Middle School Pre-Service Teachers' Mathematics Content Knowledge and Mathematical Pedagogy Content Knowledge: Assessing and Relating

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This paper examines the mathematics content knowledge of graduate entry middle school mathematics pre-service teachers at the beginning (n=105; 80%) and end of a mathematics curriculum course. It was found that mathematics content knowledge at the commencement of the course was not strong. An intervention was designed to take account of content while preparing teachers to teach the material with specific pedagogical models. On a test of similar difficulty level, the marks approximately doubled, but in areas of upper secondary mathematics, significant deficits remained. Content knowledge at the end of the course was highly predictive of measures of mathematics pedagogical knowledge including how to diagnose student errors and plan learning support. The finding have implications for teacher preparation at the study institution and potentially more broadly.

It is typical in Australia to have two pathways to mathematics teaching: an undergraduate pathway and a graduate pathway. The first pathway is via an undergraduate degree in which prospective teachers are expected to complete six university mathematics courses. The second pathway is via a graduate entry and this is the pathway that is the subject of this study. While there is some variation across the nation, in the study state the criteria for entry to middle years mathematics teacher education programs are based on the successful completion of at least four university-based subjects rich in mathematics. A number of authors (e.g., Burghes & Geach, 2011; Tatto et al, 2008) have cautioned against the use of proxy measures to evaluate the content knowledge levels of pre-service teachers. With this context in mind the paper examines middle school pre-service teachers' mathematical content knowledge (MCK) at the start and end of a mathematics curriculum intervention and relates this to a measure of pedagogical content knowledge (PCK) at the end of the intervention.

Literature review

It has been convincingly argued that high school mathematics teachers with a strong knowledge of the mathematics they are teaching are more likely to be effective in developing this knowledge in their classrooms (Australian Academy of Science, 2015; Cai, Mok, Reddy, & Stacey, 2016; Krainer, Hsieh, Peck, & Tatto, 2015). The Teacher Education and Development Study in Mathematics (TEDS-M) (Tatto et al., 2008, p. 19) summed up this argument: "Knowledge of content to be taught is a crucial factor in influencing the quality of teaching." The effects of depth of content knowledge and its relationship to effective teaching have been well researched, particularly since Shulman (1987) defined mathematics content knowledge and pedagogical content knowledge. Many scholars have subsequently refined understandings of the relationship between content knowledge and effective teaching of mathematics (e.g., Beswick & Goos, 2012; Chapman, 2015). So intertwined are these key factors that Beswick and Goos noted the "interconnectedness of MCK and PCK and the difficulty in distinguishing between them" (p. 72). In this study, the term mathematics pedagogical content knowledge (MPCK), which includes mathematics curriculum knowledge, knowledge of planning for mathematics teaching, and enacting mathematics for teaching, as defined by Tatto et al. (2008, p. 39), is used.

Australian Teaching Standards

Given the research affirming the importance of teachers' content knowledge, it is not surprising that standards for graduating mathematics teachers stipulate that prospective teachers know their content (e.g., Australian Institute for Teaching and School Leadership [AITSL], 2014). The Australian curriculum (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2012) outlines the content for Australian school children. The number and algebra strand of Year 7 includes fraction operations, ratio, and solving simple linear equations. By Year 10, more advanced students are expected to be fluent with surds, exponential and logarithmic expressions, and non-linear algebra including working with parabolas, hyperbolas, circles, and exponential functions (ACARA, 2012). Clearly, there is a great deal of mathematics between these fraction operations and logarithmic expressions; yet, a number of authors have expressed concerns about the content levels of Australian high school teachers, and therefore their ability to teach to these standards (e.g., Beswick, Callingham, & Watson, 2012; Hine, 2015).

Mathematics Curriculum Course Assessment Protocols

Henderson and Rodrigues (2008), Hine (2015), and Kotzee (2012) claim that it has become a tradition in teacher education courses at Western universities to focus on big-picture curriculum issues. Of pre-service teachers in the UK, Burghes and Geach (2011) noted a lack of relevance between theoretical studies undertaken (such as views on theories of learning assessed via essays) and school-based work. The focus on more general curriculum knowledge can be justified if it is assumed that pre-service teachers enter middle school mathematics curriculum units with a reasonable depth of mathematical knowledge. A review of assessment protocols reported by universities indicated that in Australia, almost without exception, middle years' mathematics curriculum courses are assessed via essays and the production of mathematics teaching resources. Producing teaching resources can be directly linked to teaching the subject of the resource, but essays and reflections tend to be more abstractly focused and related to generic considerations rather than the detailed pedagogy associated with specific mathematics concepts. One Australian anomaly to this general pattern of essays, report writing, and resource construction is the study institution, which had a 60%, 3-hour closed-book examination and a 40% case study research assignment involving testing, planning an intervention, and implementing the intervention.

