FOSTERING THE ARGUMENTATIVE COMPETENCE BY MEANS OF A STRUCTURED TRAINING

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We report on a long-term study which was executed in a German secondary school with 128 eights graders (ages 14 to 15) in four different classes. Two of these classes served as control groups. The mathematics lessons of the other two classes (treatment groups) were frequently enriched by distinguished phases in which structured argumentation and the use of heuristics was trained. The study aimed at investigating the development of the argumentation competence of the students over that period. For this report, the products of four different geometry tasks of 15 students from one of the treatment groups and 15 from one of the control groups respectively were evaluated.

INTRODUCTION

Both "reasoning and proof" and "problem solving" are important parts of mathematics curricula all around the world (e.g., NCTM 2000). Though both deal with aspects of producing mathematical argumentation, mathematics educators tend to compartmentalize those two domains (Mamona-Downs & Downs 2013). Problem solving is being perceived as focusing on progressing work, whereas the proof tradition highlights evaluating the soundness of the product of reasoning (cf. ibid.).

We report on a 1.5-year study covering two experimental and two control classes emphasizing reasoning and proof as well as problem solving. In this paper, we confine ourselves to the "reasoning and proof" part of this study with a focus on the methodology of rating the students' products. Additionally, we present initial results by highlighting quantitative (scores) as well as qualitative (ways of reasoning) analyses of the students' products at the beginning and at the end of the study.

THEORETICAL BACKGROUND

Reasoning and proof is a significant aspect of mathematics and therefore also important for mathematics at school. It is, however, very difficult for students of all grades up to university level to generate or even read proofs on their own. Reid and Knipping (2010, p. 68 ff.) summarize several studies regarding the construction of proofs, which all agree on the fact that most students cannot write a correct proof.

There is a need for good teaching concepts regarding reasoning and proof as well as for studies that accompany related teaching experiments. An important part of such studies are methods to measure the argumentational competencies of the participating students. These methods need to be able to account for the (partially) complex structures of proofs, to appropriately compare different approaches and levels of elaboration of proofs, and consequently to show progress in the generation of proofs.

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Many researchers studying reasoning and proof use the Toulmin (1958) model which has been developed to reconstruct arguments in different fields (cf. Knipping 2008). According to Toulmin, the basic structure of rational arguments can be described as consisting of the pair of *datum* and *conclusion*. As this step might be challenged, a *warrant* can be added to justify it. Toulmin adds additional elements to his model (like *qualifiers* that can restrict the conclusion or *backing* for warrants) as do other researchers that use it. For example, Ubuz et al. (2012) add elements to describe statements and actions of teachers in classroom situations (like *guide-redirecting*) and specifications of existing elements (like *deductive warrant* and *reference warrant*).

However, the Toulmin model has its limitations. For example, it "is not adequate for more complex argumentation structures [in classrooms]" (Ubuz et al. 2012, p. 168) and it "de-emphasises the times" (Knipping 2008, p. 439) and thus is not able to outline the development of argumentations. Most notably, the Toulmin model is not designed to analyze written argumentations such as students' solutions of proof tasks. Analyses of students' solutions with this model would mostly contain of data and conclusions, missing rebuttals of dialogue partners and according backings.

As an alternative method to reconstruct argumentation steps and streams in written work of students, we propose in this article an adapted version of the multigraph representation by König (1992). He uses different graphical elements to denote elements like "starting quantities", "solution state" and "intermediate states" as well as logical derivations between states and heuristics elements that might help proceeding from one state to another (see the Methodology part for an example of such a graph).

König had designed his method which he refers to as a "solution plan" to compare written solutions of proof tasks – be it different solutions of the same task or solutions of different tasks. The standardized way of depicting an argumentation allows for a mostly objective analysis of students' work in different states of elaboration.

Our **research intention** is to adapt the solution plan sensu König to our study and to apply it onto the written argumentations (the *products*) of students that worked on mathematical problems and proof tasks. A secondary research question deals with detecting differences between and improvements of the argumentative competence of the students that underwent our training compared to those from the control group.

