Embedding Self-Regulation Instruction within Fractions Intervention for Third Graders with Mathematics Difficulties

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Abstract

The purpose of this study was to explore the efficacy of fractions intervention, with and without an embedded self-regulation (SR) component, for 3rd-grade students at-risk for mathematics disabilities. Fractions intervention focused on magnitude understanding and word problems. Embedded SR was designed to support a growth mindset (fostering belief that intellectual and academic abilities can be developed), along with SR processes in which students set goals, self-monitor, and use strategies to engage motivationally, metacognitively, and behaviorally through challenging tasks. Students (n=69) were randomly assigned to business-as-usual control and the 2 versions of fractions intervention. Multi-level models, accounting for the nested structure of the data, identified a moderation effect on fraction word problems: For students receiving fractions intervention with embedded SR, response to intervention was robust across the continuum of students' pretest word-problem skill; by contrast, without SR, response to fractions intervention depended on students' pretest word-problem skill. On the remaining outcomes, results reflected stronger outcomes when fractions intervention embedded SR instruction without moderation.

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Competence with fractions is foundational for algebra, for success with more advanced mathematics, and for competing in the American workforce (Booth & Newton, 2012; Geary Hoard, Nugent, & Bailey, 2012; National Mathematics Advisory Panel [NMAP], 2008; Siegler et al., 2012). For this reason and because many students have difficulty understanding and operating with fractions (Namkung, Fuchs, & Koziol, 2018), the NMAP assigned high priority to improving fractions performance. As randomized controlled trials demonstrate (Fuchs et al., 2013, 2014, 2016a, 2016b), small-group intervention that focuses on fraction magnitude understanding with explicit strategy development improves understanding of and the capacity to work productively with fractions for fourth-graders at risk for mathematics learning disabilities.

Although *fourth* grade may seem like an early grade to focus intervention on fractions, the Career- and College-Ready Standards, which have been adopted in the last decade across the U.S., establish a strong emphasis on fractions starting in *third* grade. They set the expectation that third graders understand fraction magnitudes, as revealed by identifying fraction equivalencies, using reasoning to compare fractions, and placing fractions on number lines. To encourage alignment with the general education curriculum and prevent students with histories of poor whole-number learning from falling behind classroom peers on this foundational skill for advanced mathematics learning, third-grade intervention therefore requires a strong focus on fraction magnitudes.

Yet, few studies have examined whether fractions intervention can enhance performance in this critical domain at this grade level among students at-risk for mathematics difficulties due to their history of poor whole-number achievement. We identified two prior studies conducted with students identified as low-performing in math. Using a multiple-baseline study across three low-performing students, Perkins and Cullinan (1984) assessed the effects of a direct instruction intervention. Although the program improved performance, the intervention and its outcomes were of limited scope, focused dominantly on part-whole understanding (representing fractions with circles, writing numerical fractions for circle representations, and adding fractions with like denominators). Their focus on fraction magnitudes was operationalized solely as identifying whether fractions were greater than, equal to, or less than one.

Courey (2006) assessed the value of teaching visual representations of halves in the context of word problems with and without (a) teacher language designed to convey the meaning of half ("one of two equal parts”) and (b) practice identifying relevant pieces of information about half in the word problems: the number or phrases expressing "how much (number to be halved) you start with" and "a reason to find two equal parts." This component provided no added value over the same intervention without this component. Further, students in both treatment groups outperformed control students only on procedural outcomes, not on conceptual understanding. Moreover, as with Perkins and Cullinan (1984), the major emphasis was part-whole, not fraction-magnitude understanding.

Rationale and Purpose for Present Study

The field’s minimal focus on fractions intervention generally and fraction magnitude understanding specifically among at-risk third graders is likely due to the relatively recent emphasis on fractions at this grade level. It probably also stems from the field’s failure to address general education curricular targets within intervention (Powell & Fuchs, 2015). This seems like a major omission from the mathematics intervention literature given that standards reform (Edgerton et al., in press) establishes the expectation that students with disabilities will achieve college- and career-ready standards. For these reasons, the first purpose of the present study was to explore whether intervention that emphasizes fraction magnitudes improves fraction outcomes for third graders with histories of whole-number difficulty. This is an untested proposition.

At the same time, because fractions represent a major hurdle for students with histories of whole-number difficulty (Namkung et al., 2018), our second purpose was to examine whether embedding a self-regulation (SR) component within third-grade fractions intervention provides added value on student outcomes. In the context of academic learning, SR is viewed in multiple ways. This includes a growth mindset reflecting the belief that intellectual and academic abilities can be developed (e.g., Lin-Siegler, Dweck, & Cohen, 2016; Yeager & Dweck, 2012), along with SR processes in which students set goals, self-monitor, and use strategies to engage motivationally, metacognitively, and behaviorally (Cirino et al., 2017; Lezak et al., 2012; Zelazo, Blair, & Willoughby, 2016). We adopted this approach in the present study.

