REL SOUTHEAST
Regional Educational Laboratory
At Florida State University

Professional Learning Community:

IMPROVING MATHEMATICAL PROBLEM SOLVING

FOR STUDENTS IN GRADES 4 THROUGH 8

FACILITATOR'S GUIDE

U.S. Department of Education

Institute of Education Sciences

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*Dimino, J. A., Taylor, M., & Morris, J. (2015). *Professional learning communities facilitator's guide for the What Works Clearinghouse practice guide: Teaching academic content and literacy to English learners in elementary and middle school* (REL 2015–105). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Southwest. Retrieved from http://ies.ed.gov/ncee/edlabs.

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Contents

Introduction

This facilitator's guide is designed to assist Professional Learning Communities (PLCs) in using research-based strategies to help students develop proficiency in mathematical problem solving in preparation for advanced mathematics and other complex problem-solving tasks.

A PLC experience gives members of a professional community an opportunity to exercise their own expertise, professional judgement, and to share prior experiences implementing similar practices. This experience is unique in that it is intended to expand educators' knowledge base as they read, discuss, and apply the key ideas and strategies presented in the *Improving Mathematical Problem Solving in Grades 4 Through 8* (https://ies.ed.gov/ncee/wwc/PracticeGuide/16) educators' practice guide, produced by the What Works Clearinghouse, Institute of Education Sciences. The knowledge and resources provided will assist educators in meeting the rigorous requirements of the Common Core and other state standards. The facilitator will need a copy of this facilitator's guide and should study the guide and prepare all materials in advance to ensure an efficient and productive session for participants.

Even though the practice guide focuses on grades 4–8, this PLC facilitator's guide is designed to address the needs of middle school teachers and primarily includes problems from grades 6–8. However, upper elementary school teachers (grades 4–5) still may find the content of this PLC facilitator's guide useful.

Professional Learning Communities

PLCs are a form of professional development (PD) in which small groups of teachers with shared interests work together with the goal of learning and improving their teaching. Typically, a PLC consists of teacher participants that meet regularly to learn new topics, share ideas, and problem solve. Participants determine the topics they want to learn and the methods they want to use to gain knowledge. A group might read and discuss articles or books. A facilitator or group leader might guide the participants in learning a new topic, perhaps using PD materials specifically designed to guide the participants through the content. Participants might ask an expert to speak to the group, or the participants might attend trainings or conferences to deepen knowledge and understanding in an area of interest. Gathering notes from participants' experiences, their learnings, their "ah hah" moments and displaying them over the course of multiple sessions is one optional practice (either with an online document or charts shown in the room where the PLC meetings take place) that has the potential to validate participants' contributions and to honor the collective participation of members as they navigate through this valuable learning experience.

PLC participants often share the goal of improving student achievement by improving their own teaching practice. This shared interest brings coherence and continuous

learning to their PD (Vescio, Ross, & Adams, 2008). In a recent review of the research literature on learning communities, Vescio et al. (2008) note that "all combinations of individuals with any interest in schools are now calling themselves PLCs." Although the names and purposes of learning communities may vary, these groups commonly share a focus on learning, collaboration, and reflective dialogue.

The professional development (PD) setting we envision includes many elements that work together to create and sustain a social learning environment. Participants share common goals and expectations, and work creatively and productively to assist one another in accomplishing their goals. They collaborate in solving problems of practice, examine lesson artifacts, and engage in substantive conversations about teaching and student learning. They can gauge their own effectiveness by continually assessing students' progress, and reflecting on their work to make changes in instruction to improve students' performance.

Some empirical evidence exists suggesting that PLCs can foster teacher learning (Borko, 2004; Buysse, Sparkman, & Wesley, 2003; Englert & Tarrant, 1995; Little, 2002; Wilson & Berne, 1999) and improve the professional culture of a school (Vescio et al., 2008).

PD materials for use in PLCs have been generated to promote teacher learning on a variety of topics, such as mathematics (University of Massachusetts; http://www.umassmed.edu/rsrc/mlc/) and vocabulary (Dimino & Taylor, 2009), with some evidence of success. Both of those PD materials include several components of professional development that teachers invariably identify as effective—*coherence* across sessions, *concreteness* and *specificity* of discussions, and opportunities to *link the principles to the realities of the teaching* situation in the particular school or district (e.g., Desimone, Porter, Garet, Yoon, & Birman, 2002; Garet, Porter, Desimone, Birman, & Yoon, 2001; Huberman & Miles, 1984).

Forming Professional Learning Communities

There are no prescribed rules for forming PLCs and determining who will participate. For instance, they may comprise teachers from the same grade level or from across multiple grade levels. In middle schools, where departmentalization is a common practice, PLCs may be formed according to subject area. Furthermore, PLCs may consist of teachers across schools within a district. Regardless of who is identified to participate in a PLC, identifying a shared common goal helps to form a united community.

PLC Facilitators

Similar to forming PLCs, there are no prescribed rules for choosing PLC facilitators. Given the rich content in mathematics instruction addressed in each session in this guide, ideally, an educator with a strong mathematics background and good communication skills should serve as the facilitator. It is also important that facilitators be regarded as leaders in their schools and have the ability to relate well to adult learners. Alternately, there may be groups who decide that the role of facilitator should rotate among participants of the PLC.

Instructional Coaches and Mentor Teachers

There may be occasions where a PLC is made up of instructional coaches and/or mentor teachers. In many districts, coaches and mentors are support staff who work outside the classroom and, thus, do not have their own classes. During the Reflect and Plan segment under Step 5, PLC participants are asked to reflect on their current practices in light of their new learning, and then prepare a lesson to practice some of the newly learned strategies. If any of the participants are coaches or mentors, they may collaborate with classroom teachers to either teach or co-teach a lesson to students.

Five-Step Process for PLC Sessions

This facilitator's guide is based on a five-step process for collaborative learning. The process was adapted from Wald and Castleberry's (2000) five stages of work for participants engaging in a collaborative learning cycle. The five stages represent an inquiry-action cycle that encourages participants to **debrief**, **define**, **explore**, **experiment**, and **reflect and plan**. These five steps will be stretched across two days in an A–B cycle. During A sessions, the facilitator will conduct the first 3 steps which are to **debrief** the previous session's content, **define** the session goals, and begin **exploring** new learning. During B sessions, the content will focus on the same material as the A session. The B session will focus on the instructional strategies and activities that will be implemented in classrooms. This is addressed through the last two steps of the recursive process: **experiment** and **reflect and plan**. Explanations of these steps are provided below to assist facilitators in guiding PLC participants as they learn and apply the information in the *Improving Mathematical Problem Solving in Grades 4 Through 8* practice guide. As the participants explore the practice guide's recommendations and *How-to* steps, they will follow this five-step process for best results.

A Sessions: Learn

Debrief

Ask participants to share the instructional strategies and activities they implemented in their classrooms and to explain how students responded, both orally and in writing. Have them discuss how the changes they made in their teaching resulted in improved student learning; describing what worked really well; what didn't work so well as they taught the lesson. Participants may choose to describe a teaching episode or provide a video teaching vignette, and/or sample pieces of student work. Discussing all of these pieces of evidence will lend itself to a rich discussion of why they thought students responded as they did and how they might change or adapt their lessons to increase student learning. It might also lead to a discussion about the evidence they might want to collect in the future to gauge further improvement.

Define Session Goals

Identify the focus and the specific goals of the session.

Explore New Practices and Compare Them to Current Practices

Access participants' prior knowledge and experiences related to the topic of the session. Move into new learning by discussing specific features of each recommendation along with the supporting evidence. Compare current practices with newly learned concepts.

B Sessions: Apply

Experiment with Newly Learned Strategies

Collaborate with participants to practice applying activities and strategies that were addressed in the practice guide.

Reflect and Implement

Guide participants in reflecting on how the new learning relates to their previous knowledge and practice. Then, as a group, plan how the activities and strategies they studied will be implemented in participants' classrooms prior to the next PLC session. Remind the participants to be prepared to share their experiences in implementing the strategies at the beginning of the next session. End the session by giving participants the reading assignment for the next session.

Conducting the PLC Sessions

The purpose of this guide is to provide PLC facilitators with a plan for conducting each session. The directions under each step include guidance for facilitators, but typically, specific language or a script is not provided, as this defeats the purpose of a PLC. The intent is for a facilitator to read the directions and use his or her own unique style to convey the information, discuss topics, explain activities, and so forth. Reading the directions verbatim to the participants is not recommended, as doing so may reduce participants' active engagement.

We recommend that before beginning the PLC, facilitators take sufficient time in planning for the sessions. Facilitators should be familiar with all the recommendations in the practice guide, especially the 3 Recommendations of focus in the Facilitator's guide. Additionally, the facilitator should read through the entire facilitator's guide before starting the PLC. Having a "big picture" understanding of what will be covered across all eleven sessions will increase the likelihood that the sessions will go more smoothly as the facilitator will have thought about potential issues that she/he thinks may come up given his or her knowledge of the teachers attending the PLC. For example, if teachers struggle with specific math content knowledge or teaching concepts, the facilitator might want to think about ways to customize discussion or reflection questions for the participants.

To prepare for each session, facilitators should review related sections of the *Improving Mathematical Problem Solving in Grades 4 Through 8* practice guide, and read the complete session in this facilitator's guide. In addition, facilitators should study and prepare all handouts and gather any other needed materials. The facilitator should make sure that internet capabilities are available for sessions involving the presentation of videos.

Download the PLC session handouts at https://ies.ed.gov/ncee/edlabs/projects/project.asp?projectID=4566.

There are eleven sessions included in this facilitator's guide: an introductory session, five A sessions and five B sessions. A and B sessions are numbered and lettered as follows:

Session 1A; Session 1B
Session 2A; Session 2B
Session 3A; Session 3B
Session 4A; Session 4B
Session 5A; Session 5B

Sessions with the same number (e.g., 1A and 1B) focus on the same Recommendation and *How-to* steps. For example, Sessions 1A and 1B focus on Recommendation 3, *How-to* steps 1 to 3. Handout numbering corresponds with each session numbering and then goes in sequential order. For example, Handout 3A.2 is the second handout presented in Session 3A. However, sometimes handouts are revisited from prior sessions and are referred to by their original session numbering. For example, Session 3A refers to one of the Handouts from 2B, and the 2B convention remains. The numbering coincides with the session number for when it was first introduced. On the materials list for each session, handouts are included; therefore, the facilitator and the participants can see which previous handouts are required in that session along with the new set of handouts based on the numbering convention that was applied.

The authors of the PLC facilitator's guide recommend that participants download/copy all handouts for the PLC in advance and keep them in a binder for easy reference.

Whole Group and Small Group Activities

During each session, there will be whole group and small group activities. Small group activities give participants an opportunity to work in pairs or triads before sharing with the entire group. These two types of activities are clearly marked and explained in this facilitator's guide. Many of these activities rely on the Handouts explained above.

Videos

The practices addressed in selected *How-to* steps are illustrated by videos of teaching episodes that demonstrate a concept. Video Viewing Guides are provided to help participants process these teaching episodes and links to each video are provided in each module. Other videos include the chair of the practice guide, John Woodward, sharing his experiences and opinions regarding certain aspects of the recommendation.

Timeline for Completing the Sessions

The timeline for completing this set of modules is flexible. To address the content of the chosen recommendations from the practice guide, this facilitator's guide presents ten, 45–60-minute sessions. The sessions are organized into three modules that address different aspects of the recommendations. As mentioned above, each session is either an A day or a B day. Some groups may want to take longer to discuss certain topics in greater depth. Participants in the PLC can decide to allocate more than 60 minutes to a session or agree to exceed the guide's ten sessions.

The number of sessions allocated to address each practice guide recommendation is listed in Table 1. However, the session breakdown shown is only a suggested timetable. Please note that there is an introductory module that should be used prior to conducting Session 1A.

There are five recommendations in the *Improving Mathematical Problem Solving in Grades 4 Through 8* practice guide. They are:

> Recommendation 1: Prepare problems and use them in whole-class instruction.
>
> Recommendation 2: Assist students in monitoring and reflecting on the problem-solving process.
>
> Recommendation 3: Teach students how to use visual representations.
>
> Recommendation 4: Expose students to multiple problem-solving strategies.
>
> Recommendation 5: Help students recognize and articulate mathematical concepts and notation.

Recommendation 3 (visual representations) and Recommendation 4 (multiple problem-solving strategies) include a large number of novel concepts and may be less familiar to PLC participants than the content in Recommendations 1 and 2. Therefore, the ten sessions will primarily address Recommendation 3 (Sessions 1A, 1B, 2A, and 2B)

and Recommendation 4 (3A, 3B, 4A, 4B, 5A, and 5B). Recommendation 5 (articulating mathematical concepts) will be infused across all sessions because of the breadth of the topic. Session topics are summarized in Table 1.

Table 1: Overview of the Facilitator's Guide Sessions

Focus	Session
Introduction	Introduction to the Practice Guide Review the levels of evidence for Recommendations 3-5
Module 1: Selecting and Using Visuals in Mathematical Problem Solving Focus on Recommendation 3	Learning How to Select Visual Representations that are Appropriate for Students Session 1A: LEARN Session 1B: APPLY
	Using Think-Alouds and Discussions to Teach Students How to Represent Problems Visually Session 2A: LEARN Session 2B: APPLY
Module 2: Comparing Worked Examples During Instruction to Expose Students to Multiple Strategies Focus on Recommendation 4	Designing Worked Examples for Instruction Session 3A: LEARN Session 3B: APPLY
	Teaching Students to Compare Strategies in Worked Examples Session 4A: LEARN Session 4B: APPLY
Module 3: Supporting Students in Articulating Problem-Solving Strategies Focus on Recommendation 4	Supporting students in providing explanations of their problem solving Session 5A: LEARN Session 5B: APPLY

This facilitator's guide is structured so that PLCs would move through the modules in order. If the participants of the PLC want to focus only on one content area, (e.g., starting with module 2 on comparing worked examples) they can choose modules that best suit their needs. If your PLC chooses to go out of order, keep in mind that some material may have been addressed in an earlier session.

Good luck facilitating this PLC with your mathematics teachers who will be participating. We hope the organization and content are well received by you and the participants so all of you can engage in an empowering learning experience during the PLC.

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Introductory Session

Materials

Practice Guide: Improving Mathematical Problem Solving in Grades 4 Through 8

Chart paper/whiteboard

Pencil and paper

Sticky notes

Laptop, tablet, or similar device (optional)

Introductory Video
(https://youtu.be/yfEwtAkB-jY)

Professional learning communities (PLC) may vary in how they go about working together. If you have an established PLC, take some time to review the group norms. If this is your first experience working with a PLC, discuss the topics below before beginning the session:

- Roles and responsibilities of participants (e.g., facilitator, recorder, timekeeper) and whether these roles will rotate among participants from session to session
- Group norms
- Time allocated to each session
- Schedule for completing the ten (or more) sessions

Identify and explain that the five-step inquiry-action cycle (**debrief**, **define**, **explore**, **experiment**, and **reflect and plan)** will be used during the PLC sessions. Make sure to tell participants that the five steps will be stretched across two days in an A–B cycle. During A sessions, the facilitator will conduct the first 3 steps which are to **debrief**, **define** the session goals and begin to **explore** new learning. During the B sessions, the facilitator will share the instructional strategies and activities they will implement in their classrooms leading them through the last two steps, **experiment** and **reflect and plan**.

A Sessions: Learn

1. ***Debrief.*** Participants will share the instructional strategies and activities they implemented in their classrooms as a follow-up to the previous PLC session and explain how students responded, both orally and in writing. Tell participants that they will be discussing how the changes they made in their teaching resulted in improved student learning; describing what worked really well; what didn't work so well as they taught the lesson using evidence such as a teaching reflection, or sample pieces of student work. Explain to them that during this segment they will be sharing how they might change or adapt their lessons to increase student learning and discuss the evidence they might want to collect in the future to gauge further improvement in student learning.

2. ***Define Session Goals.*** The participants will identify and discuss the focus and specific goals of the session.

Introductory Session

3. ***Explore New Practices and Compare Them to Current Practices.*** Participants will access prior knowledge and experiences related to the topic of the session. Participants will move into new learning by discussing specific features of each recommendation along with the supporting evidence. Participants will then compare current practices with newly learned concepts.

B Sessions: Apply

4. ***Experiment with Newly Learned Strategies.*** With your guidance, the participants will practice applying the activities and strategies addressed in the practice guide.

5. ***Reflect and Plan.*** With your guidance, participants will think about how the new learning relates to their previous knowledge and practice. The participants will then plan how to implement the activities and strategies in their classrooms prior to the next PLC session. Participants will be instructed to be ready at the beginning of the next session to share their experiences in implementing the strategies.

Take time to focus the participants' attention on the overall goals of the ten PLC sessions:

- To build an awareness of the content in the *Improving Mathematical Problem Solving in Grades 4 Through 8* practice guide.
- To apply the research-based teaching practices to help grade 4-8 learners acquire the skills needed to be successful in mathematical problem solving.

Explain that the *Improving Mathematical Problem Solving in Grades 4 Through 8* practice guide summarizes the research on mathematics' instruction in problem solving and provides recommendations and *How-to* steps for implementing evidence-based teaching practices. The knowledge participants acquire from the guide should prove useful as they strive to meet the rigorous requirements of contemporary state standards. In addition, the materials to be shared in these sessions are flexible resources for participants, depending on their needs and those of their students. Explain that the PLC is focusing on Recommendations 3–5 (name those for easy reference OR refer them to table 2 for a list of the recommendations).