DESIGN OF THE STUDY

The HeuRekAP[1] study was launched at the beginning of the 2011/2012 school term (August 2011) in a German secondary school and lasted for one and a half years (until the end of January 2013, see Figure 1). It covered the whole eighth grade consisting of four parallel classes. Altogether there were 128 students initially aged 14 to 15. Two of these classes were continuously taught by the first author (treatment groups T_1 and T_2),

[1] ***Heu**ristisch **Rek**onstruiertes **A**rbeiten und **P**roblemlösen* means Heuristically Reconstructed Working and Problem Solving, for details of the concept of Heuristical Reconstruction see Gawlick (2013).

the two others served as control groups (C_1, C_2). Treatment group T_1 and control group C_1 were both mathematical profile classes, which implies an additional mathematics lesson per week in grades seven and nine.

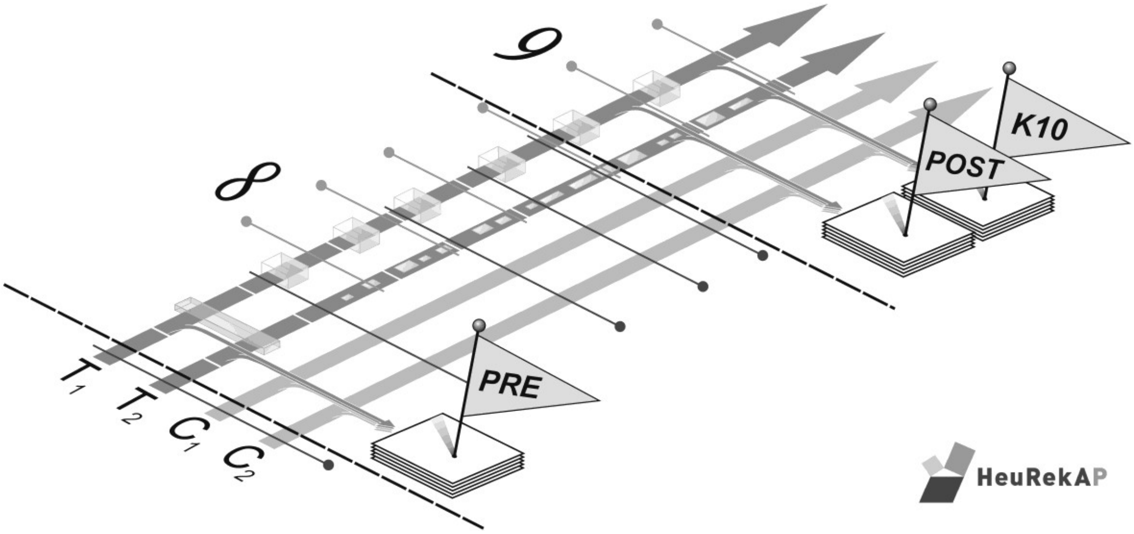


Figure 1: Overview of the study and the ascertainments relevant for this paper

For this paper, 15 students from each of the profile classes (T_1 and C_1) have been chosen by two criteria: (a) The selected students from both classes were supposed to be comparable with each other referring to their initial performance (parallelized samples). This was measured by the average school marks in Mathematics and German over the past four years before the study started. (b) The selected students should have had an above average motivation to participate in the ascertainments of the study. Therefore a survey on motivation was conducted at the beginning of the study.

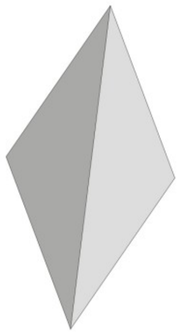
The mathematics lessons of treatment of group T_1 included distinguished phases in which structured argumentation and proving as well as the use of heuristics were trained. The students were involved into the whole process of proving according to Boero (1999) and learned to write down their proofs in the Two-Column-Format (cf. Herbst 2002). Amongst the heuristics they became familiar with are the use of auxiliary elements, principles like analogy and strategies like working backwards. See Brockmann-Behnsen (2013) for an example of a typical educational unit.

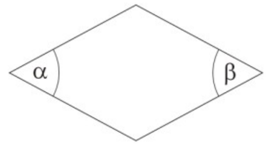
At regular intervals, sets of reasoning problems have been given to the students. Relevant for this paper are two items of the pretest, which was handed out before the treatment started, and three items of the posttest. The problems Rhombus 1, given in the pretest, and Rhombus 2, given in posttest are similar, Angle was given to the students both in the pretest and the posttest. With these pairs of problems the development of the quality of argumentation can be examined. Additionally, K10 was given to the students at the end of the study – with no matching pretest item because it is more complex than the other problems (see Table 1 for the problems).