Studies provide some support for the effects of this type of SR on mathematics learning. Most research is correlational, showing a connection between mathematics performance and active goal setting and perseverance through challenging tasks (e.g., Schunk, 1996; Park et al., 2016). A smaller body of work focused on the effects of building SR on mathematics outcomes is conducted largely with preschool children and pre-academic tasks (e.g., Blair & Raver, 2014; Schmitt et al., 2015). In a series of more relevant experimental studies, Fuchs et al. (1997) isolated positive effects for task-focused goals within classroom peer-assisted learning strategies on low-performing students' mathematics concepts, applications, and operations at grades 2-4, and Fuchs, Fuchs, et al. (2003) found added value for self-regulated learning instruction when integrated within word-problem instruction compared to word-problem instruction alone for low-performing students.

(Note that our SR regulation component differs from what's referred to as *cognitive training*. With working memory training, students practice the kinds of tasks that are assessed on working memory assessments [e.g., for a series of cards, the child counts the number of dots; after counting each card in the series, the child recalls the counts; the number of cards in the series increases when the child meets the success criterion at the present span]. See Melby-Lervag & Hulme [2013] for a meta-analysis of working memory training effects.)

In the present study, we thus compared effects of two versions of an intervention for improving at-risk third graders' fraction performance against a business-as-usual control group. Our primary purpose was to estimate the effects of our base program, a newly-developed fractions intervention. The intervention focused on magnitude understanding of fractions and schema-based instruction to teach fraction word problems using an instructional model previously validated with at-risk fourth graders (Fuchs et al., 2013, 2014, 2016a, 2016b), while simplifying the program's scope to address the third-grade curriculum. These studies limited the range of denominators to increase depth rather than breadth of conceptual understanding and to minimize the learning challenges for at-risk students with potential working memory deficits.

Furthermore, each of the components in the core program received by both treatment conditions included in the present intervention were tested as isolated intervention conditions in the aforementioned studies. These include the conceptual comparing strategies for assessing relative magnitude, fluency for implementing these strategies quickly in a timed activity, and promoting additive reasoning embedded in schema-based word problem instruction. Schema-based instruction is designed to compensate for limitations in students' cognitive resources by teaching them to identify word problems as belonging to word-problem types that share structural features (e.g., compare, change, and splitting). Students are also taught to represent the underlying structure of the word-problem type with a number sentence (e.g., Fuchs et al., 2009, 2010) or visual display (Jitendra & Star, 2012, Jitendra et al., 2011, 2009).

One of the two fraction conditions included a SR component, in which students practiced goal setting and self-directed learning activities in conjunction with ongoing progress-monitoring data, as in the Fuchs et al. (1997, Fuchs, Fuchs, et al., 2003) self-regulated learning studies. The present study expanded SR further to build a growth mindset and encourage goal-setting, self-monitoring, and perseverance through challenging tasks, with tutor-led discussions prompted via a series of stories featuring individuals who have faced school and other life hurdles. Because fractions is a challenging topic for many students, especially those with a history of poor learning with whole numbers (Namkung et al., 2018) and in this age group when a strong emphasis of fractions understanding is introduced, we hypothesized that embedding SR into a demanding fractions intervention provides added value for this population of learners.

Method

Participants

Participants were students with mathematics difficulty from 19 third-grade classrooms across six schools in a large, metropolitan school district. We conducted whole-class testing to identify students who met one of two low-math criteria at the start of the school year: (a) performance below the 22nd percentile on a broad-based calculations measure (Wide Range Achievement Test–4 [WRAT-4]; Wilkinson & Robertson, 2006) or (b) WRAT-4 performance below the 31st percentile *and* a score less than three on the *Second-Grade Calculations Battery-Minuends to 18* (Fuchs, Hamlett, & Powell, 2003). We selected these cut-points because they are frequently employed to identify students for Tier 2 intervention. Also, our cut-points are in line with schools' RTI Tier 2 system. Further, previous fractions intervention studies have used the

WRAT-4 to identify risk status (e.g., Fuchs et al., 2013, 2014, 2016a, 2016b). Of 261 students screened, 132 met one or both criteria. We randomly selected 103 for individual testing. Because this study was not about intellectual disability, we excluded six students who scored below the 9th percentile on both subtests of the 2-subtest Wechsler Abbreviated Scales of Intelligence (WASI; Wechsler, 2011). We also excluded two students who transferred to a different school before random assignment; two whose schedule precluded participation in the intervention; and three who did not provide assent to participate at the point in the study. Of the 90 remaining students, we randomly selected 73 students to meet our enrollment target for this developmental pilot study.

We randomly assigned these students at the individual level to a business-as-usual control condition (n = 26) and the two intervention conditions: (a) the base intervention focused on improving fraction magnitude understanding and fraction word-problem performance (the “base” condition; n = 24) and (b) the same base intervention with the embedded SR component (the “SR” condition; n = 23). We dropped one student from the SR condition before intervention started due to an unanticipated schedule conflict. Prior to the end of the study, one base condition and three control students moved beyond the study’s reach. Complete data were thus available for 23 base condition students, 23 SR condition students, and 23 control students. Students did not differ by condition on demographics or screening performance (see Table 1).

Measures

Screening. With WRAT-4-*Math Computation* (Wilkinson & Robertson, 2006), students complete calculations problems of increasing difficulty (median reliability at 5-12 years = .94). With *Second-Grade Calculations Battery-Minuends to 18* (Fuchs, Hamlett, & Powell, 2003), students have 1 min to complete 25 problems (α = .87). WASI (Wechsler, 2011) is a 2-subtest measure of general cognitive ability, comprising the *Vocabulary* and *Matrix Reasoning* subtests (reliability > .92). *Vocabulary* assesses expressive vocabulary, verbal knowledge, memory, learning ability, and crystallized and general intelligence. Students identify pictures and define words. *Matrix Reasoning* measures nonverbal fluid reasoning and general intelligence. Students complete matrices with missing pieces.