Research Questions

With the background above in mind the research questions for this study are as follows:

1) What was the starting content knowledge (MCK) and proficiency in basic middle school computation as measured by a written test (Author, 2017, scale).

2) What relationship exists between MCK measured at the commencement and end of the course and key components of mathematical pedagogical content knowledge (MPCK) measured at the end of the course?

Method

Overview of Methodology

The method is correlational in so much as the associations between MCK and measures of MPCK are examined. The software package SPSS was used to calculate descriptive statistics and calculate correlations that enable the reader to assess the relationships between the tests that form the basis of the data presented in this paper. The participants were most of a cohort

(n=105/131 [80%] pre-test and n=128/131 post-test) of an intake of graduate diploma in mathematics education at a reputable Australian university.

The Intervention

Three Australian professional standards for teachers (AITSL, 2014) were the target of the intervention. The first was "Know content and how to teach it" (p. 3). This was interpreted that upon graduation most of the pre-service teachers would have a reasonable grasp of a significant spread of middle years' number and algebra content. Further, they would be able to articulate this knowledge as well as a range of specific pedagogies to provide learning support, in this way demonstrating aspects of the third standard, "Plan for and implement effective teaching and learning." (AITSL, 2014, p. 3). Finally, in the assignment (40%) and for aspects of the final written test (60%), pre-service teachers were required to assess student learning and provide feedback, thus demonstrating some knowledge of the fifth standard, "Assess, provide feedback and report on student learning" (AITSL, 2014, p. 3). The course was run over 7 weeks with 28 contact hours. Learning support in the form of a 500-page text (Norton, 2014a) was supplied (given in pdf format) for the teaching of concepts from counting to simultaneous equations. A second text (2014b) covering the specific pedagogy for teaching quadratic equations was also supplied. These texts were auxiliary to lecture-captured lectures (seven) and workshops (seven) and were supported by optional video production, of which multiple copies were placed in the libraries (Norton, 2014a, b). The intent was to deepen schematic knowledge of middle school mathematics while learning specific pedagogy including error analysis.

Testing Instruments

The data reported come from two written tests where calculators were not permitted and 1 hour was allowed for completion. The intake (pre-test) test was comprised of 31 questions based on the number and algebra content the pre-service teachers were preparing to teach. Item descriptions in the results section illustrate the content validity of this assessment in that each question can readily be mapped to the current middle school mathematics curriculum. The content ranged from whole-number operations, which is primary mathematics, to quadratic equation conventions, which is Year 10 advanced content. With few exceptions, this test is simply a test of fluency with middle school content. In this regard the test items are similar to those used by Tatto et al. (2008) to assess the knowing and applying cognitive domains of mathematics. Burghes (2007) used similar items. The test was conducted during the first tutorial of the course. The Cronbach's alpha score of the pre-test MCK was 0.903 indicating very high internal consistency. The test described above is unlike the Literacy and Numeracy Test for Initial Teacher Education Students (ACER, 2017) which for most questions allows calculators and has content demands at about Year 7 level and lower.

The post-test represented the final assessment of the course; it was conducted in the 8th week of the course and had a duration of 3 hours, since MPCK was tested in addition to MCK. This test had two sections. Part A had a virtual replication of the pre-test, except that for each question a scenario was presented and the pre-service teacher was asked to identify student error and then provide a correct solution. For example, the pre-test asked for a large subtraction in context, and the replication of this was simply minor modifications in context and numbers used. The mathematics underpinning each was essentially identical.

Question 20 of the post-test assessed pre-service teachers' diagnostic capability with regard to linear algebra conventions. The mathematics with respect to context and procedures required was virtually the same as Question 21 of the pre-test. The responses below contain errors that the pre-service teachers had to describe, provide the correct solutions for, and in some instances suggest and justify grades and provide learning support. Figure 1 illustrates one
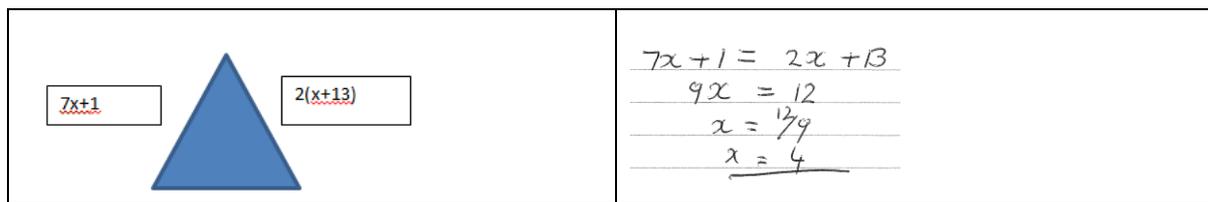
such example of student error.