<table>
<tr>
<td>Rhombus 1 (Pretest)

A rhombus is divided into two triangles by its diagonal.

Demonstrate that these two triangles are congruent.

Write down all your considerations and arguments step by step.
</td>
<td>Rhombus 2 (Posttest)

A rhombus is defined as a quadrilateral with four sides of equal length.


Given: A rhombus with opposite interior angles α and β.

Prove: $|\alpha| = |\beta|$</td>
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<tr>
<td>Source: Griesel et. al. 2006, p. 27 f.</td>
<td>Source: Beuthan 2008[3], p. 53</td>
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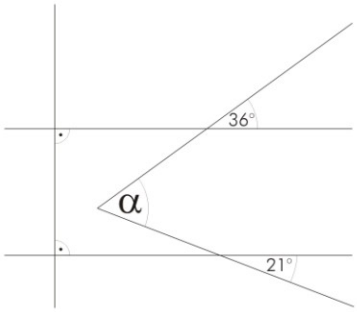
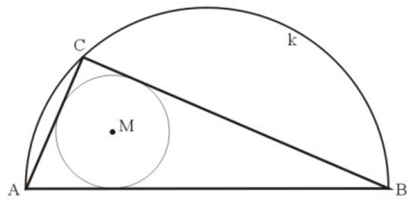
<td>Angle (Pretest/ Posttest)


Determine the value of angle α.

Write down all your considerations.</td>
<td>K10

AB is the diameter of a semicircle k, C is an arbitrary point on the semicircle (other than A or B) and M is the center of the circle inscribed into ΔABC. Determine the value of ∠AMB

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<td>Source: Lergenmüller et. al. 2006, p. 64</td>
<td>Source: TIMSS III[2]</td>
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Table 1: The four tasks selected for the analyses in this paper

METHODOLOGY

The research questions stated in this paper demand an instrument which is suitable to analyze and categorize the quality of argumentation in the students' products. These products often differ strongly in their form and structure. The spectrum ranges from disjointedly noted statements – partly written in mathematical symbols – over prosaic texts up to highly structured Two-Column-notations.

Therefore in a first step it is necessary to transform this variety of forms into one standardized format to facilitate comparability of the products. Orientated multigraph representations sensu König (1992, p. 25) serve as a basis for this standardized format. The vertices of these multigraphs comprise of the given magnitudes framed by circles, operators like Thales Theorem (TT) or the Angle Sum Theorem (AST) framed by rhombuses, intermediate aims surrounded by a mixture of rectangles and circles and the target magnitude enclosed into a rectangle.

[2] In contrast to the TIMSS III format in this study no solution alternatives were given to the students.

