanding of the mathematical corpus of knowledge. Formally speaking, all the teachers master the mathematical apparatus needed for the exercise, which does not exceed the 6th grade level requirements: data representation, arithmetic of multi-digit numbers, zero in arithmetic operations etc. Should this formal corpus be sufficient, we might expect the more-or-less homogenous results all over the group. What we are interested to appraise is how the depth of the teachers’ understanding of this corpus showed itself in the exercise performance; for this we use the notion of mathematical insight. The concise necessary description of the insight parameters appears in Appendix 2.

FINDINGS

Table 2 represents the assessment of the teachers' performance by assignment requirements and by insight components. The teachers are represented in the first column by their numbers. The assessment was validated by three experts

.	Assignment requirements					Insight components		
Nr.	DC	BA	MR	MA	EM	IA	S	ML
1.	1	1	2	1	1	1	3	1
2.	3	4	4	4	4	3	4	4
3.	4	2	3	2	1	2	2	2
4.	4	4	4	4	4	4	4	4
5.	2	3	2	2	1	2	3	2
6.	4	4	3	4	4	4	4	4
7.	2	3	3	3	3	2	4	3
8.	2	2	1	1	1	1	2	2
9.	4	4	4	4	4	4	4	4
10.	2	3	2	2	3	2	2	2
11.	4	3	3	4	4	4	3	3
12.	3	2	1	3	1	1	2	3
13.	4	4	2	4	3	4	3	4
14.	4	4	4	4	3	4	4	4

Table 2: The results of the assessment of the teachers' performance based on the assignment requirements and on the parameters of the mathematical insight.

As one may observe, the results of the group are far from homogenous. In addition, the rows of the Table indicate that the students who performed well according to the assignment requirements also demonstrated higher insight, and vice versa. Having observed that, we decided to "zoom" on the performances of some of the students in order to elucidate the differences. Table 3 enables comparison between the performances in two cases: one of a distinctly low-assessed teacher (Nr.1); one of a distinctly highly-assessed (Nr. 4).

	Nr.1	Nr.4
DC	"…I computed the average paper usage per pupil during the year…". It is unclear how the averaging was performed; the data were not used further in the model.	Based on the school working style using mainly copying: "I asked several teachers how often they hand out copied sheets to their pupils, and evaluated the total."
BA	Mutually incompatible assumptions: "during the academic year a pupil uses about 4 pages a day" (which has nothing to do with the estimate); "A pupil uses in the average at least 500 pages during an academic year" (215 days long).	All the classes are of the same size; the teachers of the same grade work similarly; only writing paper is included; the paper usage for each discipline is similar; most paper usage follows from copying.
MR	Plots a rod diagram for the monthly paper usage for each grade; then plots pie diagrams for the monthly paper usage; Does **not** plot pie or other diagram for the relative paper usage analysis.	A structured table in which the input data are presented in rounded numbers; rod diagrams; pie diagrams representing relative usage by the six grades; comparative rod diagrams
MA	A list of tools most of which were not used or used in a wrong way.	A full list of tools and notions used in the model, such as "ratio", "estimate", "negligible".
EM	"It was difficult to account for all the variables in this problem: the number of pages in a notebook, of copy sheets, etc."	Relates the diagrams to the real school life, e.g. finds real explanations for occasional increases in paper usage;
IA	The teacher clearly did not grasp the idea of mathematical model; she tries in the earnest to gain as precise and extensive information as possible on paper usage.	The newly learned concept of mathematical model is well understood; this can be observed from all the components of the exercise.
S	The teacher's skills are relatively high; e.g., she organizes the data in tables and plots diagrams using the Excel tool; but the skills usage is purely instrumental.	The teacher's skills are well developed and appropriately used; all computations and diagram are well motivated.
ML	Mathematically meaningless usage of such terms as "average", "estimation"; the reasoning comprises logically disconnected statements; uses phrases like "approximately 543 pupils", … "in the average at least".	All the terms are properly used; the reasoning in the work is coherent and consistent;

Table 3

CONCLUSIONS

One of the objectives of this research is to explore whether formal mathematical corpus of knowledge is sufficient for the successful acquiring of the concept of mathematical model. From the preliminary results it can be seen that teachers who succeeded in the exercise demonstrated also higher levels of mathematical insight in this issue. Hence, our preliminary conclusion is that in order to construct and use a mathematical model the teachers should have deeper understanding of the mathematical knowledge they possess. This conclusion is very important: if our goal as educators is developing our students' ability to work with mathematical models, we must find ways to deepen their understanding of the formal corpus of mathematical knowledge they possess.

Appendix 1: the exercise outline

In order to build the model, the students were instructed to perform the following steps (the abridged notation in the parenthesis is used in the text):

Data **c**ollection (DC) – suggest a method of obtaining the information on the paper usage at school needed, in your opinion, to provide a plausible model for evaluation of the school paper usage, such as students' writing habits, teachers' practices, sheets copying policy, etc.

On the basis of their data collection, the students were expected to propose the method for the evaluation of the paper usage at school during the calendric year.

Formulate the **b**asic **a**ssumptions used in the model construction (BA) – e.g., the paper usage in all the classes of the same grade is more or less similar; there are time periods during the calendric year when the paper usage essentially differs from average, for example, during the vacations when it is close to zero. The assumptions of the model are naturally expected to be related to the data collection method proposed by the same student.

Mathematical **r**epresentations (MR) – use the mathematical representations at the primary school level appropriate, in your opinion, for the model, e.g. structured tables; diagrams of various types, etc.

Correct identification of **m**athematical **a**pparatus (MA) – identify the mathematical tools from the primary school curriculum relevant for the assignment, such as working with big numbers (tens of thousands to millions); zero in multiplication and in addition; estimation methods; ratio and proportion; multiplication of multi-digit numbers etc.

Evaluate the **m**odel you have constructed (EM) – having constructed your model and obtained the overall results of the paper usage, indicate to what extent it adequately represents the real school situation; which of your assumptions seem to need re-adjustment; is the model really helpful in estimation of paper usage? Etc.

Appendix 2: the mathematical insight

We present here the components of mathematical insight in a form relevant for the present contents:

The **implementation ability** (IA), by which we mean the ability of a person to apply the recently acquired piece of mathematical knowledge, provided this piece is in his or her mathematical ZPD (Vygotsky, 1978). The implementation is expected to occur in the "neighborhood" of the learned issue, obviously "the farther the better".

In the present setting, the recently acquired mathematical knowledge is the concept of a mathematical model.

Skills (S); by which we mean both the variety of mathematical skills at a person's disposal, and his or her autonomy, flexibility, appropriateness and inventiveness in using them.

Extension / generalization ability (EG) by which we mean the ability to extend the acquired knowledge and / or to generalize it.

The **mathematical language** (ML) which includes the ability of a student of take in new terminology and use it appropriately, the competence in using the mathematical notation, the ability to adequately reason mathematically, etc.

We found it next to impossible to plausibly estimate the EG parameter on the basis of the exercise; hence, we did not include it in the general outline of the results.

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