Introductory Session

Video Activity

Show the Introductory video in which Barbara Dougherty, Director of Curriculum Research and Development at the University of Hawai'i Mānoa, provides a brief description of the practice guide, *Improving Mathematical Problem Solving for Students in Grades 4 Through 8*.

Lead the participants in a brief walk-through of the remainder of the practice guide to give them an overview of its contents. Make sure each person has sticky notes. As you proceed through the overview, participants should place a note on the first page of each section and label the note to indicate the topic. These notes will help them quickly find what they need as they learn about the information in the guide. Table 2 provides an overview of the practice guide and suggests a label for each sticky note.

Table 2. Overview of the *Improving Mathematical Problem Solving in Grades 4 Through 8* practice guide

Section (page)	Description
Title Page (page i)	Lists the panelists and other contributors for the practice guide.
Table of Contents (page iii)	Lists the main sections, tables, and exhibits in the guide. (Label: TOC)
Introduction (page 6)	Participants are asked to read and share the introductory information on pages 6–9 through the use of a jigsaw activity. The following can be used if further explanation seems necessary: The Institute of Education Sciences (IES), under the U.S. Department of Education, provided a rationale for the importance of improving mathematical problem solving for students in grades 4 through 8. They explained key parameters considered by the panel in developing the practice guide. This section also summarizes the recommendations for readers and concludes with a discussion of the research supporting the practice guide. (Label: Introduction)
Overview of Recommendations (page 9)	This overview lists the practice guide's five recommendations and, after each one, the How-to steps for carrying out that recommendation in the classroom. These steps provide the practical details, giving practitioners specific ideas and teaching strategies for each recommendation. If possible, create a poster-sized chart of the overview of the recommendations shown on page 8 of the practice guide. Display the chart at each session to help PLC participants remain focused on the purpose of their work. This section of the practice guide ends with a table on page 9 showing the level of evidence for each recommendation. In brief, the practice guide uses three ratings—strong, moderate, and minimal—to describe the level of research evidence available to support a given recommendation. (Label: Overview of Recommendations)
IES Levels of Evidence for Practice Guides (page 5)	Provides a summary of how the levels of evidence were determined. A table on page 5 of the practice guide describes the meaning of strong, moderate, and minimal levels of evidence. (Label: Levels of Evidence)

Introductory Session

Section (page)	Description
Recommendations 1–5 (page 8)	Each recommendation includes a description, a summary of the level of evidence, and How-to steps for carrying out the recommendation in the classroom. Each How-to step presents supporting examples, graphs, and visuals, which are referred to as exhibits. At the end of each recommendation is a section on roadblocks and solutions to help address questions and concerns readers may have. • Recommendation 1 (p. 15): Prepare problems and use them in whole-class instruction (Label: Recommendation 1) • Recommendation 2 (p. 22): Assist students in monitoring and reflecting on the problem-solving process (Label: recommendation 2) • Recommendation 3 (p. 29): Teach students how to use visual representations (Label: Recommendation 3) • Recommendation 4 (p. 38): Expose students to multiple problem-solving strategies (Label: Recommendation 4) • Recommendation 5 (p.44): Help students recognize and articulate mathematical concepts and notation. (Label: Recommendation 5)
Glossary (page 69)	Provides a glossary of terms. (Label: Glossary)
Appendices (page 47)	• Appendix A (p. 47): Provides more detail about how the Institute of Education Sciences chooses topics for practice guides, how the guides are developed, and how expert panels come to consensus on recommendations. (Label: Appendix A–Practice Guides) • Appendix B (p. 49): Provides biographical information for the panel participants and research staff. (Label: Appendix B–Bios) • Appendix C (p. 52): Addresses disclosures. (Label: Appendix C–Disclosures) • Appendix D (p. 53): Discusses the rationale for evidence ratings. (Label: Appendix D–Evidence Ratings)
References (page 104)	Provides bibliographic entries for the practice guide. (Label: References)

Engage participants in the following activities in order to give them an overview of the evidence used to support the recommendations.

1. To prepare the participants for reading about the levels of evidence assigned to the recommendations, provide the following explanation of what it means to achieve rigorous standards of research:

> It is rare to find education research that meets the same high rigorous standards that are the norm in the medical and public health professions. This practice guide is based on that same gold standard of research evidence and was developed by research experts to ensure the quality and integrity of the information presented. What does it mean to say that research meets the gold standard? First, the evidence can come from only two types of studies—a randomized controlled trial or a quasi-experimental research study. In a randomized controlled study, students, classes, or even schools are randomly assigned to either an experimental or a

control group. (Random assignment to groups is usually computer-generated but could be a roll of the dice or a flip of a coin.) In contrast, in a quasi-experimental research study, students, classes, or schools are not randomly assigned. Second, the evidence from the study must have demonstrated effectiveness. In other words, the results must show a positive impact on student achievement for one or more student populations. One study that met the gold standard was the Jitendra et al. (1998) study (see page 61 of the Practice Guide) thus contributing to the "strong" level of evidence on the recommendation for using visual representations. It was a randomized controlled study and the study results showed effectiveness. It compared groups of students in grades 2-5 who were in a treatment group who received instruction in the use of a visual representation (schematic drawing) with students in a control group receiving traditional instruction and the students in the treatment group performed at statistically significantly higher levels than their counterparts in the control group.

Explain that the levels of evidence and the criteria for each of the three ratings (strong, moderate, and minimal) which are described on pages 3 through 5 of the practice guide. Have participants read these pages. Share participants' understanding of the key aspects of the three levels of evidence.

2. Examine Table 2 on page 9 of the practice guide as a group. Ask the participants, in light of what they have learned about levels of evidence, to react to the information presented in the table, including any levels of evidence that may surprise them. Share your thoughts on recommendations with a minimal rating. Do you still think they are important when teaching problem solving? Do you think the lack of empirical evidence precludes those recommendations from being important?

3. Have participants divide into three groups to study the evidence related to each of the three recommendations used in the PLC (Recommendation 3, 4, and 5) (pp. 60–69). Each group should prepare to give a quick overview of the instructional practices with evidence using the semantic chart provided. Participants may discuss the level of evidence for the recommendation. Allow time for the small groups to provide an overview of the major findings and implications for instruction.

Assignment

Ask participants to read over pages 23–31 of the practice guide for the next session. These pages focus on Recommendation 3 which centers around visual representations in problem solving. If participants are interested, they can also glance at recommendations 1 and 2 but the focus of the next session will only be on recommendation 3. Ask participants to begin thinking about their own use of visual representations in their instruction as they read.

Session 1A

Recommendation 3

Teach students how to use visual representations

* ***How-to* Step 1: Select visual representations that are appropriate for students and the problems they are solving.**
* ***How-to* Step 2: Use think-alouds and discussions to teach students how to represent problems visually.**
* ***How-to* Step 3: Show students how to convert the visually represented information into mathematical notation.**

Materials

Handouts

Handout 1A.1: Overview of How-to Steps

Handout 1A.2: Explore Visual Representations – Schematic Diagrams, Tables, and Strip Diagrams

Handout 1A.3: Video Viewing Guide
(https://youtu.be/prPlFrDqyY8)

Handout 1A.4: Questions to Answer before Selecting Visual Representations

Additional Materials

Practice Guide: Improving Mathematical Problem Solving in Grades 4 Through 8

Video: Introduction to Recommendation 3

Chart paper/whiteboard

Pencil and paper

Laptop, tablet, or similar device (optional)

Mathematics Curriculum and/or Guiding Standards

Session 1A

Debrief

Ordinarily, team members will begin each "A Day" session by **debriefing**, which involves sharing their experiences with planning or implementing instructional approaches they learned during the previous session. Remind participants that ordinarily during this segment of the session they will discuss how the changes they made in their teaching resulted in improved student learning. The *Prepare to Share* from the previous lesson will be used to guide this discussion. Teachers are encouraged to share what they felt worked well and what did not work as well as they taught the lesson. Teachers may choose to share a teaching vignette or sample pieces of student work as evidence. Remind participants that they may also share how they could adapt their lessons to increase student learning and discuss the evidence they might want to collect in the future to gauge further improvement in student learning. Since this is the first session, this debrief step will be used to address any logistical or other issues the group may have regarding the functioning of the professional learning community (PLC). These issues may include a discussion about meeting times, location, group norms, and so forth.

Define Session Goals

Explain that over the next two sessions, the team will learn about the three *How-to* Steps for carrying out Recommendation 3, which addresses learning how to select visual representations to represent problems; using think-alouds and discussions to teach students how to represent problem information visually; and, showing students how to convert the visually represented information into mathematical notation.

* ***How-to* Step 1: Select visual representations that are appropriate for students and the problems they are solving.**
* ***How-to* Step 2: Use think-alouds and discussions to teach students how to represent problems visually.**
* ***How-to* Step 3: Show students how to convert the visually represented information into mathematical notation.**

Ask team members to review **Handout 1A.1: Overview of *How-to* Steps** that outlines some of the key components related to each of the three *How-to* Steps. Discuss how the three steps inform teachers about what is important in teaching students to use visuals. Ask teachers what they find challenging from the three *How-to* Steps and also what they hope to learn in the PLC as it relates to the content in in three *How-to* Steps.

Session 1A

Explore New Practices and Compare Them to Current Practices

Access Prior Knowledge

1. Ask participants to discuss the following questions:
   a. What purpose do visual representations serve in your classroom?
   b. Why would you use a visual representation instead of going straight to an equation or mathematical notation to represent and solve problems?
2. Accept participants' comments and highlight any of the following if they are brought forward:
   a. Visual representations help learners identify and organize important information while eliminating useless information from and problem in the problem;
   b. Visual representations help learners work with problem information by decreasing text (especially good with learners who struggle with reading);
   c. Visual representations offer learners a way to see mathematical patterns that demonstrate problem structure;
   d. Using a visual representation instead of mathematical notation might increase students' ability to engage in problem solving, especially for problems that translate to and can be solved by complicated equations; and,
   e. Visual representations provide useful assessment of students' understanding of a problem.

Think/Pair/Share

Think: Have each participant review **Handout 1A.2: Explore Visual Representations – Schematic Diagrams, Tables, and Strip Diagrams** that includes three different types of visual representations to solve the problem. Have participants think about which of the three visual representations presented are most similar to the visual representations they use in their classroom. Ask them to think about which of the three visual representations might work best for that problem or whether they might choose something different altogether. How or why did they choose the visual representation they specified?

Session 1A

Pair or Small Group: Have participants discuss the visual representations on **Handout 1A.2: Explore Visual Representations – Schematic Diagrams, Tables, and Strip Diagrams**. Ask participants to discuss the following issues with their group members:

- which visual they use in their classroom for this type of problem;
- why they would choose that visual for solving the problem; and,
- if applicable, have them explain their rationale for why they would choose something different.

Share: Lead the whole group on how and why participants selected certain visuals. Discuss some of the challenges that arise in selecting visual representations that are appropriate for students and the problems they are solving.

Move into New Learning

Remind participants that the panel members have assigned a strong level of evidence to this recommendation because the results of the research studies indicated that the use of visual representations led to improved achievement among all students, including students with learning differences (p. 9, pp. 60–62).

Use the following prompts to lead a discussion on *How-to* Step 1:

1. Refer to page 26 of the practice guide and tell participants that the guide suggests that it is not necessary to use several different types of visual representations for a particular type of problem. Rather, teachers should select a visual representation that will work well for the problem and is appropriate for students and use it consistently. Explain that the guide also suggests that students get plenty of practice with the visual representation and that after a reasonable amount time, if students continue to struggle with solving problems using a particular visual representation, then it is appropriate to try another.

2. Review the types of visual representations (e.g., schematic diagrams, percent bars and strip diagrams etc.) on page 26 of the practice guide. Tell participants that the authors suggest that some visual representations are better suited for certain types of problems (e.g., percent bars for percentage problems, schematic diagrams for ratio and proportion problems, strip diagrams for comparison or fraction problems), and this is something to consider when making your selection for what type of visual to use.

Session 1A

***Critical Learning**:* Refer participants to **Handout 1A.2 – Explore Visual Representations: Schematic Diagrams, Tables and Strip Diagrams.** Tell them that for this session you will be focusing on three types of visual representations—**Schematic Diagrams**, **Tables**, and **Strip Diagrams.** Examples are displayed on the handout for the John problem. Begin discussion about the specific qualities of these three visual representations and which aspects of the John problem they represent to make it easier to understand. Examples are below:

Facilitator should accept participants' comments and specifically look for the three listed below. If participants do not bring these ideas forward, the facilitator can raise these issues or tailor the questions below that probe for these ideas.

1. The schematic diagrams and strip representations demonstrate the temporal nature (i.e., the element of something happening over time) of the word problem. This is shown through the pattern: 15, 5, 15, 5, 15, 5, 15, 5, 15 and with the labels, *start/end* in the schematic diagram and the labels, *mile 1* and *break* in the strip diagram.

   Potential Probing Question: Do any of the visual representations facilitate understanding that the problem is about something that occurs over time?

2. The table organizes and represents the pattern (by showing 20, 40, 60…in the right-hand column) which leads to problem solving without requiring an equation or additional mathematical notation.

   Potential Probing Question: Do any of the visual representations show a pattern? Do any of the visual representations stand-alone without an additional equation needed to solve the problem?

3. All three visuals display the problem information so that the visual representation can be translated into mathematical notation. Both of these equations demonstrate the problem structure and could be generated from the schematic diagram, strip diagram, or table.

   $$x = \text{total time}$$

   $$\text{Equation 1: } x = (5 \cdot 15) + (4 \cdot 5)$$

   $$\text{Equation 2: } 4\,(15 + 5) + 15 = x$$

   Potential Probing Question: Can all three of these visual representations lead to an equation that will solve the problem?

Additional question prompts:

1. What kind of benefits do you think students receive when teachers provide instruction that utilizes the three visual representations presented?

2. After viewing the three types of visual representations selected today, ask, "Do you have a preference for which type of visual is best suited to organize the information in this problem, and if so, why?"

Use the following to move the discussion to *How-to* Step 3.

Session 1A

1. Use **Handout 1A.1: Overview of *How-to* Steps** to direct participants' attention back to the important information listed under *How-to* Step 3. Tell participants that when selecting visuals, it is also important to consider how well the relevant information in a problem can be translated from the chosen representation into mathematical notation. (This should have been touched on earlier during the Critical Learning segment of the session when discussing the John problem.)

2. Discuss the following key considerations when selecting a visual:
   a. **You should consider whether or not the visual representation you select will lead to a mathematical notation (e.g., an equation or expression).**
   b. **You may want to consider if the visual representation can lead to an accurate answer without requiring mathematical notation.**

3. Have participants refer back to the John problem **in Handout 1A.2: Explore Visual Representations – Schematic Diagrams, Tables and Strip Diagrams**. Lead a whole group discussion on the three visuals with consideration given to ***mathematical notation*** to answer questions a and b above. The facilitator should use a and b below as needed to generate discussion. Remind Participants:
   a. **The schematic diagram and strip diagram require mathematical notation to find the unknown;**
   b. **whereas, the table can lead the problem-solver to the unknown, thus solving the problem without an equation or other mathematical notation.**

Prepare to show the video *Introduction to Recommendation 3*. In this short video clip, John Woodward will give a brief introduction to Recommendation 3, including a description of the three *How-to* Steps for carrying out the recommendation. He will discuss some of the critical questions a teacher will need to answer in order to select an appropriate visual representation for solving a problem.

1. Give each team member a copy of **Handout 1A.3: Video Viewing Guide – Introduction to Recommendation 3**. Briefly review the handout. Members will take notes on the key ideas presented in the video including the three key essential ideas related to each of the *How-to* Steps.

2. As a group, view the video: *Introduction to Recommendation 3*. Have team members take notes on their video viewing guides as they watch the video.

3. Refer each team member to **Handout 1A.4: Questions to Answer before Selecting Visual Representations.** Have team members take a few minutes to review the questions that need to be answered before selecting visual representations.

**Note to facilitator: Tell participants that these questions come from the critical information explained in How-to Steps 1 and 3, and that the answers to these questions help a teacher gather important information about the math problem when selecting a visual. This same information will be used in the next session when learning to write a think-aloud to use during instruction. Tell participants that in Sessions 2A and 2B they will learn more about using think-alouds as a means to demonstrate how to select visual representations and to facilitate discussions about problem solving.*

Session 1A

Compare Current Practice

Discuss the following: **Handout 1A.4: Questions to Answer Before Selecting Visual Representations**, lists important questions to answer before selecting visual representations that will help students with solving problems (e.g., What is the problem about?, What is unknown?, How can I best represent the information?, etc.). In order to be prepared for the Review part of the next session, the facilitator might want to record responses to reflections on Handout 1A.4 so they can be brought back to the group later. The video demonstrated important issues to consider when selecting visuals. Lead the group in a discussion using the questions below:

1. How were the suggestions by John Woodward similar or different from the way you currently plan mathematics instruction using visual representations?
2. After the discussions in our earlier session and what you saw in the video, are there any changes or additions you might make when planning how to support students in solving problems? Do these changes or additions seem feasible to you? Why? Why not?

Ask the team to consider and discuss possible roadblocks when selecting visuals and possible ways to circumvent them.

Closing Comments

Tell participants that in the next session they will have a chance to apply what they have learned in the today's session by practicing how to use critical questions to guide them in selecting visual representations for certain problem types and prepare a lesson that requires them to use that same criteria for choosing appropriate visual representations.