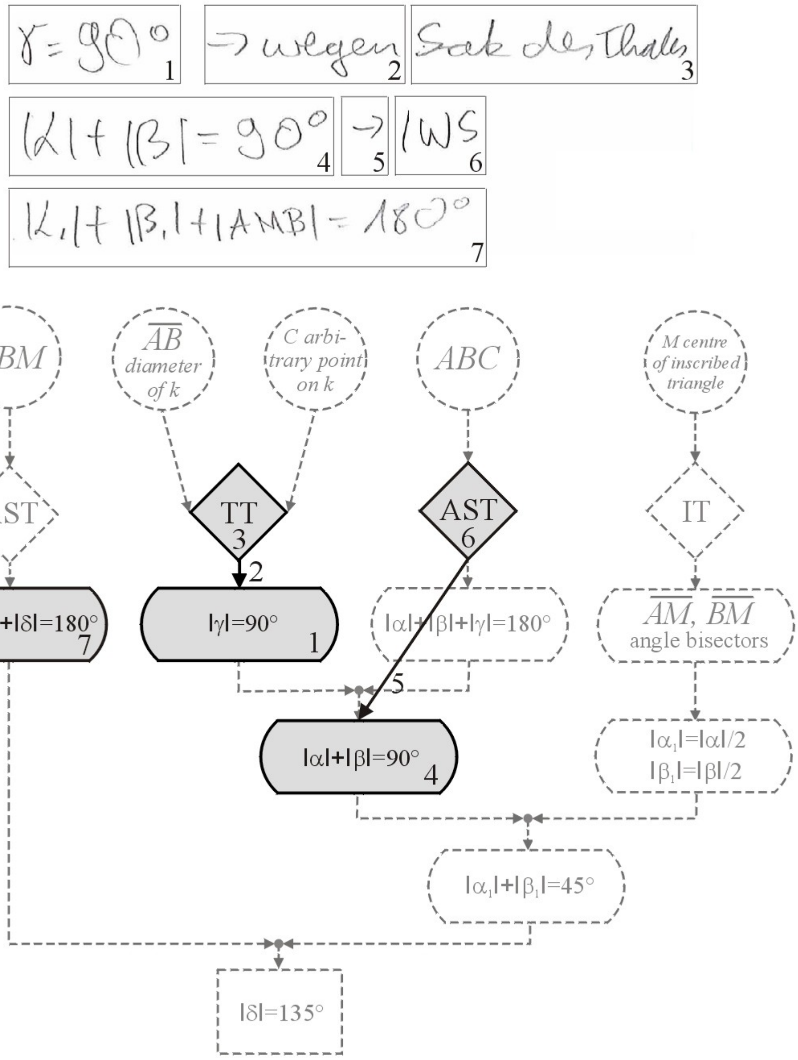


Figure 2: T_1-04-K10 original notations and standardized representation

The orientated multigraph representation depicts a survey of a complete solution of the given problem and highlights all the details reached by the student and their relation to each other. Figure 2 gives an example of such a representation. Shown are the original notations of student T_1-04 who worked on problem K10. The notations have been parsed into units that correspond to intermediate aims, identified operators or phrases that indicate connections between them. Beneath the original notations the appendant multigraph representation can be seen. The units of the original notations have been registered within this standard solution.

In a second step the quality of the students' argumentations were graded into six categories (Cat. 0 to Cat. 5) based upon the multigraph representation (see Table 2).

The notations of student T_1-04-K10 as stated in Figure 2 consist of some intermediate aims and a logical connection between the operator Thales Theorem (TT) with its conclusion $|\gamma| = 90°$. The required premises for the application of that operator are not stated. Therefore these notations were categories into Cat. 2 (Molecules).

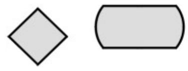
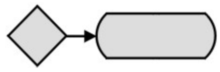
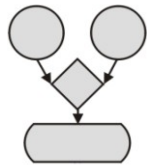
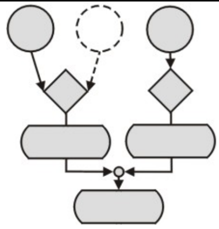
Cat. 0	**No access:** No detail of the students' notations is relevant for the solution.	Blank
Cat. 1	**Atoms:** At least one detail of the students' notations (operator, intermediate aim etc.) is *relevant* for the solution.	
Cat. 2	**Molecules:** At least two details of the students' notations are *logically connected* with each other.	
Cat. 3	**Deductive Cells:** At least *one correct and complete elementary deduction* that is relevant for the solution can be found. This is called a *Deductive Cell*. It includes the notation of the premises required for the application of an operator, the operator itself and the correct conclusion derived by the application of that operator on the stated premises.	
Cat. 4	**Deductive Torso:** At least *two deductions* relevant for the solution are *logically connected* with each other. Either one of the connected deductions or the connection itself is correct and complete (existence of at least one Deductive Cell).	
Cat. 5	**Deductive Body:** A complete solution *without any logically deficits* is being given.	Complete solution graph

Table 2: Categories for grading the students' products

RESULTS

For all tasks presented in this paper as well as additional ones within this study, the coding of students' written argumentations by representing it with an oriented multigraph and grading it into one of the six categories (Cat. 0 to Cat. 5) proved to be highly objective and reliable. Interrater correlations for 5 randomly selected students' products per task have been calculated. The percentage of agreement scores for researchers who have coded the products individually range between 65% and 100% with the median interrater correlation being 83%.

The coding of the students' products into categories via the multigraph representations allows us to compare their argumentative performances. For this report, we examined a parallelized sample of 15 students each from the treatment group T_1 and the control group C_1. Because of the fact that the category coding yields only ordinal data and because of the small sample size, in the following we use non-parametric statistical methods like interquartile ranges and chi-square-tests instead of parametric methods like standard deviation and *t*-tests.