Fractions outcomes. With the *Fraction Battery-Revised - Single-Digit Multiplication* (Schumacher et al., 2015), students have 5 min to answer 30 single-digit problems (factors 1-10) presented horizontally (α = .92). The maximum score is 30. We included this as a fractions outcome, because multiplication, as it relates to multiplicative reasoning, is foundational for identifying equivalent fractions (Lamon, 2012).

The *Fraction Battery-Revised - Comparing Fractions* (Schumacher et al., 2015) indexes magnitude understanding with 16 items. Each shows two fractions, between which students place a greater than, less than, or equal sign. Two items have the same numerator; two have the same denominator; six include ½ as one of the fractions; three require students to write a fraction equivalency for one of the fractions; and three can be solved either using ½ as a benchmark fraction or rewriting one fraction with an equivalency. The maximum score is 16 (α = .90).

The *Fraction Battery-Revised - Ordering Fractions* (Schumacher et al., 2015) measures magnitude understanding with 11 items. Each shows three fractions to be ordered from least to greatest. Five items have fractions with the same numerator. One has fractions with the same denominator. The remaining five include ½ as one of the three fractions (e.g., ¾, ½, and 2/8). The maximum score is 11 (α = .91).

With *Fraction Number Line 0-1* (Hamlett, Schumacher, & Fuchs, 2011, adapted from Siegler et al., 2011), students are presented with a 0-1 number line on a computer; endpoints are labeled. The tester models a practice item. The student practices an item and then completes the 20 test items (12/13, 7/9, 5/6, ¼, 2/3, ½, 1/19, 3/8, 1/7, 4/7, 3/4, 2/5, 9/12, 7/8, 2/6, 5/12, 9/10, 3/6, 3/10, and 1/3), presented in random order. Accuracy for each item is the absolute difference between the student's placement and the correct placement. Item scores are averaged and multiplied by 100 to derive the percent of absolute error. Lower scores indicate stronger performance (test-retest reliability = .80).

The *Fraction Battery-Revised* - *Fraction Word Problems* (Schumacher et al., 2015) includes 16 acquisition and transfer problems representing three problem types: compare (five problems), change (eight problems), and splitting (three problems). (At pretest, we used a version with eight items to limit fatigue given students' limited fraction word-problem skill at start of third grade. We did not use tested problems during intervention.) Compare word problems require students to evaluate the magnitude of fraction quantities within a narrative (e.g., Michael and Sue each baked a cake for a contest. Michael used 3/8 of a pound of flour. Sue used ¼ of a pound of flour. Who used less flour?). Some problems include irrelevant numerical information or an additional fraction that requires students to compare (order) three fractions, rather than two fractions. Change problems require students to solve for a missing Start, Change, or End amount within a cause-effect narrative (e.g., Juan had 6/8 of a pizza in his fridge. He ate 1/8 of the pizza for dinner. How much pizza is in his fridge after dinner?). Some problems include irrelevant numerical information. Splitting problems require students to make fractions from units (e.g., Matthew has 2 watermelons. He cuts each watermelon into sixths. How many pieces of watermelon does Matthew have now?). Some problems include novel vocabulary and/or novel questions (e.g., Marcus has 3 gallons of milk to make cheese. Each block of cheese needs a half gallon of milk. How many blocks of cheese can Marcus make?). Testers read each item aloud while students follow along on paper. Students can ask for one rereading. For each problem, students earn 1 point for the correct numerical answer and 1 point for the correct label (e.g., pieces of watermelon). Students can earn partial credit (0.5) points for some labeling responses (e.g., pieces). The maximum score on the 16-item test is 36 (α = .81).

To index broad fraction understanding, we administered 18 released items from 1990-2009 National Assessment of Educational Progress (NAEP): easy, medium, or hard fraction items from the fourth-grade assessment and easy items from the eighth-grade assessment. Items tap part-whole and equal-sharing conceptualization of fractions, which received greater emphases in the control than in intervention conditions. Thus, the NAEP serves as the study's distal outcome. Testers read each problem aloud, rereading up to one time upon student request. Response formats are selecting answer from four choices, writing an answer, shading a portion of a fraction, marking a number line, writing a short explanation, and writing a number. The maximum score is 24 (two items have multiple components and earn 3 to 5 points; α = .75).

Experimental Conditions

In the base condition, students received the fractions intervention, *Third-Grade Super Solvers* (Fuchs, Malone, Wang, Fuchs, Abramson, & Krowka, 2015) that included three 35-min sessions per week for 13 weeks delivered to pairs of students. It relied on the following explicit instruction principles. Tutors began new topics with worked examples, while explaining and thinking aloud each step using simple, direct language. They gradually faded worked examples as students practiced applying strategies to solve problems during guided and independent practice. Instruction incorporated efficient solution strategies to support understanding and mastery of concepts and strategies. Students had many opportunities to generate correct responses, and receive immediate feedback. Intervention included systematic, cumulative review.