Session 1B

Recommendation 3

Teach students how to use visual representations

* ***How-to* Step 1: Select visual representations that are appropriate for students and the problems they are solving.**
* ***How-to* Step 2: Use think-alouds and discussions to teach students how to represent problems visually.**
* ***How-to* Step 3: Show students how to convert the visually represented information into mathematical notation.**

Materials

Previous Handouts

Handout 1A.1: Overview of How-to Steps

Handout 1A.2: Explore Visuals: Schematic Diagrams, Tables, and Strip Diagrams

Handout 1A.4: Questions to Answer before Selecting Visual Representations

New Handouts

Handout 1B.1: Money Problem

Handout 1B.2a: Prepare to Share: Recommended Activity

Handout 1B.2b: Prepare to Share: Custom Activity

Additional Materials

Practice Guide: Improving Mathematical Problem Solving in Grades 4 Through 8

Chart paper/whiteboard

Pencil and paper

Laptop, tablet, or similar device (optional)

Mathematics Curriculum and/or Guiding Standards

**Note to Facilitator:*

In the previous session, the participants learned important content about the three How-to Steps for carrying out Recommendation 3, which addresses learning how to select visual representations to represent problems. In this session, participants will apply what they learned in the previous session by practicing how to use critical questions to guide them in selecting visual representations for certain problem types. They will also prepare a lesson that requires them to use that same criteria for choosing appropriate visual representations.

Session 1B

Review

1. Briefly review the **Handout 1A.1: Overview of *How-to* Steps** that describes the three *How-to* Steps.
2. Remind participants about the critical learning points of the activity they engaged in during the last session. Refer to **Handout 1A.2: Explore Visual Representations – Schematic Diagrams, Tables, and Strip Diagrams,** if needed.
3. Remind participants about the important questions discussed during the last session. Review **Handout 1A.4: Questions to Answer Before Selecting Visual Representations.**
4. Remind participants of the discussion they had about how these ideas compare with their current practices.
5. Ask participants if there are questions or issues they would like to address before moving on with the session.

Experiment with Newly Learned Strategies

This activity will give participants an opportunity to examine some of the critical questions teachers will need to answer before they choose a visual representation for certain types of problems.

Small Group: Form small groups to work on **Handout 1B.1: Money Problem**, involving a math problem on money and the five questions to guide you in selecting an appropriate visual representation. Discuss each question and record your responses on **Handout 1B.1: Money Problem**.

1. What is the problem about (e.g., rate, proportion/ratio, money spent, etc.)?
2. What is unknown (e.g., starting amount of money, an identified quantity, etc.)?
3. How can I best represent the important problem information? (Some examples below)
   a. Do I want to show something over time?
   b. Do I want to show proportional amounts?
   c. Do I want to show a pattern?
4. Do I want the visual representation to lead to mathematical notation or should it lead to the answer without mathematical notation?
5. Choose a visual representation keeping in mind the following:

Session 1B

a. Does it organize the information?
b. Does it display the important information, so the learner can solve for the unknown?
c. Does it simplify the problem structure for the learner?

Prepare to Share

As a group, **reflect** on the following and be prepared to tell how your visual representation helped to organize the information in the problem and effectively display the important information in the problem so that the learner can solve for the unknown. Discuss how it helped to simplify the problem structure for the learner.

Large Group: Have one participant from each small group share their assigned problem and selected visual representation. Each group will share the following:

1. Explain the decisions the group made while responding to the five important questions that need to be answered for selecting visual representations.
2. Highlight the decision about whether or not the visual representation was converted to mathematical notation, and why the group made the decision for it to lead or not lead to mathematical notation.
3. Tell whether or not the selected visual representation met the following criteria:
   a. Did the visual representation help to organize the information in the problem?
   b. Did the visual representation display the important information in the problem so that the learner can solve for the unknown?
   c. Did the visual representation help to simplify the structure of the problem for the learner?

The Facilitator has an answer key for **Handout 1B.1: Money Problem** at the end of this session on page 18 of this facilitator's guide. The answer key is provided as an example of a visual representation in case participants have trouble using a schematic diagram, strip diagram, or table. The answer key uses a strip diagram.

Session 1B

Reflect and Plan

Reflect

Ask the participants to discuss the following:

1. Is there anything they learned today that confirms or contradicts what they already knew about planning for the use of visual representations to assist with mathematical problem solving?
2. Ask participants, in light of what they have learned, to brainstorm ideas about what they would add or change as they plan instruction.
3. As a group, consider the brainstormed ideas and select the additions or changes the participants would like to implement in the classroom.
4. How will they determine whether the changes they made resulted in improving student learning? Is there any data they might collect?

Plan

To reinforce the key concepts presented in this session, assign either the following recommended activity or an activity of the participants' choosing:

Option 1: Recommended Activity

1. Each participant will choose a problem or set of problems to use with students before the next session.
2. Participants will use the questions on **Handout 1A.4: Questions to Answer Before Selecting Visual Representations** to guide them in selecting visual representations (schematic diagrams, tables and/or strip diagram). Participants will use this planning process for at least one lesson before the next PLC.
3. Each participant will complete **Handout 1B.2a: Prepare to Share (Option 1: Recommended Activity)**. At the start of the next PLC session, participants will be prepared to do the following:
   a. Share the visual representation and briefly describe the problem or set of problems they chose.
   b. Discuss the rationale for choosing the visual representation based on the criteria. Did your selection:
      i. Organize the problem information?
      ii. Display the important information in the problem (exclude irrelevant information)?
      iii. Simplify the problem for the learner?
4. What was one take away or new learning that came from this planning?

Session 1B

Option 2: Custom Activity

1. Participants may opt to develop a different activity to reinforce the content of this session. If so, consider the following questions to keep the activity on target:
   a. How does this activity relate to Recommendation 3, *How-to* Steps 1 and 3?
   b. What products will participants collect or develop and bring back to discuss during the next session?
2. Each participant should complete **Handout 1B.2b: Prepare to Share (Option 2: Custom Activity)**. At the start of the next PLC session, participants should be prepared to address the following:
   a. Explain their experiences completing the activity (e.g., level of difficulty of the activity, any problems that were encountered).
   b. Discuss what they learned from completing the activity.

The facilitator should remind the teachers that during the debrief of the next session, they can go beyond the *Prepare to Share* part of the Activity. Teachers may share what they felt worked really well and what didn't work as well as they taught the lesson. Teachers may consider sharing sample pieces of student work from the lesson. Participants are encouraged to share how they might change or adapt the lesson to increase student learning.

Assignment

Assign participants to read pages 26–29 of the practice guide before the next session. This will prepare them for their next session on learning how to use think alouds and discussions to teach students how to represent problems visually.

Session 1B

Answer Key (other visuals could be chosen)

Handout 1B.1: Money Problem

Andi spent of her money on a baseball glove. Then she spent $8 of what was remaining on a movie ticket. Now she has $32. How much money did she start with?

Answer these questions:

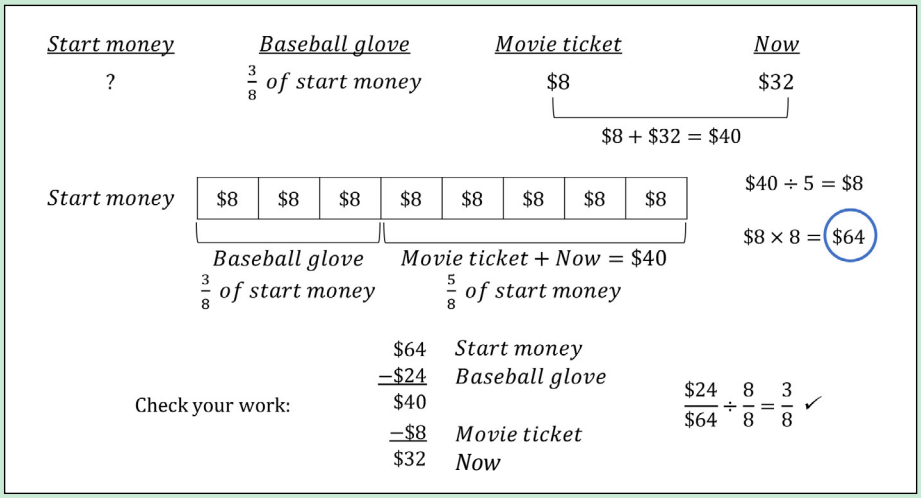
Question	Answer	
What is the problem about (e.g., rate, proportion/ratio, money spent, etc.)?	Money spent, proportion of money	
What is unknown (e.g., start amount of money? An identified quantity, etc.)?	How much money she started with	
How can I best represent the important problem information?		
Do I want to show something over time?	(circle Yes or No)	
	Yes	No
Do I want to show proportional amounts?	**Yes**	No
Do I want to show a pattern?	Yes	**No**
Do I want the visual to lead to mathematical notation or should it lead to the answer without mathematical notation?	Answers may vary	

Choose a visual representation keeping in mind the following: (visuals may vary)

Does it organize the information?

Does it display the important information, so the learner can solve for the unknown?

Does it simplify the problem structure for the learner?



Session 2A

Recommendation 3

Teach students how to use visual representations

* ***How-to* Step 2: Use think-alouds and discussions to teach students how to represent problems visually.**

Materials

Previous Handouts

Handout 1B.2a: Prepare to Share (Option 1, Recommended Activity)

Handout 1B.2b: Prepare to Share (Option 2, Custom Activity)

New Handouts

Handout 2A.1: Visual Representation Question Worksheet

Handout 2A.2: Visual Representation Think-Aloud Analysis

Handout 2A.3: Video Viewing Guide –Visual Representation Think-Alouds

Handout 2A.4: Table Representation for Ratio Problem

Additional Materials

Practice Guide: Improving Mathematical Problem Solving in Grades 4 Through 8

Grade-level Videos: Model of Visual Representation Selection using a Think-Aloud

Grade 7 video (https://youtu.be/5rH8Uq41p3c)

Grade 8 video (https://youtu.be/fXYFpE2hL5w)

Chart paper/whiteboard

Pencil and paper

Laptop, tablet, or similar device (optional)

Mathematics Curriculum and/or Guiding Standards

Session 2A

Debrief

Ask participants to **debrief** the follow-up activity from Session 1B. Participants were to complete either the recommended or a custom activity.

Option 1: Recommended Activity

The recommended activity was to choose a problem or problem set and select a visual representation (tables, schematic diagrams, and/or strip diagram) by using **Handout 1B.2a: Prepare to Share (Option 1: Recommended Activity)**. If the group completed this activity, have participants refer to their completed copies of **Handout 1B.2a: Prepare to Share (Option 1: Recommended Activity)**.

Use the completed Handout 1B.2a: Prepare to Share (Option 1: Recommended Activity) to address the following:

- Briefly describe the content of the problem selected.
- Discuss why you chose that problem.
- Identify the visual representation that you chose, and the criteria used to select it.
- ***Optional:*** Share all the visual representations you chose and discuss the criteria used to select them.

Option 2: Custom Activity

If the group opted to do a different activity, ask participants to refer to their completed copies of **Handout 1B.2b: Prepare to Share (Option 2: Custom Activity)** to address the following:

- Explain their experience completing the activity (e.g., level of difficulty of the activity, any problems that were encountered).
- Discuss what they learned from completing the activity.

When applicable, teachers may choose to share beyond the *Prepare to Share* prompts from the Activity. Teachers may share what they felt worked really well and what didn't work as well as they taught the lesson. Teachers may consider sharing sample pieces of student work from the lesson. Participants are encouraged to share how they might change or adapt the lesson to increase student learning.

Define Session Goals

Tell participants that the goal of the next two sessions is to teach them how to develop think-alouds for showing students how to select visual representations and to develop think-alouds they will use with their students. Participants will identify similarities and differences among types of visuals and will evaluate the appropriateness of alternative

Session 2A

visual representations for different types of problems.

Recommendation 3 addresses teaching how to use visual representations.

***How-to* Step 1:** Select visual representations that are appropriate for students and the problems they are solving.

- **a.** Select appropriate visuals for problem structure.
- **b.** Use visuals consistently. Select a few good visual representations rather than using several different types for a particular type of problem.
- **c.** Remember that some visual representations work well for some problems but not necessarily for all problems.
- **d.** Provide time for students to practice visuals.
- **e.** Only after a reasonable amount of time, if students still struggle, try a different visual representation.

***How-to* Step 3:** Show students how to convert the visually represented information into mathematical notation.

- **a.** Determine whether the quantities and relationships in the visual representation need to connect to mathematical notation. This is really up to the teacher and her determined learning goal.
- **b.** Determine if the visual representation can easily translate to an equation.
- **c.** Participants should connect the visual representation to mathematical notation when appropriate, which can be accomplished through think-alouds (the focus of *How-to* Step 2, and today's session).

Spend time drawing the participants' attention to the new learning contained in the second *How-to* step for carrying out Recommendation 3.

***How-to* Step 2:** Use think-alouds and discussions to teach students how to represent problems visually.

Explore New Practices and Compare Them to Current Practices

Access Prior Knowledge

Explain that *How-to* Step 2 provides examples of think-alouds and discussion questions that teachers can use to help students choose visual representations and solve problems.

Small Group: Working in groups of two to three, have participants describe their current instructional practices for using think-alouds or discussion techniques for teaching students how to select visual representations and solve problems.

Whole Group: Lead a whole-group discussion of the assumptions about student learning

Session 2A

that drive participants' current practices and ask them to share their thoughts with the group on using think-alouds in teaching problem solving.

> ***Ask:*** What challenges are you currently facing when teaching students to select visual representations for solving problems?

Move into New Learning

Take a minute to have participants read over the key ideas in *How-to* Step 2 (Use think-alouds and discussions to teach students how to represent problems visually) on pages 26–29 of the practice guide.

Discuss the following:

1. Multiple studies show that selecting visual representations that are appropriate for students and the problems they are solving and teaching students how to use visual representations leads to increased skill in problem solving. Another key finding is that teaching students the use of visual representations requires a direct and explicit approach to instruction. Researchers in several studies reported that they used think-alouds and discussion as instructional techniques when teaching students how to represent problems visually (pp. 60–62).
2. The first paragraph under *How-to* Step 2 on page 26 states that when teachers are teaching a new visual representation or type of problem, they should demonstrate how they selected the visual to represent the problem and take students through what they are thinking as they solve it. Remember, in the first session participants learned that it is important to answer a number of questions about the problem to determine the fit between the problem and its visual representation.
3. Engage participants in the following activities as a way of analyzing the teacher's selection of a visual representation in a problem involving ratios.

Think/Pair/Share

Think: Have each participant read the ratio problem including the visual representation at the top of **Handout 2A.1: Visual Representation Question Worksheet** and then answer the questions on the Handout. (Refer to pp. 27-28 in the practice guide for the more comprehensive view of the visual.)

Pair or Small Group: Have participants discuss their answers to the questions on **Handout 2A.1: Visual Representation Question Worksheet**. Ask group members to share their answers to the questions on the handout.

Share: Lead a whole-group discussion of how well the visual representation organizes information in the problem; how well it displays the important information; and, if it simplifies the problem structure for the learner.

Session 2A

Explain to participants that the authors of the practice guide stated that after teachers choose an appropriate visual representation, they should use a think-aloud to make their thinking process public to show how they select a visual representation to solve a problem. The authors remind teachers that a think-aloud is more than just telling students what they are doing, but that it also involves teachers expressing their thoughts as they approach the problem, especially as it relates to the decisions they make about selecting the visual representation; why they thought it was a good one; and, how they would use that tool to either create an equation that will lead to the problem's solution or directly solving it. As participants prepare to deliver a think-aloud, they should outline the key decisions they make as they connect the problem to the visual representation.

Engage participants in the following activity to give them a clear picture of the critical elements necessary in developing a think-aloud.

Think/Pair/Share

Think: **Use Handout 2A.2: Visual Representation Think-Aloud Analysis** to analyze the think-aloud used by the teacher in the same ratio problem. Participants should refer to pages 27–28 (Example 11) of the practice guide to view the think-aloud for this activity. The think-aloud accompanies the visual representation that was analyzed on **Handout 2A.1: Visual Representation Question Worksheet** and is also in the practice guide on page 28.

Pair or Small Group: Have participants discuss how they answered the questions on **Handout 2A.2: Visual Representation Think-Aloud Analysis.**

Share: Lead a whole-group discussion of how well the think-aloud displays the participant's thinking as they were selecting a visual representation. Were all of the questions addressed? If not, how would they have included those missing elements?

**Note to Facilitator: Participants may disagree on whether items 5 and 6 were fully addressed in the practice guide think-aloud script. Expect participants to have varying responses on those items which may fuel the whole-group discussion.*

Prepare to show one of the grade-level videos: *Model of Visual Representation Selection and Think-Aloud*.

A set of two grade-based videos are available for review (listed below and printed on **Handout 2A.3: Video Viewing Guide – Visual Representation Think-Alouds**). Choose the video that coincides to the grade-level span for most of the participants, or choose which problem the group is most interested in. Alternatively, if your setting allows, different groups could view different videos based on the grade level they teach. The seventh-grade problem is solved with a table and the eighth-grade problem is solved with a strip diagram.

1. Before viewing the video, explain that its purpose is to demonstrate how to use a think-aloud to teach students to select a visual representation. Teaching think-alouds circumvents any misconceptions students may acquire before they participate in

activities that increase their ability to choose visual representations effectively.

2. Refer to **Handout 2A.3: Video Viewing Guide – Visual Representation Think-Alouds**.

3. As a group, view the video the PLC participants selected from the Think-Aloud Grade-Level Video Options below. Have participants complete their video viewing guides as they watch the video.

Table 2A.1. Think-Aloud Grade-Level Video Options

Grade 7 Video
Problem presented: Sally likes to exercise. For every 10 minutes that she runs, she does jumping jacks for 2 minutes. If she exercises for 1 hour, how many minutes did she do jumping jacks?
Grade 8 Video
Problem presented: Andi spent $\frac{3}{8}$ of her money on a baseball glove. Then she spent $\frac{1}{5}$ of what was remaining on a movie ticket. Now she has $32. How much money did she start with?