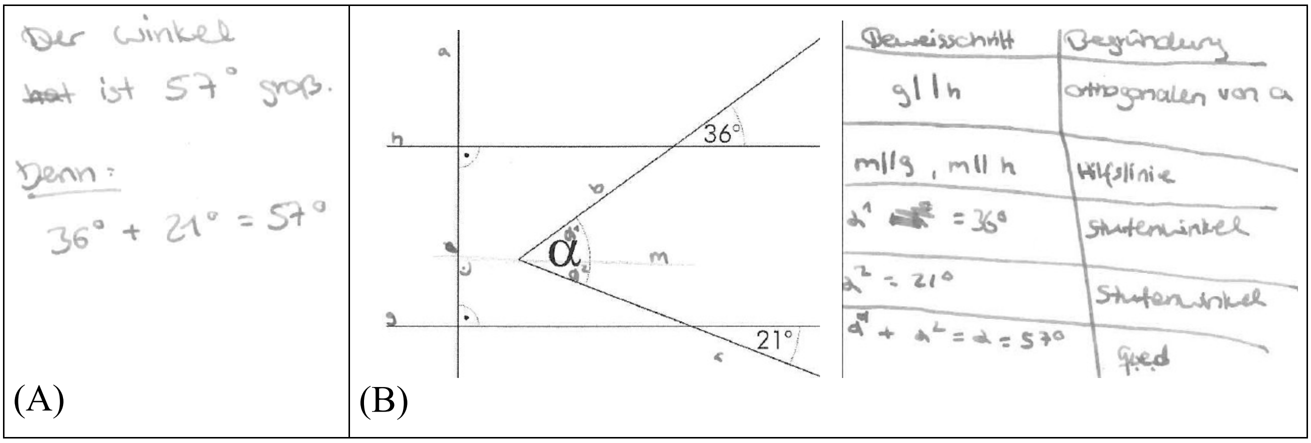
Comparing the two groups shows that they scored equally at both pretest items as it was expected because of the parallelization with regard to previous achievement. The three posttest items, however, show a significant difference in favor of the treatment group (see Table 3). This was proven by chi-square-test ($\chi 2 = 19,72$, $p < 0,0001$).

	Rhombus 1	Rhombus 2	Angle (pre)	Angle (post)	K10
T_1: median (interquartile range)	2 (1)	4 (1)	2 (3)	4 (0.5)	2 (0.5)
C_1: median (interquartile range)	2 (1)	1 (2)	2 (2)	2 (2)	1 (1)

Table 3: The mean results of the students for each task

This result can be supported by an analysis of the individual development of the students between the two matching pairs of pre-posttest items (Rhombus 1/2 and Angle pre/post). From the tasks of the pretest to the tasks of the posttest only 5 out of 30 products of the treatment group had no change or even a decline in their categories, whereas 21 products ascended by two or more categories. In the control group, 18 out of 30 products had no change or even a decline in category from the pretest tasks to the posttest tasks and only 5 products increased by two or more categories.

We like to illustrate the development of the students argumentative competence exemplarily by the elaborations of student T1-15 working on the Angle Problem in the pretest (A) and in the posttest (B). In the pretest the student merely states the correct result with the argument: “Denn: (Because:) 36°+21°=57°”. No mathematical connections between the given and the demanded angles are being drawn. In the posttest the solution is structured by a Two-Column-System and heuristic elements such as auxiliary lines and notations can be found.



DISCUSSION

We introduced a study to foster the argumentative competencies of eighth graders. To examine such competencies and possible advancements, we developed a method based upon multigraph representations that enabled us to categorize and thereby compare written products of students working on mathematical problems and proof tasks. We challenged the objectivity of this method by measuring its interrater reliability and gained very satisfactory results.

With the help of this method, we were able to grade the students’ argumentations before and after the 1.5-year period of our study. In accordance with the literature, most of the students scored quite bad results in proof tasks previous to the study. The control group (with no special training in heuristics and argumentational strategies) showed equally poor results at the posttest. The treatment group, on the other hand, reached significantly better results after the training. Ongoing research has to further demonstrate the effectiveness of the teaching method elaborated in this study.

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