Super Solvers included four activities: *Problem Quest*, *Fraction Action*, *Math Blast*, and *Power Practice*. *Problem Quest* instruction included multiplication and fraction word problems (compare, change, and splitting problem types). *Fraction Action* focused on comparing, ordering, and placing fractions on a 0-1 number line. *Math Blast* comprised fluency-building activities for comparing fractions and multiplication. *Power Practice* was independent student practice. (For a full description of this intervention program, see Supplemental Materials, including Lesson Activity Schedule and Skill and Sequence. Due to limited space, this publication focuses on the SR condition, which distinguished the two intervention conditions.)

In the SR condition, students received the same fractions intervention along with the embedded SR component. Instructional time across the two intervention conditions was held constant (see below). Tutors implemented the bulk of SR instruction at the start of each lesson, and then provided some additional SR instruction prior to independent practice (i.e., *Power Practice*). Time spent on SR components averaged 4-7 min per lesson. At the start of each lesson, tutors presented and led discussion about a SR topic using scripted SR lessons (i.e., which were studied, not read). Scripted topics addressed key SR concepts such as self-sufficiency (using help cards and other tools only when necessary), partner support (asking for and providing help), goal-setting, taking responsibility for planning one's own learning activities, and tracking one's own progress. Tutors relied on a catalogue of SR terms to guide responses and prompts, including references to "brain power" (e.g., "Your brain gets stronger when you work hard to learn new things"), how you learn when you help your partner, and the importance of mapping your progress, self-regulating, staying on task, and perseverance.

During Weeks 1-2, SR instruction focused on teaching students about “brain power” and how to apply their “brain power.” In Week 3, SR instruction shifted to goal-setting instruction. Students learned how to set goals, track their progress, and change their plans to meet those goals. In Weeks 4-13, SR instruction heavily emphasized student progress-monitoring. Using scripted guided questions and discussion points, tutors assisted students in identifying strengths and weaknesses using students’ scores on curriculum-based measures.

Tutors conducted *Super Challenge* curriculum-based measures (CBMs), which tapped core skills from the fractions intervention, bi-weekly in both conditions. In the SR condition, tutors led discussion about results to help students evaluate their progress and adjust plans to reach their self-set goals. (In the base condition, tutors scored assessments but did not guide reflection on progress or discuss implications for SR behavior.) Every two weeks following the CBM, SR students set goals to beat their highest score, developed and discussed strategies to meet goals, and tracked progress on graphs (see Supplemental File). The graph included a chart grouping the 20 problem types to help students assess progress on skill groups and identify areas of strength and in need of improvement. After identifying skills in need of improvement, SR students selected a *Super Solvers Homework* sheet to practice their selected problem types. *Super Solvers Homework* involved practice sheets organized by problem types. Tutors guided each student to select a *Super Solvers Homework* sheet aligned with the student’s own plan for improvement. (In the base condition, students completed tutor-selected sheets at random during the session.)

Tutor Training and Fidelity

Six tutors were research grant employees. None was a licensed teacher; six had completed a bachelor’s degree; two a master’s degree. Each tutor was responsible for 2-4 groups, distributed across the base and SR conditions. To avoid contamination across conditions, we color coded materials, conducted periodic live observations, and monitored fidelity of implementation audiotapes. Tutors also attended biweekly meetings to receive condition-specific training for upcoming sessions, engage in problem solving, and receive feedback.

To quantify fidelity of implementation, we digitally audio recorded all sessions. Of the 936 tutoring sessions, 20% were randomly sampled to ensure comparable representation of conditions, tutors, and lessons. Research assistants (RAs) listened to recordings while completing a checklist of essential points addressed during each lesson. For the base program component, the mean percentage of points addressed was 91 (*SD* = 10.24) in the base condition; 96 (*SD* =5.33) in the SR condition. For the SR component, the percentage of points addressed was 95 (*SD* = 12.86). Two RAs independently re-coded 20% of sessions, with 96% agreement for the base component and 94% for the SR component. A within-tutor paired *t*-test indicated higher fidelity of the base component for the SR condition than the base condition ($p = 0.04$). Note, however, that fidelity percentage was high in both conditions. Also, the mean difference of five percentage points between conditions for the base program is explained by the difference in a single item. Thus, it does not appear meaningful.

Mathematics Instructional Time for Two Intervention Conditions versus Control

The 19 classroom teachers completed a survey on their instructional time and practices. They reported that math instruction occurred in 80-90-min math block five days per week. Eight (35% of) control students received the school’s supplemental math intervention (mean 97.50 min [*SD* = 46.52] per week). The study’s intervention typically occurred during part of classroom math instruction or the school’s intervention period, and 11 (24% of) intervention students also received supplemental math intervention from their school (mean 97.27 min [*SD* = 44.96] per week). On average, students in the intervention and control conditions received the same amount of mathematics instruction (including minutes of classroom and supplemental intervention provided by the study or school).

Distinctions between Classroom and Intervention Fraction Instruction

As part of the survey, teachers also provided information about the schools' fraction instruction. Two of 19 teachers reported it was based largely on the district's mathematics program (enVisionMATH; Scott Foresman-Addison Wesley, 2011), 17 on state standards, and two a combination of both. See Table 2 for teacher questionnaire responses about the control group's fraction instruction, as contrasted to *Super Solvers* fraction intervention. Overall, there were three major differences between the schools' and *Solver Solvers* fractions instruction. First, schools focused on part-whole understanding of fractions; *Super Solvers* on magnitude understanding. Next, school instruction did not restrict the range of fractions taught; *Super Solvers* limited the range of denominators (1, 2, 3, 4, 5, 6, 8, 10, 12). Last, for word problems, school instruction focused more on operational procedures and key words, whereas *Super Solvers* focused on more identifying and strategic solving of problem types.