Lead a discussion about the video:

- What did they notice about the teacher's think-aloud?
- What steps did the teacher take to reach their goal?
- Did the teacher follow similar steps put forth in the practice guide?
- Was the think-aloud taught explicitly?
- Did the teacher reflect on whether the visual organized the problem information?

For more practice in selecting visual representations, tell participants that they will be going back to the ratio problem on **Handout 2A.4: Table Visual Representation for the Problem**. Tell them that for this activity they will be looking at an alternative visual representation: a table.

***Small Group**:* Have participants examine the table visual representation on **Handout 2A.4: Table Visual Representation for Ratio Problem**. Ask them to respond to the questions below the table and discuss answers to the following two questions.

1. Does the alternative visual representation (table) lead to a mathematical notation/ equation and should it?

2. Would they agree or disagree with the teacher's selection of the table visual representation for this problem, rather than the schematic diagram with proportions they viewed earlier? Why or why not?

***Large Group**:* Regroup to discuss each group's conclusions about the table as a visual representation. Note: Facilitator should accept participants comments and add the pros of each visual below, if they are not addressed by participants:

1. The table displays the pattern for how the ratio of red/yellow roses builds to 24

Session 2A

roses using a pattern where a consistent amount is added each time. This could help students visualize the part-part relationship in the problem.

2. The schema set up is a precursor for solving ratio/proportion problems when finding an unknown quantity, *x*.

Compare Current Practice

As a group, discuss the following:

1. In what ways are their teaching practices similar to or different from those that they saw in the think-aloud videos and from the visual representations they have discussed today taking into consideration the set of criteria they've discussed?
2. In light of what they learned from the practice guide and these videos, are there any changes or additions they might make to the way they use think-alouds in their classroom for representing problems with visual representations? Do these changes or additions seem feasible to them?
3. Ask the participants to consider and discuss possible roadblocks:
   a. **Do you foresee roadblocks to implementing the changes or additions?**
   b. **Can you think of a plan to work around any roadblocks?**

Closing Comments

Tell participants that in the next session they will have a chance to apply what they learned in today's session by developing a think-aloud to guide students in selecting visual representations for certain problem types. They also will prepare a lesson that requires them to use that same criteria for choosing appropriate visual representations.

Session 2A

Answer Key (these are potential answers)

Handout 2A.2: Visual Representation Think-Aloud Analysis

Answer these questions (refer to the think-aloud in the practice guide on pp. 27–28):

Question	Yes/No
Did the teacher in her think-aloud tell what problem is about (rate, proportion/ratio, money spent, etc.)?	Yes
Did the teacher in her think-aloud tell how she figured out what the problem type is about?	Yes
Did the teacher in her think-aloud explain how she determined the unknown?	Yes
Did she reread parts of the problem to confirm the unknown and/or the problem type?	Yes – unknown No – problem type
Did the teacher explain why she selected the visual? If so, what did she say? Draw a diagram that helps me understand the problem. These diagrams show relationships between two quantities.	Yes
Did she tell how the visual simplified, organized, and displayed the important information?	Yes No Answers may vary
Did the teacher in her think-aloud sum up and review her problem solving?	No
Was there anything critical you think she forgot to say in her think-aloud? If so, list in the box below:	Answers will vary.

Session 2B

Recommendation 3

Teach students how to use visual representations

* ***How-to* Step 2: Use think-alouds and discussions to teach students how to represent problems visually.**

Materials

Previous Handouts

Handout 2A.1: Visual Representation Question Worksheet

Handout 2A.2: Visual Representation Think-Aloud Analysis

Handout 2A.4: Table Representation for Ratio Problem

New Handouts

Handout 2B.1: Preparing a Think-Aloud

Handout 2B.2: Table Visual Representation for Monica and Bianca with Altered Numbers

Handout 2B.3: Preparing a Think-Aloud for Instruction (Option 1, Recommended Activity)

Handout 2B.4a: Prepare to Share (Option 1, Recommended Activity)

Handout 2B.4b: Prepare to Share (Option 2, Custom Activity)

Additional Materials

Practice Guide: Improving Mathematical Problem Solving in Grades 4 Through 8

Chart paper/whiteboard

Pencil and paper

Laptop, tablet, or similar device (optional)

Mathematics Curriculum and/or Guiding Standards

**Note to Facilitator:*

In the previous session, the participants learned important content on developing think-alouds for showing students how to select and solve problems with visual representations. We also discussed similarities and differences among types of visuals and evaluated the appropriateness of alternative visual representations for different types of problems. In this session, participants will apply what they learned in the previous session by developing think-alouds to show students how to represent problems visually. Participants will prepare a lesson that requires them to use their think-aloud with students in an upcoming lesson.

Session 2B

Review

1. Briefly review *How-to* Step 2 and its key features.
2. Briefly review the critical questions that should be used before selecting a visual. Refer to **Handout 2A.1: Visual Representation Question Worksheet** and **Handout 2A.2: Visual Representation Think-Aloud Analysis.** These were completed in the previous session.
3. Remind participants about the importance of answering critical questions before selecting a visual representation.
4. Remind participants about the critical elements of a think-aloud. Discuss when think-alouds might be embedded into their teaching practice.
5. Remind participants about the discussions they had regarding the appropriateness of the use of the table versus a strip diagram or schematic diagram and how they make decisions about what to use.

Experiment with Newly Learned Strategies

This activity will give participants an opportunity to write a think-aloud for the ratio problem where a table was used as a visual representation. Have participants refer to their completed **Handout 2A.4: Table Representation for Ratio Problem** and give each participant a copy of **Handout 2B.1: Preparing a Think-Aloud**.

Small Group: Give each participant a copy of **Handout 2B.1: Preparing a Think-Aloud**. Have participants work in small groups of two to three. Using the information they learned from completing **Handout 2A.4: Table Representation for Ratio Problem** and referencing the think-aloud guiding questions on **Handout 2B.1: Preparing a Think-Aloud**, have participants write a think-aloud for the table solution of the Ratio problem on **Handout 2B.1: Preparing a Think-Aloud** in the space provided. To save time, you may want to recommend that groups write down bullet points on the important elements they want to include in their think-aloud. Participants could then use those notes as they record it on their phones or a tape recorder.

Whole Group: Ask each small group to share its think-aloud with the whole group. Facilitator should lead a discussion to compare the think-alouds.

A final important point that was highlighted in this *How-to* Step was the importance of allowing students to share their work so that students can learn from others in the class. Tell participants that this important idea will be discussed in other sessions and it also relates to Recommendation 5 which suggests that articulating mathematical concepts and ideas is critical for problem solving.

Session 2B

Before moving on to planning, the facilitator should revisit the idea of selecting an appropriate visual representation. Have participants read the ratio problem where the numbers given in the problem are altered. Refer to **Handout 2B.2: Table Visual Representation for Ratio Problem with Altered Numbers**.

The facilitator should draw participants' attention to ten dozen in the problem. Have participants engage in the following discussion.

Whole Group: Discuss the usefulness of the table as a visual representation now that the numbers in the problem have been altered. Discuss the following three questions:

1. Does the table lead to a mathematical notation/equation and should it?
2. Would they use the table for this problem now that the numbers are changed? Or do they think the schematic diagram with proportions we viewed earlier might be more appropriate? Why or why not?
3. Did they select the same preferred visual representation for Monica and Bianca regardless of the altered numbers in the problem?

Facilitator should accept participants' comments and add those listed below if they are not addressed by participants:

1. The table is labor intensive with the changed numbers, so it might take students too much time.
2. The schematic diagram using ratios in the earlier problem might make more sense with larger numbers.
3. Completing the table may help reinforce the pattern and show that the ratio remains constant.

Session 2B

Reflect and Plan

Reflect

Ask the participants to discuss the following:

1. Is there anything they learned over the past two sessions that either confirms or contradicts what they already knew about using think-alouds to demonstrate the appropriate selection of visual representations?
2. Ask participants, in light of what they have learned, to brainstorm ideas about what they would add or change as they plan instruction.
3. As a group, consider the brainstormed ideas and select the additions or changes the group would like to implement in the classroom.
4. How will they determine whether the changes they made resulted in improving student learning? Is there any data they might collect?

Plan

To reinforce the key concepts presented in this session, assign the following recommended activity or an activity of the participant's choosing:

Option 1: Recommended Activity

1. Each participant should choose a mathematics problem that they are currently working on with their students. They will solve this problem using a visual representation and present it to their class with a think-aloud.
2. Using **Handout 2B.3: Preparing a Think-Aloud for Instruction (Option 1, Recommended Activity)** participants should develop a think-aloud for the problem they selected. Participants should then use the think-aloud to model for students.
3. Each participant should complete **Handout 2B.4a: Prepare to Share (Option 1, Recommended Activity)**. At the start of the next PLC session, participants should be prepared to respond to the following:
   a. Share the problem and describe the problem structure.
   b. Tell about the visual representation they used and how they figured out that it was the best choice.
   c. Describe the lesson they taught. Remember to share the think-aloud that they used for the lesson. Participants are encouraged to bring a video or audio recording of their lesson if they so choose.
   d. Did they teach the lesson as planned? If not, describe any adjustments they made and why they made them.
   e. How did their students respond to their instruction? What evidence did they observe or collect?

Session 2B

f. What conclusions did they make based on this evidence?

g. How did using visual representations and a think-aloud help their students solve the problem?

Option 2: Custom Activity

1. The participants may opt to develop a different activity to reinforce the content of this session. If so, consider the following questions to keep the activity on target:
   a. How does this activity relate to Recommendation 3, *How-to* Step 2?
   b. What products will participants collect or develop and bring back to discuss during the next session?
2. Each participant should complete **Handout 2B.4b: Prepare to Share (Option 2, Custom Activity)**. At the start of the next PLC session, participants should be prepared to address the following:
   a. Explain their experiences completing the activity (e.g., level of difficulty of the activity, any problems that were encountered).
   b. Discuss what they learned from completing the activity.

The facilitator should remind the teachers that during the debrief of the next session, they can go beyond the *Prepare to Share* part of the Activity. Teachers may share what they felt worked really well and what didn't work as well as they taught the lesson. Teachers may consider sharing sample pieces of student work from the lesson. Participants are encouraged to share how they might change or adapt the lesson to increase student learning in the future.

Assignment

Assign participants to read pages 32–38 of the practice guide before the next session. This will prepare them for their next session on learning how to teach students to compare solution strategies and for generating and sharing multiple solution strategies.

Session 3A

Recommendation 4

Expose students to multiple problem-solving strategies

* ***How-to* Step 1: Provide instruction in multiple strategies.**
* ***How-to* Step 2: Provide opportunities for students to compare multiple strategies in worked examples.**

Materials

Previous Handouts

Handout 2B.4a: Prepare to Share (Option 1, Recommended Activity) (completed)

Handout 2B.4b: Prepare to Share (Option 2, Custom Activity) (completed)

New Handouts

Handout 3A.1: Video Viewing Guide – Introduction to Recommendation 4

Handout 3A.2: Choosing Worked Examples and Understanding the Benefits

Handout 3A.3: Guiding Questions for Designing Worked Example Comparisons

Handout 3A.4: Examining Example 15

Handout 3A.5: Designing a Worked Example for Fraction Division (Optional Activity)

Answer Keys (end of Session 3.4)

Answer Key: Examining Example 15

Answer Key: Designing a Worked Example for Fraction Division (Optional Activity)

Additional Materials

Practice Guide: Improving Mathematical Problem Solving in Grades 4 Through 8

Video: Introduction to Recommendation 4 (https://youtu.be/bshAa2SJHQ4)

Chart paper/whiteboard

Pencil and paper

Laptop, tablet, or similar device (optional)

Mathematics Curriculum and/or Guiding Standards

Session 3A

Debrief

Ask participants to **debrief** the follow-up activity from Session 1B. Participants were to complete either the recommended or a custom activity.

Option 1: Recommended Activity

The recommended activity was to choose a math problem that can be visually represented and develop a think-aloud for the problem. If the participants did this activity, have them refer to their completed copies of **Handout 2B.4a: Prepare to Share (Option 1, Recommended Activity)** to address the following:

1. Share the problem and describe the problem structure.
2. Tell about the visual representation they used and how they figured out that it was the best choice.
3. Describe the lesson they taught. Remember to share the think-aloud that they used for the lesson. Share video or audio recording of the lesson if applicable.
4. Did they teach the lesson as planned? If not, describe any adjustments they made and why they made them.
5. How did their students respond to their instruction? What evidence did you observe or collect?
6. What conclusions did you make based on this evidence?
7. How did using visual representations and a think-aloud help their students solve the problem?

Option 2: Custom Activity

If the participants opted to do a custom activity, ask participants to refer to their completed copies of **Session 2B Handout 4b: Prepare to Share (Option 2, Custom Activity)** to address the following:

1. Explain their experience completing the activity (e.g., level of difficulty of the activity, any problems that were encountered, and so on).
2. Discuss what they learned from completing this activity.

When applicable, teachers may choose to share beyond the *Prepare to Share* prompts from the Activity. Teachers may share what they felt worked really well and what didn't work as well as they taught the lesson. Teachers may consider sharing sample pieces of student work from the lesson. Participants

Session 3A

Define Session Goals

The goal of this session is to learn how to design an activity for comparing "worked examples." A "worked example comparison" is defined in this facilitator's guide as "a problem with at least two correct and complete solution methods." An activity for comparing "worked examples" occurs when teachers and/or students analyze and compare the two solution methods and discuss how they are similar or different; how both solution methods arrive at the same answer; and whether a student has a preference or affinity for one strategy over the other.

Explain that in the remaining two modules (six sessions), the participants will learn about Recommendation 4: Expose students to multiple problem-solving strategies.

Sessions 3A and 3B will focus on *How-to* Step 1: Provide instruction in multiple strategies and *How-to* Step 2: Provide opportunities for students to compare multiple strategies in worked examples. This session focuses on worked examples as one method of providing instruction in multiple strategies.

Have participants generate questions they would like to have answered in this session that focus on *How-to* Steps 1 and 2, especially those related to comparing worked examples.

During the session, refer to these questions and address them as appropriate. Some of the questions may not be answered in the session. In this case, participants may choose to research the topics on their own or in collaboration with other participants and report back to the group.

Explore New Practices and Compare Them to Current Practices

Access Prior Knowledge

Explain that *How-to* Step 2 focuses on comparing worked examples to promote the idea that problems can be solved in more than one way. This ties into the big idea that students should be exposed to multiple strategies which is the focus of Recommendation 4.

***Whole Group*:** Have participants discuss teaching students to use multiple strategies when solving problems. Use the questions below for discussion prompts.

Session 3A

Ask:

- Do you encourage or require students to generate more than one solution for a problem? Why or why not?
- What challenges do you currently face when teaching students to solve problems in multiple ways?
- Do you include worked example comparisons in your instruction?

Move into New Learning

Explain that *How-to* Step 1 focuses on the importance of providing instruction in multiple strategies. These can be problem-specific strategies, (i.e., strategies that they would use that are specific to the problem) or more general strategies, (i.e., those that can be applied across various problem types). The practice guide panel recommends when providing instruction in multiple strategies, that teachers occasionally demonstrate employing an unsuccessful strategy coupled with think-alouds to show students how an alternate strategy may end up being more successful.

Remind participants that eight studies found positive effects of teaching and encouraging students to use multiple problem-solving strategies and that these findings have implications on the type of instruction that is provided to students. For example, some studies examined the effects of direct instruction, comparing worked examples, and strategic prompting when problem solving. These studies are the basis of Recommendation 4, which the panel agreed had moderate supporting evidence. Refer participants to the material on pages 32–33. If participants are interested in a more comprehensive review of the research base for Recommendation 4 refer them to pages 62–64 (Appendix D).

As a whole group, discuss the following three questions pertaining to Recommendation 4: *How-to* Step 1 and *How-to* Step 2. (Allow 3-5 minutes for participants to review Recommendation 4 in the practice guide. Have participants review text features such as headings, bolded words, and illustrations on pp. 32–35 of the practice guide.)

Participants should reference the practice guide when discussing the questions below.

Question 1: What are the advantages of teaching multiple strategies for problem solving?

**Note to Facilitator: Bring out the following points if they are not generated by participants:*

- Students can become more efficient at selecting appropriate ways to solve problems.
- Learning more than one strategy can increase flexible thinking.
- Discuss potential strategies (i.e., using visual representations, tables, and charts; finding patterns, algebraic notation, guess and check).

Session 3A

Question 2 (*How-to* Step 1): The authors recommended employing unsuccessful strategies for solving problems. What are some of the benefits to students in doing that? Do they agree or disagree?

**Note to Facilitator: Bring out the following points if they are not generated by participants:*

- Changing strategies if one is not working demonstrates that problems may not always be solved easily the first time.
- Trying multiple strategies can teach persistence.

Question 3 (*How-to* Step 2): What are the instructional benefits of comparing worked examples? And how might this approach benefit their students?

**Note to Facilitator: Bring out the following points if they are not generated by participants.*

Comparing worked examples allows teacher to:

- Highlight specific concepts;
- Provide a quick and efficient way for students to see similarities and differences among strategies, especially when presented side-by-side;
- Decrease the cognitive load for students who struggle because they can compare solutions generated for them without solving the problem;
- Promote positive discussion about mathematics and problem solving;
- Enhance analytical skills for determining efficient and sophisticated solutions paths; and
- Offer teachers a way to expose students to multiple solutions without overtaxing students.

Prepare to show the video *Introduction to Recommendation 4*. In this short clip, John Woodward, the chair of the practice guide panel, presents his perspective on Recommendation 4, which includes a description of the three *How-to* Steps for carrying out the recommendation along with suggestions for addressing potential roadblocks.