Figure 1. Example of algebra convention post-test assessment of MPCK (<u>describe student thinking</u>) and MCK (<u>present the correct solution</u>).

For all questions in the post-test, pre-service teachers were asked to <u>describe children's errors and provide the correct solution</u>. Analysing or evaluating students' solutions and diagnosing students' responses falls into the MPCK defined by Tatto et al. (2008). The Cronbach's Alpha statistic for the post-test MCK 27 items was 0.881 indicating strong reliability. The scale measuring diagnostic capability was based on the same 27 items on the post-test and had a Cronbach's Alpha of 0.839. The method of testing enabled both the content (MCK) and an ability to diagnose and describe children's errors, an aspect of MPCK, to be documented over a range of middle years' concepts.

In the second section of the post-test there were additional dimensions including assessing pre-service teachers' planning to provide learning support and justification of grading of children's written mathematical work. There were four questions of this nature related to teaching whole-number division, fraction subtraction, quadratic problem solving, and quadratic conventions. There was one generic MPCK question asking pre-service teachers to describe factors that challenge children's learning of mathematics and how the resultant incidence of misconceptions can be reduced through specific learning support. By way of example with respect to assessing specific MPCK, Question 3, related to assessing and providing learning support in the topic of quadratics, illustrates the question format. "*The base of a right angle triangle is 4 cm longer than its height, and the hypotenuse is 4 cm longer than the base. Find the height*." The Year 10 student provided the response shown below in Figure 2:
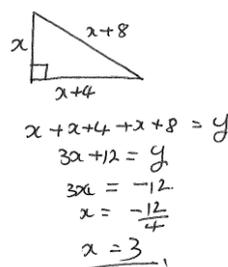

Figure 2. Year 10 student sample solution related to working with quadratics requiring pre-service teachers to <u>diagnose children's misconceptions</u>, grade the response, and provide learning support.

For the question illustrated in Figure 3, the pre-service teachers were asked to grade the work (out of 5) and provide a justification for the grade including identifying any procedural or conceptual errors in the solution to justify the grade. The second part of this question required the pre-service teacher to provide a step-by-step solution using "completing the square" method. Questions such as this enable scoring on three aspect dimensions: provision of the correct solution (an aspect of MCK), diagnostic capability (an aspect of MPCK), and provision of learning support (an aspect of MPCK). This last aspect of the test aligns with "Enacting Mathematics for Teaching and Learning," in particular, "Explaining or representing mathematical concepts or procedures" (Tatto et al., 2008, p. 39). It also demonstrates aspects

of the AITSL (2014) standard "know content and how to teach" (p. 3). The scale designed to assess pre-service teachers' capacity to provide specific learning support had a Cronbach's Alpha score of 0.608.

Analysis

SPSS was used to calculate descriptive data (means, minimum, maximum, standard deviations) and correlational data sufficient to answer the research questions.

Results and Analysis

Descriptive summary data on the tests assist in answering the research questions. Detailed success rates on particular items add meaning to these statistics, presented in Table 1.

Table 1.

Descriptive Statistics Related to the Test Instruments

Test	Minimum score	Maximum score	Mean score	Standard deviation
Pre-test content /31 (n=105)	1	29	10.34 (33%)	6.641
Post-test content /53 (n=128)	9	53	35.62 (67%)	10.82
Post-test diagnosis/27 (n=128)	9	27	21.61 (80%)	4.28
Post-test learning support (n=128) /25	7	25	15.15 (60%)	5.70

The pre-service test data presented in Table 1 indicate that while there is a considerable spread and some students have attained nearly perfect scores on the pre-test, the means are not flattering. Table 2 documents MCK in middle school content areas assessed. The measured MCK doubled subsequent to the course intervention.

Table 2.