Procedure

We conducted whole-class testing in one 45-min session late-August to early-September, in which we administered the screening measures *WRAT-4 Arithmetic* and *Minuends to 18*, and the pretest measure NAEP. Students who met at least one entry criterion (WRAT-4 or *Minuends to 18*) were tested individually in one 60-min session in mid-September to early-October on pretest *Fraction Number Line 0-1*, and the screeners *WASI-Vocabulary* and *WASI-Matrix Reasoning*. Students not excluded due to the *WASI* or other exclusion criteria were further tested in two small-group sessions, each lasting 45 min, on *Multiplication*, *Comparing Fractions*, and

Fraction Word Problems-Pretest. From late October to early February, intervention occurred. In early March, we posttested students on NAEP and Ordering Fractions in a whole-class testing session and on *Fractions Word Problems* and *Multiplication* in two small-group sessions. We administered *Fraction Number Line 0-1* individually. At pretest, we administered *Comparing Fractions*; at posttest, *Ordering Fractions* instead of *Comparing Fractions*. Testers were graduate RAs who received training and passed fidelity checks on testing procedures prior to administering tests. Two independent RAs scored and entered data for each test. RAs discussed and resolved all scoring discrepancies. All testing sessions were audiotaped; 20% of tapes were randomly selected, stratifying by tester, for accuracy checks by an independent scorer. Agreement on test administration accuracy was 98%. Testers were blind to conditions when administering and scoring tests.

Data Analysis

Table 3 shows the adjusted posttest means by intervention condition. Multilevel models including random intercepts for schools and classrooms were estimated to obtain point estimates, *p*-values, and effect sizes (ESs) associated with intervention conditions (vs. control) on NAEP, Ordering, Word Problems, Multiplication, and Number Line. (We did not compare base condition vs. SR condition, as this was not a planned contrast and would require correcting the alpha value for multiple comparisons.) Random intercepts for intervention pairs were not included because residual design effects were less than 2 across all outcomes. The design effect equation incorporates information about the intra-class correlation coefficient as well as the average cluster size. Levels with design effects less than 2 can be safely excluded without overly biasing the results (Muthén & Satorra, 1995). We compared each intervention condition separately to control by entering two dummy codes, Base (1 = base program, 0 = otherwise) and

SR (1 = SR condition, 0 = otherwise). The pretest score corresponding to each dependent variable was also entered to reduce residual error. We used the *Comparing Fractions* test score as the covariate in the Ordering model. All models were first run with pretest score (centered) as a moderator of intervention effects to check the robustness of findings. The pretest score was centered because centering a continuous moderator improves the meaningfulness of the intercept term, increases the accuracy of main effect estimates, and reduces multicollinearity. Where a significant interaction effect occurred, both interaction terms were retained in the model to correctly estimate intercept; where both interaction terms were simultaneously not significant, both were dropped.

The small number of clusters in the sample necessitated restricted maximum likelihood estimation with a Kenward-Roger adjustment (McNeish & Stapleton, 2016). Where model estimates of level-1 variances differed markedly by condition (by a factor of 2 or more), separate level-1 variance components were estimated for each condition to account for heteroscedasticity across conditions (Roberts & Roberts, 2005). Level-1 residuals were checked for normality and homoscedasticity. Across all outcomes, no problematic violations were noted except for the word-problem outcome. The normality assumption was violated in that the distribution was skewed with an abundance of scores at the low end of the range of scores. Note that the fraction word-problem measure was the most difficult outcome in the study. We used the square-root transformation for the word-problem outcome and re-ran the model. Afterwards, residuals met assumptions.

Results

Across the outcomes, we detected one significant interaction effect, between the pretest word-problem score X the effects of base program versus control (Coeff. = 0.42, *SE* = 0.17, *p* =

.021) on the word-problem outcome. Figure 1 reveals that the base program was more effective than control for students who began the study with higher as compared to lower word-problem scores. Typically, significant interactions are followed up by calculating a region of significance. In this case where the outcome was transformed, the specific value associated with a region of significance may not generalize to other contexts, even though the direction of the interaction shown in Figure 1 likely holds. Results for the fixed effects, along with ESs (Hedges *g*) for intervention main effects (those without associated significant interaction terms), are in Table 4. ESs were calculated from model coefficients (What Works Clearinghouse, 2013). The base condition produced significantly higher scores than control on Ordering (ES = 1.29) but not NAEP, Multiplication, or Number Line. The SR condition produced significantly higher scores than control on NAEP (ES = 0.56), Ordering (ES = 1.76), Word Problems (ES for the square-root-transformed outcome = 0.73), and Multiplication (ES = 0.78), but not Number Line. Though we did not contrast the two intervention conditions as already mentioned (base vs. SR), we provide ESs in Table 3 contrasting these two conditions.