1. Give each participant a copy of **Handout 3A.1: Video Viewing Guide – Introduction to Recommendation 4**. Briefly review the handout.
2. Tell participants that the discussion they just had as a group may have similar points to the ones John Woodward makes in his video.
3. Show the video *Introduction to Recommendation 4*. Have participants complete their video viewing guides as they watch the video.
4. Ask participants to refer to their video viewing guides. Ask each participant to share one piece of important information about Recommendation 4 that was new to them.

Session 3A

Critical Learning

Authors of the practice guide recommended that teachers model more than one solution for solving the same problem. Comparing worked examples is one way to accomplish this. In designing an activity for comparing worked examples, teachers select a problem and generate two or more solutions to compare. Solutions should be presented side-by-side so that differences in the solutions are presented clearly to the students when drawing comparisons between them.

Tell participants that when designing worked examples for comparison, they must consider its purpose. Are they introducing a new concept or skill? Are they are using the activity to review previously taught solution methods? Or are they introducing a new solution method through connecting it and comparing it with one that is familiar?

Participants also must consider whether they want to compare two solutions that both rely on mathematical notation using a procedure or an algorithm or two solutions where one solution uses a visual representation and the other solution uses an algorithm. They must also consider whether they are comparing skills, conceptual ideas, or both when preparing their activity.

Have participants look at **Handout 3A.2: Choosing Worked Examples and Understanding the Benefits.**

Take a few minutes to review the diagram on **Handout 3A.2: Choosing Worked Examples and Understanding the Benefits**. Explain that there are two foci to choose from (introducing a new concept or reviewing previously taught concepts) and two solution types to choose from (two algorithms or a visual compared to algorithm). Hence, there are four combinations with a variety of benefits to the students (1A, 1B, 2A, 2B). As participants consider these four combinations, they should also think about whether they want to compare skills or concepts. As the facilitator, you should:

- Review the benefits under 1A, 1B, 2A, and 2B and the overall benefits to worked example comparisons (displayed at the bottom of the handout).
- Draw participants attention to the benefits associated with introducing a concept (1A and 1B) versus reviewing a concept (2A and 2b).
- Explain to participants that the diagram is intended to support their thinking when designing worked example comparisons and selecting solutions.

Draw participants' attention to Session **Handout 3A.3: Guiding Questions for Designing Worked Example Comparisons.** These six questions can guide participants as they select a mathematical skill or concept and the two worked example solutions. Explain that the diagram on **Handout 3A.2: Choosing Worked Examples and Understanding the Benefits** is intended to support participants when answering these questions as they design worked example comparison activities:

1. When selecting the mathematical skill or topic and word problem:
    a. **Am I introducing something new or reviewing?**
    b. **What mathematical concept(s) or skills do I want to highlight?**

Session 3A

   c. What background knowledge do my students already have (i.e., what do my students already understand; what do they not understand)?

2. When selecting the solution methods for the topic:
   a. Should I include at least one solution method that includes a visual representation? Why or why not?
   b. What features of each solution might I focus on to highlight the mathematical concepts and skills included in each solution?
   c. In what ways is the comparison of these two solutions important for students' progress in problem solving and mathematical development?

Explain that today participants will **explore** applying these questions from **Handout 3A.3: Guiding Questions for Designing Worked Example Comparisons** to support their thinking about designing worked example comparisons.

Think/Pair/Share

Think: Have participants examine the problem; its two solutions and the six questions listed in the table on **Handout 3A.4: Examining Example 15.**

Pairs or Small Group: Have participants complete **Handout 3A.4: Examining Example 15**. Have participants examine the problem; its two solutions and the six questions listed in the table. Notice some of the questions have already been answered. Have participants answer the remaining questions in the table using the information provided in the problem, the two solution methods that are provided, and from the responses to the questions that were already answered. Refer to page 35 in the practice guide, as needed. Remember, the participant choses two solutions to teach and highlights the concept of "composite variables" to his or her students. (Mention that some mathematicians prefer to call this a "composite quantity" since (y+1) includes a variable and a quantity.)

Share: Discuss the answers to the questions on **Handout 3A.4: Examining Example 15**. Make the point that these questions help them when thinking about selecting solutions that address their learning goal for worked example comparison activities.

Think/Pair Share

Think: Have participants examine the activity for **Handout 3A.5: Designing a Worked Example for Fraction Division**. Read the situation.

Pairs or Small Group: Explain to participants that they will select the two solutions they wish to compare and then answer the six questions for designing a worked example activity for the problem $5 \div \frac{1}{2}$. During this activity, participants should use the diagram on **Handout 3A.2: Choosing Worked Examples and Understanding the Benefits** to select the two solution strategies for the problem on **Handout 3A.5: Designing A Worked Example for Fraction Division**.

Share: Allow small groups time to share their responses with the whole group.

Session 3A

Note to Facilitator: An answer key for **Handout 3A.5: Designing a Worked Example for Fraction Division is provided on the last two pages of this session (pp. 42-43). It is provided to guide you in preparing for the session by helping you to think about potential answers participants might provide or in helping you to think about questions you might want to raise to encourage the whole group discussion. It is in no way intended to limit you on how you would like to have discussions regarding these examples.*

Compare Current Practice

Ask participants to think about *How-to* Step 2's recommendation that a teacher can use worked examples to facilitate comparing strategies with interesting contrasts and not just minor differences. Ask the following questions:

1. Do you structure worked example comparison activities in a way that is similar to or different from the approach presented in *How-to* Step 2 of the practice guide?
   - **a. Do you put worked examples side by side for comparison?**
   - **b. Do you use worked examples to highlight a new learning concept by comparing a visual to an algorithm?**
   - **c. Do you use worked examples to highlight a new learning concept by comparing two algorithms?**
   - **d. Do you structure your own comparisons, or do you rely in comparisons generated in your text? What are the benefits to either?**
2. In light of what you have learned today, what changes or additions might you make to your current practice in providing problem-solving instruction on multiple solutions to your students?

   Ask the participants the following questions:
   - **a. Do you foresee roadblocks to designing worked example comparisons?**
   - **b. Can you think of a plan to work around any roadblocks you might foresee?**

Before next session, ask participants to:

- Bring their text, curricular materials, or standards to the next PLC in case they need to refer to these during activities.
- Think about a concept they might want to teach through comparing worked examples.

Closing Comments

Tell participants that in the next session they have a chance to apply what they learned in the today's session by designing an activity for comparing worked examples and prepare a lesson that requires them to use the strategy of comparing worked examples with their students.

Session 3A

Answer Key

Handout 3A.4: Examining Example 15

When selecting the mathematical skill or topic	Participant Answer
Is the teacher introducing something new or reviewing material?	*Introducing something new*
What mathematical concept(s) is the teacher highlighting?	*Composite variables or* *composite quantities*
What background knowledge do you think these students already have (i.e. what do they already understand)?	Potential Responses: We need to create an answer key for the ones that are blank – Distributive property Meaning of the equal sign Four operations Order of operations
Did she/he choose solutions that include: • Two algorithms? • A visual and an Algorithm? Why do you think she/he made that choice?	*Two algorithms.* Potential Responses: my goal is to highlight different algorithms for this problem by highlighting composite variables. Comparing the solution with the composite variable to a visual would not highlight the different types of algorithms and mathematical notation that could be used to solve this equation. Two algorithms will expand students' repertoire of solution methods that use mathematical notation or symbolic notation.

Session 3A

When selecting the mathematical skill or topic	Participant Answer
What features of each solution should the teacher focus on when comparing these two solutions?	Potential Responses: Mandy: step 1 (distribute) Erica: step 1 (subtraction on both) Emphasize how both first steps are mathematically correct but different from each other. Both will lead to finding a value for y. Mandy: divide on both sides [2y = 6; y = 3] Erica: divide on both sides [2(y+1) = 8; y = 1 = 8] Emphasize that both solutions required dividing at some point during the solution method; but they look very different within the solutions.
How does comparing these two solutions support students' progress in problem solving and mathematical development?	Potential Response: Flexible thinking Introducing composite variables allows students to expand their problem-solving approaches.

Answer Key

Handout 3A.5: Designing a Worked Example for Fraction Division

When selecting the mathematical skill or topic	Participant Answer
Am I introducing something new or reviewing?	*Reviewing fraction division dividing a whole number by a fraction*
What mathematical concept(s) do I want to highlight?	*Concept of division with fractions*
What background knowledge do my students already have (i.e. what do my students already understand; what do they not understand)?	Potential Response: whole number division since this is review they understand dividing a whole number by a fraction they understand the concept of dividing something into fractions
Should I compare • Two algorithms? • A visual and an algorithm?	*Two algorithms.* Potential Response: By reviewing two algorithms, students will be prepared to select and use an algorithm when dividing a fraction by a fraction in the next lesson. *One visual and one algorithm.* Potential Response: By selecting a visual and one algorithm, the concept of division is being reviewed so that students are reminded what is happening when dividing a quantity by a fraction, which will support future learning when dividing a fraction by a fraction.

Session 3A

When selecting the mathematical skill or topic	Participant Answer
What features of each solution should I focus on when comparing the two solutions?	*Two algorithms.* Potential Responses: the parts of the algorithm that are similar and different and explain why each algorithm works. *One visual and one algorithm.* Potential Responses: The steps in the algorithm should be directly connected to the steps in the visual that are the same so that students see why the procedure works to solve the problem.
How does comparing these two solutions support students' progress in problem solving and mathematical development?	Potential Responses: Students build analytical skills. Students continue to build their understanding of fraction division before moving toward more difficult problems. Fraction division is notoriously difficult so this comparison activity reviews concepts and procedures without requiring students to solve the problem.

Session 3B

Recommendation 4

Expose students to multiple problem-solving strategies.

* ***How-to* Step 1: Provide instruction in multiple strategies.**
* ***How-to* Step 2: Provide opportunities for students to compare multiple strategies in worked examples.**

Materials

Previous Handouts

Handout 3A.2: Choosing Worked Examples and Understanding the Benefits

Handout 3A.3: Guiding Questions for Designing Worked Example Comparisons

New Handouts

Handout 3B.1a: Designing a Worked Example Comparison (Concept A)

Handout 3B.1b: Designing a Worked Example Comparison (Concept B)

Handout 3B.2: Solution Bank

Handout 3B.3a: Prepare to Share (Option 1, Recommended Activity)

Handout 3B.3b: Prepare to Share (Option 2, Custom Activity)

Additional Materials

Practice Guide: Improving Mathematical Problem Solving in Grades 4 Through 8

Chart paper/whiteboard

Pencil and paper

Laptop, tablet, or similar device (optional)

Mathematics Curriculum and/or Guiding Standards

**Note to Facilitator:*

In the previous session, the participants learned important content on how to design an activity for comparing worked examples. They also discussed the instructional benefits to comparing worked examples such as highlighting specific concepts; providing a way for students to see similarities and differences among strategies; and, promoting positive discussion about mathematics, especially when presented side-by side, thereby decreasing the cognitive load for students who struggle.

In this session participants will apply what they learned in the previous session by ***experimenting*** *with designing worked example comparisons. They will select a problem and develop two solutions that align to what they are currently teaching so they can try this activity with their students.*

Session 3B

Review

1. Review *How-to* Steps 1 and 2, their key features, the research base, and roadblocks.
2. Briefly review the diagram on **Handout 3A.2: Choosing Worked Examples and Understanding the Benefits** and the guiding questions on **Handout 3A.3: Guiding Questions for Designing Worked Example Comparisons.**
3. Review Example 15 and the fraction division worked examples using **Handout 3A.4: Examining Example 15** and **Handout 3A.5: Designing a Worked Example for Fraction Division** if needed to remind participants of what they learned from Session 3A.
4. Answer any outstanding questions from the participants about comparing worked examples.
5. Ask participants if there are questions or issues they would like to address before moving on with the session.

Experiment with Newly Learned Strategies

This activity will give PLC participants an opportunity to **experiment** with designing worked example comparisons to teach mathematical concepts and highlight multiple solutions.

Small Group: Have participants work in pairs or triads. Each group will have the opportunity to choose from the two different mathematical concepts. Be sure that across groups at least one group is covering concept A and one group is covering concept B.

- **Concept A** focuses on extending the fraction division concept, which is important for 6th-grade standards and also relevant for solving some 7th- and 8th-grade mathematical content.
- **Concept B** focuses on a word problem on ratios and proportions, which address 7th- and 8th-grade standards.

Session 3B

Small Group: Have each group choose from either the fractions division problem on **Handout 3B.1a: Designing a Worked Example Comparison (Concept A)** or the ratios problem on **Handout 3B.1b: Designing a Worked Example Comparison (Concept B)**.

- For each concept, participants may select two of the three solution methods presented on **Handout 3B.2: Solution Bank**. Alternatively, they may create their own solution method if they prefer.
- Tell participants they will use **Handout 3A.2: Choosing Worked Examples and Understanding the Benefits** and/or **Handout 3A.3: Guiding Questions for Designing Worked Example Comparisons** to help make decisions on which solutions to choose.

Have participants write their two solutions and record their responses on **Handout 3B.1a: Designing a Worked Example Comparison (Concept A)** or **Handout 3B.1b: Designing a Worked Example Comparison (Concept B)** for the problem they selected (Solution methods may be developed by participants or chosen from **Handout 3B.2: Solution Bank**).

After groups have completed the handouts, they will spend 2–3 minutes reviewing their decisions before presenting to the whole group. Each group will **reflect** on what led them to the selection of the two solutions. Participants will also **reflect** on which steps in the solutions they would highlight during the comparison.

***Whole Group*:** Have one participant from each small group share their work. Each group will share their completed work from **Handout 3B.1a: Designing a Worked Example Comparison (Concept A)** or **Handout 3B.1b: Designing a Worked Example Comparison (Concept B)** to guide them. Ask the participants to respond to the following two questions while sharing:

- What are the two solutions you selected? Why did you select them?
- What parts of the solutions do you view as critical to highlight when comparing and discussing the two solutions with your students?

Session 3B

Reflect and Plan

Reflect

Ask the participants to discuss the following:

1. Is there anything they learned across the two sessions that confirms or contradicts what they already knew about using worked examples comparisons in their problem-solving instruction?
2. Is there anything they learned across the two sessions that confirms or contradicts what they already knew about the benefits in selecting certain types of solutions (e.g., visual vs. algorithm; comparing two algorithms)?
3. Ask participants, in light of what they have learned, to brainstorm ideas about what they would add or change as they **plan** instruction.
4. As a group, consider the brainstormed ideas and select the additions or changes the participants would like to implement in the classroom.
5. How will they determine whether the changes they made resulted in improving student learning? Is there any data they might collect?

Plan

To reinforce the key concepts presented in this session, assign either the following recommended or custom activity.

Option 1: Recommended Activity

1. Each participant will select a problem and develop two solutions that make sense for the instructional goals. Solutions should be presented side-by-side so that comparison is easy for students. Participants should use **Handout 3A.2: Choosing Worked Examples and Understanding the Benefits** and **Handout 3A.3: Guiding Questions for Designing Worked Example Comparisons** to help design the activity.
2. Using **Handout 3B.3a: Prepare to Share (Option 1: Recommended Activity)**, each participant should select a math concept to teach that is relevant to their mathematics classroom. This can be guided by:
   a. One of the state standards,
   b. Their district pacing guide, or
   c. A concept that is typically challenging for students to understand.

Session 3B

3. Each participant should be prepared to share to the following:
   a. The mathematical concept or skill and the two solutions.
   b. Was selecting a problem to highlight this concept easy for them? Hard for them? What are the issues?
   c. What questions and considerations do they find most useful when designing the activity?
   d. What do they expect might be the response they will get from students when presenting this activity? What evidence will they observe or collect?
   e. What conclusions did they make based on this evidence?

Option 2: Custom Activity

1. The participants may opt to develop a different activity to reinforce the content of this session. If so, consider the following questions to keep the activity on target:
   a. In what way does this activity relate to Recommendation 4, *How-to* Step 2?
   b. What products will participants collect or develop and bring back to discuss during the next session?

2. Each participant should complete **Handout 3B.3b: Prepare to Share (Option 2, Custom Activity)**. At the start of the next PLC session, participants should be prepared to address the following:
   a. What were their experiences completing the activity (e.g., level of difficulty of the activity, any problems that were encountered)?
   b. What did they learn from completing the activity?

The facilitator should remind the teachers that during the debrief of the next session, they can go beyond the *Prepare to Share* part of the Activity. Teachers may share what they felt worked really well and what didn't work as well as they taught the lesson. Teachers may consider sharing sample pieces of student work from the lesson. Participants are encouraged to share how they might change or adapt the lesson to increase student learning.

Assignment

Assign participants to read page 35 of the practice guide before the next session. This will prepare them for learning instructional strategies they will use with their students on comparing multiple solutions.

Session 4A

Recommendation 4

Expose students to multiple problem-solving strategies

* ***How-to* Step 2: Provide opportunities for students to compare multiple strategies in worked examples.**

Materials

Previous Handouts

Handout 3A.2: Choosing Worked Examples and Understanding the Benefits

Handout 3B.3a: Prepare to Share (Option 1, Recommended Activity) (completed)

Handout 3B.3b: Prepare to Share (Option 2, Custom Activity) (completed)

New Handouts

Handout 4A.1: Comparing Solutions Framework and Graphic Organizer

Handout 4A.2: Video Viewing Guide – Teaching Student to Compare Solution Methods for the Rate Problem

Handout 4A.3: The Rate Problem and Anticipatory Questions

Additional Materials

Practice Guide: Improving Mathematical Problem Solving in Grades 4 Through 8

Video: Teaching Students to Compare Solution Methods (https://youtu.be/_kDXx3ixKso)

Chart paper/whiteboard

Pencil and paper

Laptop, tablet, or similar device (optional)

Mathematics Curriculum and/or Guiding Standards

Session 4A

Debrief

Ask participants to **debrief** the follow-up activity from Session 3B. Participants were expected to complete either the recommended activity or an activity of the participants' choosing via the Custom Activity.