Discussion

We investigated the effects of an intervention designed to improve at-risk third graders' fractions understanding and word-problem performance. We adopted the approach used in prior research at fourth grade, demonstrating the efficacy of focusing on magnitude understanding (Fuchs et al., 2013, 2014, 2016a, 2016b). While examining effects of the base fraction magnitude program, we explored the added value of a SR component, operationalized in terms of a growth mindset with active goal setting and perseverance through the challenging tasks, as required for this domain for third-grade students with histories of poor learning with whole numbers

(Namkung et al., 2018). We hypothesized that embedding a SR component into a demanding mathematics intervention may hold added value for this population of learners.

Previous studies provide the basis for anticipating SR instruction's added value when embedded within mathematics instruction (Fuchs et al., 1997; Fuchs, Fuchs, et al., 2003). In the present study, we extended this line of work to fraction magnitude understanding, a domain potentially well suited for SR due to demands on a positive mindset that supports perseverance and metacognition through challenging tasks (De Corte et al., 2000). Further, whereas the previous Fuchs et al. studies delivered intervention in a whole-class peer-mediated format, we explored SR's value within a small-group intervention framework.

The present study's results indicated an advantage for the embedded SR condition. Most notably, the embedded condition outperformed the control group on the distal outcome measure, a set of 18 fraction items from the NAEP, whereas the base program alone did not (effect size [ES] = 0.56 vs. 0.10). The SR condition's added value over the control is particularly noteworthy because NAEP, which measures generalized fraction knowledge and covers forms of fraction knowledge and response formats not explicitly taught during intervention, is equally distal for the intervention and control conditions.

Not surprisingly, therefore, the instructional guidance unique to the SR condition, which addressed perseverance in the face of learning challenges and goal-directed behavior, also supported higher performance on proximal measures of fraction understanding. For example, the same pattern of results on the distal NAEP measure recurred on multiplication. Intervention allocated only minimal focus on multiplication: 5 min in the first nine lessons; then fluency practice every other week for two min and one or two multiplication problems in each lesson's independent practice. Yet, we found a significant effect between the SR program and control group, whereas base program students performed comparably to the control group (ES = 0.78 vs. 0.26). What distinguished the two intervention conditions was that SR students had repeated opportunities, via their CBM graphs and skills charts, to assess whether multiplication was in need of improvement. SR encouraged students who performed poorly to focus on this skill and work hard on this skill during and beyond intervention to meet their *Super Challenge* goals. In fact, it was only on the ordering task, on which intervention focused heavily, that the base intervention group (without the SR component) outperformed the control group. Note, however, that the ES was 1.29 for the base program but 1.76 for the SR condition, again suggesting the SR's added value.

At the same time, results on the word-problem outcome perhaps provide the most interesting insights into the value of incorporating SR training into intervention for at-risk learners. As with fractions, difficulty with word problems is prevalent, even when other forms of mathematical cognition are intact (Fuchs et al., 2008). Thus, the challenge associated with fractions presented in the context of word problems is multiply determined. Therefore, as expected, the embedded SR condition outperformed the control group on the word-problem solving outcome, with a large ES of 0.73.

Moreover, although a main effect also favored the base program over the control group, this base condition effect over the control group was moderated and superseded by a significant interaction. As visualized in Figure 1, the moderation effect reveals that base condition students with stronger pretest fraction word-problem performance responded more adequately than did base condition students with weaker pretest fraction word-problem performance. Of note, this moderation effect was not significant for the contrast between the embedded intervention versus control. Thus, the SR component appears to have “protected” students with low pretest skill from inadequate responsiveness to fractions word-problem intervention.

This may seem surprising given that word problems were not included on CBMs. So we note that SR students were regularly encouraged to apply their SR strategies on fraction word problems, for example by completing word-problem strategies without the use of help cards and by persevering through difficult problems. Also, solving for unknown values within nonstandard equations, a skill needed to solve the most difficult word-problem type addressed in *Super Solvers* (the change word-problem type), was included on CBMs. Similarly, comparing fractions, which was required for compare word problems, was also featured on the CBMs. Thus, SR students had multiple opportunities to identify weaknesses in knowledge and skills foundational for fraction word problems, thereby developing an advantage over base students on the word-problem outcome.

Before closing, we note four important study limitations, which should be considered when interpreting study findings. First, our study was limited in size, which precluded direct contrasts between the two intervention conditions. Future research with larger samples is needed to permit such comparisons. Second, this study’s design does not provide insights into which SR components were most effective in increasing student performance. Future studies designed to test this question would be of value. Third, our study did not collect data on teachers’ use of SR instruction within classrooms. Future studies should incorporate SR activity and instruction questions within teacher surveys. Fourth, due to resource constraints, we were unable to plan for delayed posttest or fadeout of intervention effects.

We also note that neither intervention condition outperformed the control group on the fraction number line task, and ESs, although favoring each intervention condition over control, were small (0.15 for base; 0.35 for SR). This runs contrary to findings from previous randomized control trials conducted at fourth grade, where large effects favored fractions magnitude intervention over control students (Fuchs et al., 2013, 2014, 2016a, 2016b). The present study's lack of significant effects at third grade may be due to the demands of the computer number line task. That is, *Super Solvers* number-line activities were designed to compensate for the working memory limitations at-risk students frequently demonstrate (Fuchs, Fuchs, Seethaler, & Barnes, in press) by having students use pencils to apply benchmarking strategies to paper number lines. By contrast, the computerized pre- and posttest required mental benchmarking, which may have overburdened third-graders' working memory in ways that students a year older can overcome.

With these caveats in mind, our main conclusion is that, within the context of a highly explicit and structured fractions intervention focused largely on magnitude understanding, embedding a strong focus on positive mindset, goal-setting, evaluation, and planning appears important for improving at-risk third graders' fractions performance. Even so, as results on NAEP, multiplication, ordering, and word problems suggest, future work should examine ways for enhancing the base intervention program's strength so it reliably promotes more adequate learning, even without the SR component. It is also possible that stronger outcomes are achieved when a *strengthened* base program is used with embedded SR instruction.

Therefore, subsequent studies should test the effects of stronger and improved versions of the base program. The present study provides the following insights for such improvement: (a) adding opportunities for direct practice across all types of fraction problems, (b) slowing the pacing of fraction instruction to allow third-grade students time to achieve firm mastery, and (c) reducing the number of problem subtypes taught to teaching for depth rather than breadth. With these improvements, the base program may exert a sufficiently strong effect on students' fractions learning to eliminate the need for embedded SR instruction. Future studies should test this possibility. Further, although the present study's ESs contrasting the two intervention conditions, which ranged from 0.23 to 0.45, suggest an advantage for SR over the base condition, an adequately powered randomized controlled trial is necessary to explicitly test this possibility.

Conclusions and Implications for Practice

In these ways, the present study provides support for an explicit small-group intervention approach focused on magnitude understanding of fractions, with schema-based instruction on fraction word problems for improving the fraction outcomes of third graders with histories of mathematics learning difficulties. This conclusion, which corroborates studies at fourth grade (e.g., Fuchs et al., 2013, 2014, 2016a, 2016b) does, however, require an important qualification: At third grade, this approach needs to embed SR instruction for building students' growth mindset, goal-setting, planning, and perseverance.

Our focus on fractions in the at-risk population is innovative. It reflects the need to address the general education state standards within intervention, even as interventions remediate students' foundational skills. We achieved this dual focus by restricting the range of fractions used to teach challenging content, even as we incorporated instruction to address students' limitations in whole-number knowledge and operations. The goal was to better prepare at-risk students to keep pace with classroom peers and return from intervention ready to succeed in the general education programs.

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Table 1

Student Demographics and Descriptive Data at Pretest by Tutoring Condition

	Base Condition (*n* = 23)		SR Condition (*n* = 23)		Control Condition (*n* = 23)	
Variable	*n*	%	*n*	%	*N*	%
Males	9	39.1	11	47.8	13	56.5
Race						
African American	12	52.2	14	60.9	10	43.5
White	2	8.7	2	8.7	6	26.1
Hispanic	7	30.4	7	30.4	5	21.7
Asian	2	8.7	--	--	--	--
Kurdish	--	--	--	--	1	4.3
Other	--	--	--	--	1	4.3
Economically disadvantaged	22	95.7	21	91.3	22	95.7
School-identified disability						
Learning disability and behavior disorder	2	8.7	--	--	--	--
Speech/language delay	1	4.3	--	--	1	4.3
Speech/language delay and developmental disorder	--	--	--	--	1	4.3
English-language learner	5	21.7	8	34.8	--	--
Screening Measure	*M*	*SD*	*M*	*SD*	*M*	*SD*
Minuends to 18 (Subtraction)	2.00	2.78	2.48	3.10	1.78	2.26
WRAT4 Arithmetic	20.87	2.32	21.48	1.78	21.04	1.94
WASI Matrix Reasoning	11.17	3.94	9.83	3.89	9.43	3.79
WASI Vocabulary	19.61	5.03	17.57	3.00	18.48	4.27

Table 2

Fraction Instruction: Classroom vs. Intervention

Domain	Dimension	Method	Classroom (%)	Intervention (%)
Fractions	Fraction Interpretation	Part-Whole	74.66	25.00
		Measurement	25.34	75.00
	Fraction Representation	Fraction Tiles	14.62	20.00
		Fraction Circles	15.49	10.00
		Pictures with Shaded Regions	31.84	10.00
		Blocks	10.08	0.00
		Number Lines	25.34	60.00
		Other	2.63	0.00
	Fraction Magnitude	Number Lines	24.00	20.00
		Drawing Pictures	24.61	0.00
		Referencing Manipulatives	10.76	10.00
		Benchmark Fractions	8.42	25.00
		Understanding Numerator and Denominator	11.88	25.00
		Finding Common Denominator	5.26	20.00
		Cross-multiplying	13.73	0.00
		Other	1.05	0.00
Multiplication		Manipulatives	11.17	0.00
		Graph Paper	1.58	15.00
		Drawing	17.80	0.00
		Skip Counting	18.60	40.00
		Decomposition	11.39	0.00
		Memorization	15.72	15.00
		Trick	9.10	20.00
		Fact Families	13.59	10.00
		Other	1.05	0.00
Algebra	Equations	Standard Equation (e.g., 3 + __ = 5)	94.73	--
		Double Operation (e.g., 3 + __ = 4 + 4)	68.42	--
		Double Operation (e.g., 3 + 2 = __ - 4)	47.37	--
		Standard Equation (e.g., __ - 2 = 5)	84.20	--
Word Problems		Comprehension	15.75	70.00
		Operational Procedures	44.83	20.00
		Communication	17.42	10.00
		Keywords	21.47	0.00
		Other	0.53	0.00

Note. % refers to percentage of emphasis reported by teachers versus as reflected in *Super Solvers*.