Option 1: Recommended Activity

The recommended activity was to select a math concept, and then choose a problem and develop two solutions that address this math concept. If the participants did this activity, have participants refer to their completed copies of **Handout 3B.3a: Prepare to Share (Option 1, Recommended Activity)** to address the following:

1. Share the mathematics concept or skill that they chose.
2. Discuss why they chose the problem.
3. Discuss why they chose the two solutions.
4. Explain the features of the solution they would highlight with their students.

Option 2: Custom Activity

If the participants opted to do a different activity, ask them to refer to their completed copies of **Handout 3B.3b: Prepare to Share (Option 2, Custom Activity)** to address the following:

1. Explain their experience completing the activity (e.g., level of difficulty of the activity, any problems that were encountered).
2. Discuss what they learned from completing this activity.

When applicable, teachers may choose to share beyond the *Prepare to Share* prompts from the Activity. Teachers may share what they felt worked really well and what didn't work as well as they taught the lesson. Teachers may consider sharing sample pieces of student work from the lesson. Participants are encouraged to share how they might change or adapt the lesson to increase student learning.

Define Session Goals

Spend time drawing the participants' attention to the goal of the session which is to continue to focus on *How-to* Step 2: Provide opportunities for students to compare multiple strategies in worked examples.

Tell participants that they will learn about using worked example comparisons in their

Session 4A

classroom. Participants will focus on developing questions that will support students as they identify what is similar or different about the key mathematical concepts or skills in the solution methods they are comparing.

Ask participants to discuss what they hope to learn in this session as it relates to *How-to* Step 2 and what further learning related to designing worked example comparisons they might find interesting.

Explore New Practices and Compare Them to Current Practices

Access Prior Knowledge

Explain that in the previous session, participants discussed how to choose a problem and two solution methods to address a specific mathematics learning goal in their classroom. Remind participants of the overall benefits of using worked example comparisons during mathematics instruction by reviewing the diagram on **Handout 3A.2: Choosing Worked Examples and Understanding the Benefits** from the previous sessions.

Some benefits to using worked examples are that they will:

- Expose students to multiple, correct solution strategies without overtaxing students' cognition;
- Promote positive discussion about mathematics and problem solving;
- Enhance analytical skills; and
- Build flexibility for problem solving.

Ask: What challenges do you anticipate, or have you experienced when having your students compare solution methods?

**Note to Facilitator: Wrap up the whole-group discussion stating that the discussion of the solutions during the comparison is critical for students' learning.*

Move into New Learning

Facilitator reminds participants of the research in the practice guide.

1. ***Benefits to Struggling Learners:*** In many studies, authors found that comparing worked examples was especially beneficial for struggling students. Presenting worked examples can reduce the cognitive load for learners so that solutions can be learned more efficiently. Evidence has also shown that superior learning occurs when solutions are presented side-by-side and can be viewed simultaneously. Therefore,

when modeling and providing students with practice on comparing worked examples, teachers should always present solutions that are correct and present them side-by-side. Additionally, comparing two correct solutions supports the learning of students who struggle more than comparing a correct solution to an incorrect solution. The thinking is that presenting incorrect solutions may inadvertently encourage their use when students solve problems on their own.

2. ***Integrating Worked Example Comparisons into Instruction:*** Problem solving instruction is enhanced when teachers use worked example comparison activities iteratively throughout lessons. For example, rather than presenting problem after problem that students are asked to solve, teachers should weave in comparing worked examples. This will then expose learners to multiple solutions for solving a problem. The practice guide suggests integrating worked examples into instruction by providing them with every couple of problems students are asked to solve.

Tell participants that comparing worked examples is likely not something with which their students will have much experience. It is suggested that a participant model comparing solution methods with their students using a think-aloud. This think-aloud is similar to what was practiced when selecting visuals in Sessions 2A and 2B. Remember, think-alouds provide students with a look into what the teacher thinks about when making comparisons between two solutions and provides an example for what students should be thinking about when they are asked to compare solutions. Before modeling, teachers should determine the important mathematical concepts or skills they plan to highlight during the comparison.

While modeling a think-aloud, participants should:

- Read the problem and state what the problem is asking (note that this is most important with word problems because students are required to comprehend a mathematical situation before viewing solution methods).
- Discuss each solution separately, first while noting the process and mathematics that underlie each solution. Make clear why both solutions are mathematically correct and note they both arrive at the same answer.
- Analyze and compare the solutions by highlighting what is similar and/or different. Discuss what is happening mathematically at different steps to show how the solutions are similar or different, and note also how both are correct.

In practice, after modeling the comparison of worked examples with a think-aloud, participants should support students in comparing and analyzing two solutions methods with worked examples. The practice guide includes three sets of questions teachers can pose to support students as they compare. Have participants refer to page 35 of the practice guide where questions for comparing examples are listed.

These include:

- How are the strategies similar? How are they different?
- Which method would you use to solve the problem? Why would you choose this approach?

Session 4A

- The problem was solved differently, but the answer is the same. How is that possible?

Tell participants that the first and third questions are critical for comparing the solutions, while the second question addresses students' opinions of the solutions which may lead to a discussion about how students may solve a similar problem in the future.

Group Discussion – Ask: What do you foresee as some problems in asking these questions:

- How are the strategies similar? How are they different?

**Note to Facilitator: Receive responses and add the following if not mentioned:*

Students may focus on superficial features of the solutions rather than on mathematics. For example, "Both solutions have plus signs and equal signs." While that may be true, that answer does not focus on the mathematical content. Students will need to be supported as they compare solution methods, especially at first.

Because it is not easy for students to identify which aspects of the solutions are similar or different without some additional prompting/questioning by the teacher (especially for struggling students), teachers must think ahead about prompts and questions they may want to pose.

Direct participants to look at **Handout 4A.1: Comparing Solutions Framework and Graphic Organizer**. Tell them that this graphic organizer is a framework that students can use to organize their thinking as they go through the process of comparing solution methods in two worked examples. In particular, it can help them answer the question, "How are the two solution methods similar or different?" These three additional questions can be used as a follow up to the questions on how the two worked examples are similar or different. It is likely, however, that participants will need to anticipate additional questions they will ask students, ones that are specific to the problem and the two solution methods in each worked example comparison activity.

Share the outline below with participants. It addresses how a participant might use **Handout 4A.1: Comparing Solutions Framework and Graphic Organizer** with their students when comparing worked examples to help them answer the question, "How are the strategies similar and how are they different?"

1. **First**, participants ask the student to read the problem and assess whether their student understands what the problem is about.

2. **Second,** the participant asks the student to go through each step of the first solution, and then each step of the second solution and discuss what is happening in each one separately. They should look to see if the student understands each solution method as the student describes each solution separately so that the participant can support the student in identifying what is happening in each solution method before comparing them.

3. **Third**, the participant moves the student toward comparing the worked examples. The participant asks, "How are the solutions similar or different?" Some students will likely have trouble identifying similarities and differences in each solution method, therefore, the participant will need to support them as they compare. Additional

Session 4A

questions will likely be necessary to draw out the mathematical comparisons that were intended by the participant as they designed the activity. There are two types of questions participants should consider: general and problem-specific.

Prepare to show the video, *Teaching Students to Compare Solution Methods*. Complete the following activities prior to viewing the video.

1. Tell participants that in this video clip, a teacher demonstrates how to question her student as she goes through the process of comparing two solution methods. The teacher asks the student to compare and asks how the solution methods are similar and different. Some of the additional questions the teacher asks are similar to the ones the participants reviewed in **Handout 4A.1: Comparing Solutions Framework and Graphic Organizer**.

2. Refer to **Handout 4A.2: The Rate Problem and Anticipatory Questions Part A and Part B** and **Handout 4A.3: Video Viewing Guide: Teaching Student to Compare Solution Methods for the Rate Problem.**

   a. Individually or as partners, have the participants review the two solution methods for the rate problem on Part A of **Handout 4A.2: The Rate Problem and Anticipatory Questions.**

   b. Tell participants they will be viewing a video where two different types of questions will be used by the teacher. Share with participants the following two types of questions teachers can ask a student when having them compare solution methods:

      i. ***General Questions:*** Some questions may be general and applicable for most any worked example comparison activity. The first three questions in the Features of the Solution Strategies found in **Handout 4A.1: Comparing Solutions Framework and Graphic Organizer** are intended to provide students with a broad questioning framework such as how the problem solved, what operation was used, etc.

      ii. ***Problem-specific Questions:*** Some questions will need to be problem-specific and more strategic in nature. They are questions that point students toward the critical mathematics the teacher wants the students to identify through the worked example comparison. Teachers should also pose these types of questions in order to support students in analyzing the different mathematics used in each solution method and will likely build off the broader-type, general questions. Problem-specific questions direct students toward specific features in the solution methods and will vary depending on the problem and solution methods that were chosen.

Key Idea: Teachers should try to anticipate both general and problem-specific questions prior to teaching their lessons. When designing the comparison activity, a teacher should consider their learning goals and anticipate questions that will help students compare the solution methods. Teachers must spend time unpacking the mathematics in the two solutions methods so they can anticipate questions and potential directions of discussion.

Think/Pair/Share

Think: Refer participants back to Part A of **Handout 4A.2: The Rate Problem and Anticipatory Questions**. Ask them to also examine Part B of **Handout 4A.2: The Rate Problem and Anticipatory Questions**.

Session 4A

Pairs or Small Groups: Have participants work together to anticipate some general and problem-specific questions they might pose to students for the rate problem and the two solution methods they reviewed on Part A. List questions on Part B of **Handout 4A.2: The Rate Problem and Anticipatory Questions.**

Share: Share the questions your group developed.

As a group, view the video *Teaching Student to Compare Solutions for the Rate Problem*. Have participants complete **Handout 4A.3: Video Viewing Guide: Teaching Student to Compare Solution Methods for the Rate Problem** as they watch the video. Ask participants to refer to their video viewing guides and prepare to share one piece of information they learned.

Ask participants the following questions about instructions involving comparing worked examples in their instruction:

- Do you foresee roadblocks to implementing worked example comparisons?
- Can you think of a plan to work around any roadblocks?

Compare Current Practice

Ask participants to consider what they have learned about *How-to* Step 2, in Session 4A. What changes or additions might they make when writing lesson plans to include worked example comparisons?

- Does this seem feasible to them?
- Do they see value in including worked example comparisons in their instruction?
- Are they comfortable covering fewer problems in depth rather than covering several problems more quickly?

Closing Comments

Tell participants that in the next session they will have a chance to apply what they learned in today's session by designing an activity for comparing worked examples and prepare a lesson that requires them to ask probing questions as students engage in activities involving comparing solution strategies in worked examples.

Session 4B

Recommendation 4

Expose students to multiple problem-solving strategies.

* ***How-to Step 1: Provide instruction in multiple strategies.***
* ***How-to Step 2: Provide opportunities for students to compare multiple strategies in worked examples.***

Materials

Previous Handouts

Handout 3A.2: Choosing Worked Examples and Understanding the Benefits

Handout 4A.1: Comparing Solutions Framework and Graphic Organizer

New Handouts

Handout 4B.1: Video Viewing Guide – Comparing the Fractions Problem

Handout 4B.2: Fruit Strips Problem

Handout 4B.3: Fruit Strips Problem – Comparing Solutions Framework and Graphic Organizer

Handout 4B.4: Problems and Solutions

Handout 4B.5: Designing a Worked Example Lesson Plan (Option 1: Recommended Activity)

Handout 4B.6: Comparing Solutions Framework and Graphic Organizer

Handout 4B.7a: Prepare to Share (Option 1: Recommended Activity)

Handout 4B.7b: Prepare to Share (Option 2: Custom Activity)

Answer Keys (end of Session 4B)

Answer Key: Fruit Strips Problem – Comparing Solutions Framework and Graphic Organizer

Session 4B

Additional Materials

Practice Guide: Improving Mathematical Problem Solving in Grades 4 Through 8

Video: Comparing the Fractions Problem
(https://youtu.be/lw8L2w4iGvA)

Chart paper/whiteboard

Pencil and paper

Laptop, tablet, or similar device (optional)

Mathematics Curriculum and/or Guiding Standards

**Note to Facilitator:*

*In the previous session, participants learned important content on instructional activities for comparing worked examples. They discussed general and problem-specific questions to support students in identifying what is similar or different in the two solution methods and the importance of **planning** questions ahead of time so that they can support their students in unpacking the mathematics in the solution methods. In this session, participants will **experiment** with previously learned content from Sessions 3A, 3B, and 4A. They will develop a worked example comparison activity including the preparation of general and problem-specific questions that they anticipate asking their students.*

Review

1. Discuss the video from previous day's Session 4A.
   a. The teacher was asking probing questions to elicit students' comparisons.
   b. How might a teacher get students to move beyond superficial comparisons?
2. Remind participants that designing a worked example comparison requires forethought for the mathematical concepts or skills they are addressing and their learning goals.
3. In Session 4A, participants worked with a Framework and Graphic Organizer (**Handout 4A.1: Comparing Solutions Framework and Graphic Organizer**). The first three questions on the handout were general questions about the solutions (e.g. How was the problem solved; which operation was used; did one solution method rely on a pattern). These are leading questions that teachers can use to help students identify what is similar and different about two different solutions. Discuss those three questions and how they lead into identifying similarities and differences between solutions.
4. Sometimes general questions are not sufficient for supporting students in effectively

Session 4B

comparing two solution methods. When participants **experiment** today, they will practice more with developing general questions and how they might frame problem-specific questions to move beyond "What is similar or different about the solution methods?"

5. Discuss any of the additional key points included in the Framework and Graphic Organizer that may fuel discussion and remind participants of the learning from the previous session.

6. Ask participants if there are questions or issues they would like to address before moving on with the session.

Experiment with Newly Learned Strategies

Video Activity

Prepare to show the video, *Comparing the Fractions Problem*. In this video clip, a teacher supports her student in comparing two solutions. When the participants watch the video, look for how the teacher encourages the student's skill in comparing, and notice what she does to promote students in advancing their mathematical thinking.

Give each participant a copy of **Handout 4B.1: Video Viewing Guide - Comparing the Fractions Problem**.

Individually or as partners: Have the participants review the problem and the two solution methods for the Fractions problem on **Handout 4B.1: Video Viewing Guide – Comparing the Fractions Problem**. Think about the features of each of the solution methods. What do you suspect students might say about how the two solutions are similar and different? What mathematical ideas might be drawn out through this comparison?

Whole group: View the video *Comparing the Fractions Problem*. Have participants write a **reflection** on the interaction after viewing the video. Participants should think about the following questions:

- What does the teacher do to encourage the students' skill in comparing?
- What does the teacher do to promote students in advancing their mathematical thinking?

Ask participants to refer to their video viewing guides and each share one of their **reflections** with the group. Ask participants if the mathematical ideas they anticipated were drawn out during this comparison.

Session 4B

Comparing Solutions Framework and Anticipating Questions Activity

In this next activity, participants will practice applying the graphic organizer and framework for comparing worked examples. They will also anticipate questions they may pose to their students that are based on what they identify as critical learning from the problem and the two worked examples.

The first three questions in the graphic organizer framework from **Handout 4A.1: Comparing Solutions Framework and Graphic Organizer** are intended to follow-up on how solution methods are similar or different. These additional questions are also general and can be applied to most comparisons.

The participant will often need to ask additional questions that are more problem-specific so that the class discussion may lead to a deeper analysis of the underlying mathematics in both solution methods of the worked examples.

Small Group: In pairs or triads, participants will explore how they might use general and problem-specific questions when supporting students as they compare worked examples. Distribute **Handout 4B.2: Fruit Strips Problem**. Tell participants to complete **Handout 4B.3: Fruit Strips Problem--Comparing Solutions Framework and Graphic Organizer** for the Fruit Problem. Participants will answer the first three general questions; will write what is similar and different between the two solution methods; and will use the graphic organizer to record and write additional general and problem-specific questions that they anticipate will lead students to these learning goals.

Whole Group: Participants should share their responses on **Handout 4B.3: Fruit Strips Problem – Comparing Solutions Framework and Graphic Organizer**. An Answer Key is found on pages 62-63 of this Facilitator's Guide. These are not, however, the only goals or questions participants might want to ask. Be flexible and encourage discussion among the participants.

The facilitator should ask:

1. What did you think of the general questions provided on the table? Did they help you determine what was similar or different between the solution methods?
2. Did you think it was hard to identify learning goals and corresponding questions?
3. Did you identify any additional learning goals (in addition to the ones provided on the left-hand column) that you would want your students to discuss while they compare the solution methods?
4. Did your use of the table to identify the features of the solution strategies and your work on identifying learning goals and thinking of anticipatory questions assist you in completing the table on what is similar or different about the two solution methods?

Session 4B

Reflect and Plan

Reflect

Ask the participants to discuss the following:

1. Is there anything they learned over the past two sessions that confirms, contradicts, or extends what they already knew about designing worked example comparisons and supporting their students when comparing worked examples?
2. Ask participants to consider what they have learned across the last four sessions and to brainstorm ideas about what they would add or change as they **plan** instruction on problem solving. Specifically, would they include worked example comparisons?
3. As a group, consider the brainstormed ideas and select the additions or changes the participants would like to implement in their mathematics classrooms.
4. How will they determine whether the changes they made resulted in improving student learning? Is there data or specific information that can support this?