Table 3

Adjusted Posttest Means, Standard Deviations, and Effect Sizes Between Base and SR Conditions

	Base Condition (*n* = 23)		SR Condition (*n* = 23)			Control Condition (*n* = 23)	
	M	*SD*	*M*	*SD*	ES	*M*	*SD*
0-1 Number Line	0.25	0.10	0.23	0.07	0.23	0.26	0.10
Word Problems	7.35	5.00	7.18	4.12	--	3.76	2.54
Multiplication	17.69	7.88	20.62	5.55	0.43	16.60	7.21
Ordering	5.61	4.05	6.52	3.55	0.24	1.57	1.67
NAEP	8.91	3.45	10.61	4.18	0.45	8.50	3.73

Note. Each *M* calculated as adjusted posttest with pretest as a covariate. Effect size is reported as Hedges *g*. 0-1 Number Line is Number Line Estimation 0-1 (Hamlett, Schumacher, & Fuchs, 2011), where lower scores represent stronger performance. Word Problems, Multiplication, and Ordering are from the Fractions Assessment Battery-Revised (Schumacher et al., 2015). NAEP is released fraction items from the National Assessment of Educational Progress.

Table 4

Fixed Effects from Multilevel Models by Outcome

	Intercept			Pretest Score			Base Condition v. Control				SR Condition v. Control			
Outcome	Coeff.	*SE*	*p*	Coeff.	*SE*	*p*	Coeff.	*SE*	*p*	*ES*	Coeff.	*SE*	*p*	*ES*
NAEP	5.31	1.00	<.001	0.80	0.15	<.001	0.37	0.87	.675	0.10	2.24	0.86	.011	0.56
Ordering	1.05	1.75	.554	0.06	0.19	.742	4.08	0.91	<.001	1.29	4.96	0.83	<.001	1.76
SQRT WP (Tot Combined)	1.82	0.16	<.001	0.07	0.10	.468	0.91	0.21	<.001	—	0.54	0.20	.007	0.73
Multiplication	12.51	1.78	<.001	0.47	0.11	<.001	2.01	1.47	.178	0.26	4.26	1.50	.006	0.78
0-1 Fraction NL[a]	0.17	0.05	.001	0.27	0.12	.030	-0.02	0.03	.569	-0.15	-0.03	0.03	.209	-0.35

Note. Pretest score = pretest corresponding to each outcome. SR is self-regulation. ES is Hedges *g*. NAEP is released fraction items from the National Assessment of Educational Progress. Ordering, Word Problems, and Multiplication are from the Fractions Assessment Battery-Revised (Schumacher et al., 2015). SQRT is square-root. NL is Number Line Estimation 0-1 (Hamlett, Schumacher, & Fuchs, 2011), where lower scores represent stronger performance.

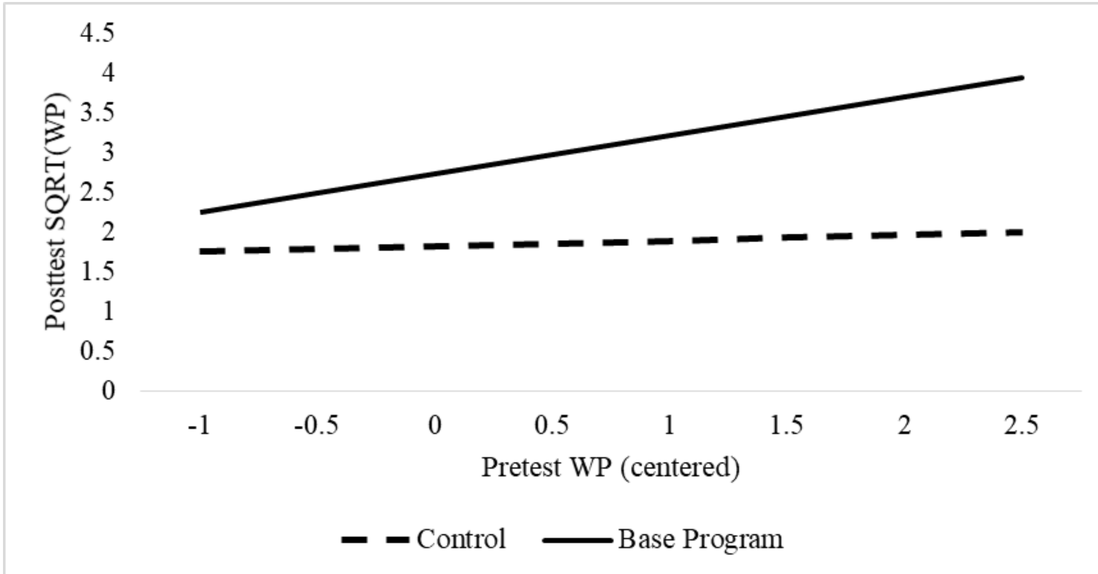

Figure 1. Graphical display of significant interaction between pretest word-problem (WP) score and regular treatment (base condition). The effect of the base program versus control is stronger at higher pretest WP scores.