Plan

To reinforce the key concepts presented in this session, assign either the following recommended activity or an activity of the participants' choosing:

Option 1: Recommended Activity

1. Participants can work independently or with a partner to select a problem and develop two solutions that make sense for the instructional goals of an upcoming mathematics lesson. Or, if participants choose, they can opt to use one of the grade 6 or 7 problems and select two of the solutions provided in **Handout 4B.4: Problems and Solutions**. Solutions should be presented side-by-side so that comparison is easy for students. Participants should use **Handout 4B.5: Designing a Worked Example Lesson Plan** to design the worked example comparison and refer to **Handout 3A.2: Choosing Worked Examples and Understanding the Benefits** when making their decisions about the problem and solution methods to use.
2. Additional preparation for the lesson will require participants to record their responses on **Handout 4B.6: Comparing Solutions Framework and Graphic Organizer** for the problem and its two solutions. Participants should answer the first three general questions and complete the table for telling what is similar and different between the two solution methods and additional general- and problem-specific questions that they anticipate will lead to the accomplishment of the mathematical goals of the lesson.
3. Teach a lesson using this problem and its two solutions. By now they should have already modeled for students how to compare solution methods as required by the

Session 4B

recommended activity in Session 3B. Now they are ready to turn this over to students by providing them with an opportunity to compare solution methods accompanied by feedback from them. This lesson includes a) writing the problem on the board with both solutions, b) guiding students through comparing the solutions by asking them to review each solution individually, c) supporting them in noting that both solutions are correct and in discussing/explaining the mathematics that underlie the solutions, d) asking what is similar and different about each solution method, and e) asking additional questions that support mathematics learning that you prepared ahead of time on **Handout 4B.3: Comparing Solutions Framework and Graphic Organizer**. Participants are encouraged to bring a video or audio recording of their lesson if they so choose.

4. After implementing the lesson, complete **Handout 4B.7a: Prepare to Share (Option 1: Recommended Activity).** Each participant should be prepared to share to the following:
   a. How did they select the problem and two solutions?
   b. Describe the lesson they taught. (Use Handouts 4B.4 and 4B.5 as a guide.)
   c. Did they teach the lesson as planned (e.g., did they end up using the probing questions they anticipated)? If not, describe any adjustments they made and why they made them.
   d. How did their students respond to their instruction? What evidence did you observe or collect?
   e. What conclusions did you make based on this evidence?

Option 2: Custom Activity

1. The participants may opt to develop a different activity to reinforce the content of this session. If so, consider the following questions to keep the activity on target:
   a. In what way does this activity relate to Recommendation 4, *How-to* Step 2?
   b. What products will participants collect or develop and bring back to discuss during the next session?

2. Each participant should complete **Handout 4B.7b: Prepare to Share (Option 2, Custom Activity)**. If this activity is chosen, participants do not need to complete **Handout 4B.3: Fruit Strips Problem – Comparing Solutions Framework and Graphic Organizer**. At the start of the next session, participants should be prepared to address the following:
   a. Explain their experiences completing the activity (e.g., level of difficulty of the activity, any problems that were encountered).
   b. Discuss what they learned from completing the activity.

The facilitator should remind the teachers that during the debrief of the next session, they can go beyond the *Prepare to Share* part of the Activity. Teachers may share what they felt worked really well and what didn't work as well as they taught the lesson. Teachers may consider sharing sample pieces of student work from the lesson. Participants are encouraged to share how they might change or adapt the lesson to increase student learning.

Assignment

Assign participants to read pages 36–38 and pages 40–42 of the practice guide for the next session. These pages focus on sharing and explaining solution methods.

Session 4B

Answer Key

Handout 4B.3: Fruit Strips Problem – Comparing Solutions Framework and Graphic Organizer

Features of the Solution Strategies	Solution 1	Solution 2
How was the problem solved? (visual representation, algebraic equation, table, etc.)	3 Tables	Mathematical notation Least common denominator
Which operation(s) were used in each solution?	Repeated addition	Division Equivalent ratios
Did one of the solution methods rely on a pattern?	Yes – pattern of repeated addition to find the cost of 24	No

Use the table above to write some ways the two solution methods are similar and/or different. Remember to focus on how the mathematics skills and concepts underlying each strategy are similar or different.

Similar	Different
24 is used as a target number in both solutions. It is a common multiple for 4, 8, and 12, and was used in Angela's solution and found to be the least common denominator in Reece's solution. In both solutions, they find the cost of fruit strips for each color separately first. Then, both Angela and Reece compare the colors to find the best deal. Both solutions find purple as the best deal.	Angela relies on a pattern of the number of strips to money to find how many strips can be bought for $24. Reece finds equivalent ratios for the price of $24 to decide on the best deal.

Session 4B

What do I want the students to learn or notice when comparing these solutions?	What probing questions could be posed to elicit this learning?
1. 24 is a common multiple for 4, 8, and 12 and was used in both solution methods.	**1.** Do you see any numbers that are the same in both solution methods that weren't given in the problem?
2. Common multiples are used to find common denominators; 24 is a critical number in each solution method.	**2.** 24 is circled in Angela's solution for each color. It is used as the denominator for each color in Reece's solution. Why did both problem solvers need $24?
3. Reece used equivalent ratios and Angela used a common multiple approach.	**3.** What mathematical ideas led each problem solver in their solution method?
4. Angela uses a table to organize her information.	**4.** Which of the solutions used a visual representation to help solve and organize the problem?
5. Both solutions solve each fruit strip color separately to find how many can be bought for the same amount of money ($24).	**5.** Why can't we compare the fruit strip deals with the information given on the table? Why can't we compare them right away? Why do we have to do something to each fruit strip in order to answer the question about which fruit strip is the better deal?

Session 5A

Recommendation 4

Expose students to multiple problem-solving strategies

* ***How-to* Step 3: Ask students to generate and share multiple strategies for solving a problem.**

Materials

Previous Handouts

Handout 4B.4a: Prepare to Share (Option 1, Recommended Activity) (completed)

Handout 4B.4b: Prepare to Share (Option 2, Custom Activity) (completed)

New Handouts

Handout 5A.1: Examining Example 17 – Fraction Strip

Handout 5A.2: Questioning Framework for Mathematics

Handout 5A.3: Probing Questions for Example 9

Additional Materials

Practice Guide: Improving Mathematical Problem Solving in Grades 4 Through 8

Chart paper/whiteboard

Pencil and paper

Laptop, tablet, or similar device (optional)

Mathematics Curriculum and/or Guiding Standards

Debrief

Ask participants to **debrief** the follow-up activity from Session 4B. Participants were to complete either the recommended activity or an activity of the participants' choosing.

Option 1: Recommended Activity

The recommended activity was to choose a math problem with two solutions, analyze them, and develop probing questions to support their students in comparing during guided practice. Then participants were to present the worked example comparison to their students and implementing the probing questions as needed based on student responses.

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If the participants completed this activity, have them refer to their completed copies of **Handout 4B.6a: Prepare to Share (Option 1, Recommended Activity)** to address the following:

1. Share the problem and the two solutions they chose.
2. Discuss why they chose them.
3. Share the learning goals they identified (what they wanted students to learn from the comparison).
4. Share the probing questions they designed to support these learning goals and move beyond the bulleted questions presented in the practice guide.
5. Describe the lesson. Did they teach the lesson as planned? If not, describe any adjustments they made and why they made them. Did the questions they anticipated end up being used during the lesson? Share video or audio recording of the lesson if applicable.
6. How did comparing worked examples help their students learn more about mathematics and problem solving?
7. What evidence did you observe or collect?
8. What conclusions did you make based on this evidence?

Option 2: Custom Activity

If the participants opted to do a custom activity, ask them to refer to their completed copies of **Handout 4B.4b: Prepare to Share (Option 2, Custom Activity)** to address the following:

1. Explain their experience completing the activity (e.g., level of difficulty of the activity, any problems that were encountered).
2. Discuss what they learned from completing this activity.

When applicable, teachers may choose to share beyond the *Prepare to Share* prompts from the Activity. Teachers may share what they felt worked really well and what didn't work as well as they taught the lesson. Teachers may consider sharing sample pieces of student work from the lesson. Participants are encouraged to share how they might change or adapt the lesson to increase student learning.

Define Session Goals

The goal of the final two sessions (5A and 5B) is to **explore** probing questions that support students when explaining their solutions. Participants will focus on material primarily from Recommendation 4, *How-to* Step 3: Ask students to generate and share multiple strategies

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for solving a problem. This *How-to* Step focuses on sharing and explaining solutions for problem solving rather than on teaching students to generate multiple solutions on their own. The hope is that by exposing students to multiple strategies (the core of Recommendation 4) students will have a larger pool of problem solving choices when they approach novel problems. Having students share their solution methods in class is another way to expose students to multiple problem-solving strategies.

**Note to Facilitator: The participants will refer to material from Recommendation 2 and Recommendation 5 of the practice guide as it relates to supporting students in their explanations of problem solving.*

Participants will **explore** ways to support their students as they explain their strategies when solving mathematical problems. The session will focus on probing questions that they might pose to help them understand the students' thinking and/or to help students clarify or refine their own understanding of their approach.

Participants will discuss the following ideas across these two sessions:

1. Why it's important for students to explain and share their problem solving.
2. How participants can support students' explanations through probing questions.
3. How probing questions during problem solving may help redirect students toward a correct solution or help them clarify the solution procedure they were using.

Have participants discuss initial thoughts regarding sharing solutions for problem solving in their classrooms. As the facilitator, you will record participants' thoughts, so you can refer to them as the session evolves and address them when appropriate.

Explore New Practices and Compare Them to Current Practices

Access Prior Knowledge

Whole Group: Have participants discuss the following questions:

1. Do they have their students share their problem-solving solutions in class? If so,
   - **a. How do they support students in explaining their problem-solving process?**
   - **b. Do they ask prompting or probing questions to elicit explanations or to extend an explanation that is not complete?**
2. If not,
   - **a. Are there reasons that they do not have students practice sharing solutions with the class?**

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b. Do they see a benefit in adding this to their instruction?

Move into New Learning

Refer participants to pages 64–66 in the practice guide to view the research on **Recommendation 4 on *How-to* Step 3**.

1. Research shows that generating multiple solution strategies in peer-tutoring dyads is more beneficial than having students generate multiple strategies individually. Four studies reviewed for the practice guide support this approach. Authors of the practice guide reported that based on the research results, it was found that students will develop a better understanding of mathematical concepts when they are asked to explain the steps used to solve a problem in a worked example, and this understanding will help them solve problems successfully. (Endnote 138 from practice guide.)
2. In one study that had positive effects, students solved problems and generated solutions in a collaborative fashion. Findings showed that students who shared strategies and answers after solving problems outperformed students who did not discuss their work with peers.

Whole Group Discussion:

Question 1: Why do you think it's important for students to share their solution strategies with other students?

**Note to Facilitator: You should mention the following if participants do not.*

- *Student explanations for strategies help create a positive sharing environment among students in the whole class.*
- *Sometimes students access what other students present better than what the teacher presents—students' language sometimes helps other students understand.*
- *The students have more buy-in during instruction and may enjoy presenting their strategy to the class.*

Question 2: Why do you think it's important for students to present an explanation for their solution when they present it to their peers?

**Note to Facilitator: You should mention the following if participants do not.*

- *It can promote discussion on key mathematics concepts among the teacher and all the students in the classroom.*
- *Teachers can have a deeper understanding of what each student is thinking as they approach problems and can be a valuable informal assessment tool to see if students understand the concepts or if there are misunderstandings.*

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Think/Pair/Share

Think: Have participants examine **Handout 5A.1: Examining Example 17-Fraction Strip**. Point out that they should be looking at all four parts of the handout.

Pairs or Small Group: Have participants complete **Handout 5A.1: Examining Example 17 – Fraction Strip**. Work together to answer the question below.

Part 1: Looking at the solutions alone, can you draw any conclusions about the students' thinking and understanding just from the equations?"

Part 2: After examining **Example 17** (p. 37) of the practice guide to view the explanations that Student 1 and Student 2 provided for their work on the problem, what conclusions about each student's strategy can be drawn from their explanations? Participants should record their responses on the lines under Part 2 of **Handout 5A.1: Examining Example 17 – Fraction Strip**.

Part 3: On page 37 of the practice guide, the panel suggests asking additional questions to Student 1 and Student 2. Do you have any additional insights participants may gain from posing these questions? Participants should record their responses on **Handout 5A.1: Examining Example 17 – Fraction Strip**.

Student 1: I see that on the left, the green part is $\frac{1}{3}$. The teacher could ask, "How do you know that? And also, "How did you know that the green part on the right is half the area?"

Student 2: "How did you know that the green part is the same as the area colored black?"

Part 4: Participants answer the final question on **Handout 5A.1: Examining Example 17 – Fraction Strip**. What additional questions might they pose to each of these students to elicit more information about the strategy they used? Additional questions should be accompanied with the aspect of the strategy the participants might like to learn more about.

Share: Ask the small groups to share with the whole group their thinking on **Handout 5A.1: Examining Example 17 – Fraction Strip**. Go through the four parts of the handout. To help drive discussion, the facilitator can refer to the Answer Key on pages 71-72 at the end of the session for potential responses.

In wrapping up the activity, state that this example demonstrates how explanations students provide offer the teacher a look into the students' strategy and understanding. It also demonstrates how probing questions can elicit additional information from students and questions can be tailored to assess key understandings.

Questioning Framework Activity

Refer participants to **Handout 5A.2: Questioning Framework for Mathematics**. State that The National Center for Teachers of Mathematics (NCTM) suggests that there are four types of questions teachers may pose to their students. Review the four question types

Session 5A

from NCTM with participants.

Four types of probing questions are listed and explained on **Handout 5A.2: Questioning Framework for Mathematics**. Participants will focus on asking probing questions, which is one of four question types from NCTM. Review the four types of probing questions with participants.

Probing Questions Activity

Now, let's look at an interaction that is scripted in **Recommendation 2, *How-to* Step 3 in Example 9** (p. 21 of the practice guide). Even though this interaction is included in Recommendation 2, it demonstrates how questioning is used to support the learner during the problem-solving process. The panel believes that student-teacher dialogue during problem solving can help students clarify and refine their thinking on their solution method and is one way to encourage and support explanations in mathematics classes.

This example focuses on the teacher using probing questions to help clarify, refine, and direct the problem-solving process with the student. Take a minute to review the script and think about how the teacher supported the student in her thinking.

Whole Group: The facilitator should lead a discussion on the teacher-student dialogue from Example 9 using **Handout 5A.3: Probing Questions for Example 9.** Participants should record examples of these four types of questions in each box during the discussion of Example 9. After the participants categorize the questions from the dialogue, they should discuss how the teacher used each question to support the student in their problem solving.

The facilitator should lead the group in designating the questions in the following categories. The facilitator should also lead a discussion about what the teacher was trying to do when asking each question.

1. **Clarifying Questions:** "Why did you try those numbers?"
    a. Discussion about its use: Motivates the students talking more about why they took their approach rather than just stating what they did.
2. **Confirmation Questions:** "So, you knew they couldn't all be bigger than 15. Or they couldn't all be smaller either?"
    a. Discussion about its use: The teacher posed this question to make sure that the student was explaining that the numbers could not all be bigger or smaller than 15 or the average would not be 15. This question was used to confirm what the teacher suspected the student understood about averages.
3. **Leading questions:** "Where was 15 in your planning?"
    a. Discussion about its use: It prompted the student to think about the average of the target number 15, without explicitly providing the student with how 15 should be used in planning how to solve the problem.
4. **Elaboration or Extended Feedback:** "So the next step is to think about how much bigger some have to be, and how much smaller the others have to be. Okay?"

Session 5A

   a. Discussion about its use: This elaboration or extension of their discussion moves the student to the next step in trying a new set of five numbers since the original set did not average to 15. It uses the information from the entire discussion to get the student back on track.

Many teachers have developed skills for posing questions to their students that provide clarity on how students are solving problems. Questions are also used to help stimulate students' mathematical thinking and to encourage discourse in the mathematics classroom. Generally, it is important for teachers to ask question that will support and enhance students' thinking.

Compare Current Practice

Discuss the following:

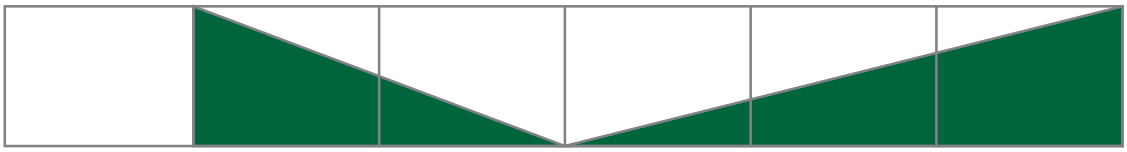
1. The participants discussed examples about questioning students during problem-solving. They discussed different types of questions to facilitate dialogue that will support students—confirmation questions, clarification questions, leading questions, and elaboration or extended feedback questions.
   a. Do the participants include these types of questions in their teaching?
   b. Do the participants feel that these questions come naturally as they support students in explaining problem solving?
2. After today's discussions, are there any changes or additions they might make when teaching problem-solving instruction?
3. Do these changes or additions seem feasible to them? Why? Why not?
4. Ask the participants to consider and discuss possible roadblocks and possible ways to circumvent them.
5. Review the Roadblocks 4.1, 4.2, and 4.4 of the practice guide. Ask the participants to discuss the following questions:
   a. Can you think of a plan to work around these roadblocks?
   b. Do you like the suggestions the panel members recommended to work around these roadblocks in their classrooms?

Session 5A

Answer Key

Handout 5A.1: Examining Example 17 – Fraction Strip

What fraction of the whole rectangle is green?



Student 1:

$(\frac{1}{3} \times \frac{1}{2}) + (\frac{1}{2} \times \frac{1}{2}) = \frac{1}{6} + \frac{1}{4} = \frac{2}{12} + \frac{3}{12} = \frac{5}{12}$ of the entire rectangle.

Student 2:

$\frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

Part 1: Can you draw any conclusions about the students' thinking and understanding just from the equations?

Not definitively. I can guess that Student 1 viewed $\frac{1}{2}$ of the strip separately and that Student 2 looked at $\frac{5}{6}$ of the strip. Without their explanation, it's hard to draw firm conclusions.

Part 2: Now look at the two explanations the students provided (Example 17, p. 37). What conclusions can you draw about the strategy the student used to solve the problem?

Student 1's explanation:

The student looked at the left side of the strip separately from the right side of the strip. She split it in half. She knew $\frac{1}{3}$ of the left half was shaded and that $\frac{1}{2}$ of the right half was shaded. She can write an equation to show how much of the whole strip is shaded.

Student 2's explanation:

She knew that $\frac{1}{2}$ of the rectangle that included shaded parts was shaded and she knew that only $\frac{5}{6}$ of the rectangle included shaded parts. She can write an equation to show how much of the whole strip is shaded.

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Part 3: Now look at the suggested questions in the practice guide on page 37 and answer the questions below:

For **Student 1's** response, the practice guide recommends asking the student how they knew ⅓ was the green part on the left. It also suggests asking, "How did you know the green part on the right is half the area?"

For **Student 2's** response, the practice guide recommends asking the student, "How did you know that the green part is the same as the area colored black?"

What insight about each student's strategies is the teacher trying to uncover by posing these additional questions to her students?

She is trying to get the students to discuss their understanding of the area of rectangles and triangles and that a triangle drawn from corner to corner of a rectangle is $\frac{1}{2}$ of the area of the rectangle.

Part 4: What additional questions might you pose to each of these students to elicit more information about the strategy they used?

Student 1's explanation:

Why did you use the middle (half) to solve this problem?

How did you know the left side is $\frac{1}{3}$ of $\frac{1}{2}$?

Student 2's explanation:

What prompted you to color the other part of the $\frac{5}{6}$ portion of the strip black?

What were you thinking when you chose this strategy?

Session 5A

Answer Key

Handout 5A.3: Probing Questions for Example 9

Clarifying Questions	Leading Questions
Why did you try those numbers?	Where was 15 in your planning?
Confirmation Questions	**Elaboration or Extended Questions**
So, you knew they couldn't all be bigger than 15. Or they couldn't all be smaller either?	So, the next step is to think about how much bigger some have to be, and how much smaller the others have to be. Okay?

Session 5B

Recommendation 4

Expose students to multiple problem-solving strategies

* ***How-to* Step 3: Ask students to generate and share multiple strategies for solving a problem.**

Materials

Previous Handouts

Handout 5A.1: Examining Example 17 – Fraction Strip

Handout 5A.2: Questioning Framework for Mathematics

New Handouts

Handout 5B.1: Proportion Problem I

Handout 5B.2: Handout 5B.2: Video Viewing Guide Proportion Problem I – Focusing on Student Explanation

Handout 5B.3: Proportion Problem II

Handout 5B.4: Video Viewing Guide Proportion Problem II – Focus on Student Explanation

Handout 5B.5: The Bead Problem

Handout 5B.6: Bead Problem – Categorizing Teacher Comments in the Script

Handout 5B.7: Cycle Center Problem

Handout 5B.8: Cycle Center Problem – Categorizing Teacher Comments in the Script

Handout 5B.9: Reflection

Answer Keys (end of Session 5B)

Answer Key: Bead Problem

Answer Key: Cycle Center Problem

Session 5B

Additional Materials

Practice Guide: Improving Mathematical Problem Solving in Grades 4 Through 8

Video: Proportion Problem I
(https://youtu.be/Ste6U-3uLHk)

Video: Proportion Problem II
(https://youtu.be/ZGdAYKtH39g)

Video: Bead Problem
(https://youtu.be/i4KMStSQ2Ls)

Video: Cycle Center Problem
(https://youtu.be/_F15PwxsjCo)

Chart paper/whiteboard

Pencil and paper

Laptop, tablet, or similar device (optional)

Mathematics Curriculum and/or Guiding Standards

**Note to Facilitator:*

In the previous session, the participants learned important content on how to question and respond to students as they share their strategies and give explanations. Participants explored ways to support students when analyzing solutions to help them learn about multiple ways to solve problems focusing on developing questions that will encourage students to identify critical mathematical content when comparing solutions so that they expand their mathematical thinking and understanding.

In this session, participants will apply what they learned in Session 5A by engaging students in activities where they solve problems and share their solutions with others. Participants ask probing questions to expand their mathematical thinking and understanding and elicit more clarity in their explanations.

Review

1. Review *How-to* Step 3 including the key concepts regarding the research, approaches, and benefits of providing students with experiences using multiple methods to solve problems.
2. Briefly review the diagram on **Handout 5A.2: Questioning Framework for Mathematics**.
3. Review key learnings from the **Fraction Strip Problem – Example 17** activity in Session 5A.

Session 5B

4. Answer any outstanding questions from the participants about sharing multiple strategies including students giving explanations and teachers using effective questioning strategies to foster growth in students' mathematical thinking.

5. Ask participants if there are questions or issues they would like to address before moving on with the session.

Experiment with Newly Learned Strategies

Video Activity 1

Prepare to show two videos that show two different students giving an explanation of their problem-solving methods on the Proportion problem. Each student is presenting a solution to the Proportion Problem on pages 34 and 35 of the practice guide.

Proportion Problem

> *Ramona's furniture store has a choice of 3-legged stools and 4-legged stools. There are five more 3-legged stools than 4-legged stools. When you count the legs of the stools, there are exactly 29 legs. How many 3-legged and 4-legged stools are there in the store?*

Before viewing the first video, have participants take a few minutes to review the two ways that the student solved the problem in the video participants are about to see. The problem can be found on **Handout 5B.1: Proportion Problem.**

Distribute **Handout 5B.2: Video Viewing Guide Proportion Problem I – Focusing on Student Explanation**. Tell participants that they will be viewing a student solving the Proportion Problem. Explain to them that they will be looking for key features of how the student provided an explanation of their solution. The student behaviors they are going to look for are listed below and are also located in **Handout 5B.2: Video Viewing Guide Proportion Problem I – Focus on Student Explanation**. Discuss with participants the directions for completing the video viewing guide. Remind participants that they are looking for the student to address the following:

- Did the student state what the problem was about?
- Did the student state what she is trying to find?
- Did the student talk about which information is important for solving the problem?
- Did the student explain her strategy as it relates to key information in the problem?
- Did the student explain why she thinks her answer is reasonable?

Have participants complete their video viewing guide as they watch the video.

Session 5B

Whole Group: Conduct a whole group discussion on the student's explanation of the Proportion Problem. Also, discuss how the teacher supported the student in explaining their solution.

Distribute **Handout 5B.3: Proportion Problem II and Handout 5B.4: Video Viewing Guide Proportion Problem II – Focus on Student Explanation.** Have participants view the second video of another student giving an explanation. Again, this video focuses on the key features of a student providing an explanation for her solution method.

Whole Group: Conduct a whole group discussion on the second student's explanations of the Proportion problem. Discuss the student's explanation. Also discuss how the teacher supported the student in explaining her solution.

Video Activity 2

Prepare to show two additional videos. These videos will focus on teacher questioning that was intended to advance the student's mathematical thinking.

Bead Problem

> *Cindy is making a necklace and using a pattern with her beads. For every 3 jade beads she puts on the necklace, she adds 10 magenta beads. If she has 9 beads, how many magenta beads will she need to make the necklace.*
>
> *Follow up question: How many beads are on the necklace?*

Before viewing the video, have participants take a few minutes to read the problem and think about the important mathematics in the problem. The problem can be found on **Handout 5B.5: Bead Problem.** Participants should write a few anticipatory questions that they may pose to students as they consider key mathematical ideas from the problem. Then have participants look at how the student solved the problem in the video and have them take note of the teacher's questioning.

After viewing the video, distribute **Handout 5B.6: Bead Problem – Categorizing Teacher Comments in the Script**. Participants will take some time to review the script of the instructional talk that occurred during the video. Then, they will categorize the type of questions or comments the teacher made by marking them as either clarifying, confirmation, leading or elaboration/extended feedback.

Afterwards, discuss the different types of questions and elaborations the teacher made. In addition, discuss which question types or elaborations were posed most and least often. Discuss the teacher's questions/comments and the participants initial thoughts about the interaction. Also discuss any additional questions or comments that could have been posed to this student.

**Note for the Facilitator: It is not necessary for everyone to agree on the question types. There may be varying opinions. The purpose is to notice the different types of questions that can be asked and how they help students move forward in their mathematical knowledge and skills.*

Session 5B

Cycle Center Problem

> *The Cycle Center has bicycles and tricycles in the storeroom. There are at least two of each, and there are more bicycles than tricycles. There are 23 wheels altogether. How many bicycles and tricycles are in the storeroom?*

Before viewing the video, have participants take a few minutes to read the problem and think about the important mathematics in the problem. The problem can be found on **Handout 5B.7: The Cycle Problem – Focus on Teacher Questioning.** Participants should write a few anticipatory questions that they may pose to students as the students consider key mathematical ideas from the problem. Then have participants look at how the student solved the problem in the video and have them take note of the teacher's questioning.

After viewing the video, distribute **Handout 5B.8: The Cycle Center Problem – Categorizing Teacher Comments in the Script**. Participants will take some time to review the script of the instructional talk that occurs during the video. Then, they will categorize the type of questions or comments the teacher made by marking them as either clarifying, confirmation, leading or elaboration/extended feedback.

Afterwards, discuss the different types of questions and elaborations the teacher made. In addition, discuss which question types or elaborations were posed most and least often. Discuss the teacher's questions/comments and the participants' initial thoughts about the interaction. Also discuss any additional questions or comments that could have been posed to this student.

Note to Facilitator: It is not necessary for everyone to agree on the question types. There may be varying opinions. The purpose is to notice the different types of questions that can be asked and how they help students move forward in their mathematical knowledge and skills.

Reflect and Plan

Reflect

Ask the participants to discuss the following:

1. Is there anything you learned across the two sessions that either confirms or contradicts what you already knew about supporting students when sharing and explaining solution strategies?
2. Ask participants, in light of what they have learned, to brainstorm ideas about what they would add or change as they plan instruction.
3. As a group, consider the brainstormed ideas and select the additions or changes the participants would like to implement in the classroom.

Session 5B

4. How will they determine whether the changes they made resulted in improving student learning? Is there any data they might collect?

Plan

To reinforce the key concepts presented in this session, assign the following recommended activity. In this final session, the activity below is a direct extension of the learning from 5A and 5B. Since there is not a session following, the intention is that teachers implement questioning in their classroom and **reflect** on the lesson for future improvements.

Activity

Each participant should choose a word problem that they will pose to their students. This problem should be novel to their students and capitalize on mathematical skills and understandings they have been taught or are working on. Non-routine problems, like the stool problem from the video, are good for challenging students and seeing how they can apply their learning.

Participants should pose the problem to their students and ask them to solve it independently. They should walk around the room looking for students who have solved the problem correctly to share their solution and process with the class.

When students explain and share how they solved the problem, the participant should support the students' explanation by asking probing questions. The participant should think about the four types of probing questions that have been discussed over the last two sessions.

After teaching the lesson, each participant completes **Handout 5B.9: Reflection**. As this is the final session on this topic, there will not be a formal time to share how your lesson went with other participants, unless this is an ongoing PLC where an extension is natural. However, encourage the participant to continue to discuss problem solving instruction with their colleagues as they move forward with implementing the strategies that have been covered across all sessions of the PLC.

Items in **Handout 5B.9: Reflection** include:

1. Did the students who explained the solution need a lot of probing questions from you?
2. Did any of your students approach the problem in an unexpected way?
3. Were you able to think of probing questions that worked well with your students' approaches to the problem?
4. How did your students respond to your questioning? What evidence did you observe or collect?
5. What conclusions did you make based on this evidence?

Session 5B

Answer Key

Handout 5B.6: The Bead Broblem

Clarify (CL) **Confirming (CF)** **Elaboration (E)** **Leading (L)**

1. CL **T:** Can you explain some of your thinking and how you went through this problem?

S: *I started off making the pattern, so I could visually see it and I did some multiplication to find the numbers, but then I realized there's a better way of doing it.*

2. CL **T:** Can you explain how you started making the pattern? What made you make the decision to put circles and the squares and the number of circles and squares you used?

S: *I made a key that says that one circle makes one jade bead and a square equals magenta beads. I did three because it says three jade beads. And I put three circles and then it says she puts on the necklace she adds in magenta beads and so I added ten squares and then I repeated that three times.*

3. CL **T:** And what made you decide not to continue with this way of solving the problem?

S: *Because there was another question that said how many beads are on the necklace and so I thought because I really didn't know what you called the method, so I thought of a thing we do in school which is called R.I.C.E.*

4. CF **T:** So down here it looks like you used the R.I.C.E. method and you thought that might be better than what you were doing up here to solve this follow up question? Is that why you chose to shift from solving it this way to solve it this way (points to each of the two different methods)?

Note: Some people may categorize this is a clarification question. Authors settled on "Confirming" because it seemed like the teacher confirms what the student did and asks a confirming question of why they used two different methods.

Session 5B

5. **CL** **T:** Can you explain what you did for your R.I.C.E. method. What were each of these pieces?

S: *The first one is Read so I put the important details and then the second one is for Read I underline for every three jade beads she puts on the necklace she puts on 10 magenta beads b/c I thought that was one of the important parts to know.*

6. **E** **T:** And I noticed that you did that up here too when you were (points to underlined important information the student identified) first reading it in your first method, you also underlined these, so you were highlighting the important information when you were going through it. Very good.

Note: Some participants may categorize this as a clarification question. Authors chose elaboration here because we thought the teacher was elaborating on what the student said about highlighting important information in the problem.

S: *For the second one, its Illustrate, so I did kind of the same pattern, except I made it simpler instead of one square equaling two magenta beads. I did one square with a line equals 2 magenta beads and I just continued with the pattern.*

7. **CF** **T:** And did you use this method with one square equaling two magenta beads?

8. **CF** **T:** Was that to save you some space as you were working?

S: *Yes.*

T: All right good thinking.

S: *And then for the C is Calculating.*

T: I see right here it says to calculate. Good.

S: *Up here it says 3 jade beads she put on the necklace. On here it says if she has 9 jade beads, how many magenta beads would she need to make so I did 3 x 3 was 9 and then 3 x 10 using the 3 because I used 3 here. I would use the 3 again to multiply it by 10 so I got 30 and then I added that to get 39. And then for the other question of how many beads are on the necklace. (student pauses)*

9. **CL** **T:** So, you showed you calculated here. It looks like this—part of R.I.C.E. is the equation. Can you explain how you built up that equation?

S: *So, for the first one-30 magenta beads and for the second one it was 39 magenta beads.*

Session 5B

10. **CF** **T:** So, what I'm thinking is that you had 30 magenta beads and you knew that you needed the nine more so is this where you showed that this was 30 and the 39?

S: *Yah*

11. **L** **T:** Can you think of a way you could have gotten to this solution of 39 beads with the information you got here? (points to 000 box box box, etc.)

S: *I could have counted by 10s for this and then I would have gotten to 30 and then I could have just added 9 to it.*

12. **L** **T:** And if you take a look at the work you did here, you're thinking 3 x 3 = 9 and also the 3 x 10 = 30, so you would have arrived at 39 by doing what?

S: *Adding*

T: You've got it very good.

Session 5B

Answer Key

Handout 5B.8: Cycle Center Problem

Clarify (CL) **Confirming (CF)** **Elaboration (E)** **Leading (L)**

1. CL **T:** Can you say a little about how you solved this problem?

S: *So we started out with 23 total wheels and there's 3 wheels on the tricycle so I started taking away 3 wheels and that led me to 20 wheels and I took away 3 more and that got me to 17 wheels and take away 3 more which is 14 and this is an even number so I can use that so I divide 2 b/c there are 2 wheels on a bicycle which leaves me with 7 and so I have 7 bicycles and 3 tricycles.*

2. L **T:** Can you say a little bit more—why didn't you just stop at 1 tricycle. You mentioned that it came down to an even number so often we had one tricycle why didn't you just stop and divide by 2 there?

Authors saw this as leading—asking the student to think alternatively why they did not do something else rather than what they did.

S: *So, in the problem it said there were at least two of each, two bicycles and tricycles so there would only be one tricycle if I left it at 20.*

3. L **T:** Oh, so one of the parameters of the problem was that there had to be at least two of each. Why didn't you stop the second time around?

4. L **T:** How did you know not to stop at 2 tricycles?

S: *So, 17 is not an even number. It's an odd number so if I would have divided it by 2, it would be a fraction which you really can't have a fraction of a bike, it needs to be a whole bike.*

T: That is true.

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5. L **T:** If we were going to extend this kind of pattern you got going, could we get another answer where we still had more bikes than trikes—if we had a least two of each but we had a different answer than this?

S: *Yes. We could because if you subtracted 3 from here (points to 14) that would be 12…no 11 and if you take 3 away from 11 that would be 8 and you could do 8 divided by 2 which is four bikes wait…*

6. L **T:** And how many tricycles would that be?

S: *That would be more tricycles so no you could not do that.*

T: Great work and high five!


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