Summaries of Average Success on Different Content Domains for the Pre-test (n=105)

Content domain/total score	Minimum score	Maximum score	Mean score	Standard deviation
Whole number computation /4	1	4	2.22 (55%)	1.194
Fraction computation /6	0	6	2.99(59%)	1.975
Index and logarithm conventions /9	0	9	2.29(25%)	2.072
Linear algebra conventions/6	0	6	1.82(30%)	1.692
Quadratic conventions /6	0	6	0.99(16.5%)	1.417

Data in Table 2 indicate that abstractness resulted in greater challenges. Specific results give the reader added insight with respect to the nature of the pre-test. Whole-number computation included subtracting 2,147 from 50,000 (85% success rate; Year 6 Standard; ACARA, 2012) and dividing 18,354 by 23 (40% success rate; Year 6 standard). The best done fraction problem necessitated the addition of mixed numbers ($3\frac{5}{7} + 2\frac{4}{6}$.: 58% success rate; Year 7) and the most difficult computation was to find the area of a silicon chip "*0.2mm wide and 0.3mm long*" (39% success rate-Year 7 standard). Question 19 involved problem-solving by repeated division by two; "*Say I had 64 biscuits and ate half of the number/amount each time. How many/much biscuits would I have on the 8th feed?*" had a success rate of 66% (Year 9 standard). The most difficult index/log question was "*Solve for x in: $4^x = 8$*" (success rate 7%; Year 10A standard). The most successfully done linear algebra question was to solve for "n" in *5n+2=9n-26* (44% success rate; Year 8 standard) and the most difficult was related to simultaneous equations: "*There are 10 more men than women at a party. If one more woman joined the party, there would be twice as many men as women. How many men and how many women at the party?*" (success rate 19%; Year 10 standard). The most successfully completed quadratic question was "*factorise x^2-x-12*" (28% success rate; Year 10 standard) and the most challenging question was to state the equation of a graphed quadratic where the roots, turning point, and y intercept were clearly identified (6% success rate; Year 10A standard).

Table 3 indicates the correlations between the four scales; this helps to answer the second research question.

Table 3.

Correlation Coefficients between Tests of Content Knowledge, Diagnostic Capability, and Descriptions of Provisions of Learning Support

Test	Pre-test content	Post-test content	Post-test diagnosis	Post-test learning support
Pre-test content	1	.791	.709	.640
Post-test content	.791	1	.877	.694
Post-test diagnosis	.709	.877	1	.646
Post-test learning support	.640	.694	.646	1

Each correlation was significant at 0.01 level (2 –tailed).

The correlation coefficients between pre- and post-test MCK was reasonably high at .791. This means that about 63% in variance of post-test content is predicted by pre-test content scores. Unsurprisingly, post-intervention MCK was more strongly correlated to diagnostic capability than demonstrated content at the commencement of the course, having a correlation coefficient of .877. The correlation between post-test MCK and capacity to describe learning support was relatively strong at .694; this means about 48% of the variance in provision of learning support was explained by the post-test content score. The correlation statistics associated with learning support can be explained by the lower reliability Cronbach's Alpha score for that scale (.608). The limited number of items used to test learning support provision may have been a factor in these weaker relationships.

Discussion

The data in Table 1 on pre-test scores in MCK and the detail of pre-test scores documented in Table 2 effectively answer the first research question. The level of MCK of the enrolling pre-service teachers was varied, but overall very poor. In this regard the data support earlier questioning of the wisdom of using proxy measures such as numbers of courses completed to assume reasonable content levels prior to enrolment (Burghes & Geach, 2011; Tatto et al., 2008). The relatively high correlation between pre- and post-test MCK supports the notion that selecting candidates with strong mathematics into courses is a worthy endeavour; however, as noted above, it is best to avoid the use of proxy methods to assess MCK. Assuming that MCK is central to the enactment of effective teaching, as suggested by a range of authors (e.g., Cai et al., 2016; Krainer et al., 2015; Tatto et al., 2008) the teacher-preparation processes might consider providing learning support and focusing on ensuring pre-service teachers graduate with reasonable levels of MCK for middle school teaching. The content associated with simultaneous equations, index notation, and quadratics was in urgent need of support in the study institution.

The correlations presented in Table 3 suggest that MCK, especially MCK at the end of a content and specific mathematics pedagogy course, is highly predictive of capacity to diagnose student thinking, an important aspect of MPCK (Tatto et al., 2008). In this study MCK was moderately correlated with the measure of provision of learning support, another aspect of MPCK. This reduced predictability is likely explained by the reduced Cronbach's Alpha reliability of the post-test learning support test. The practicalities of testing that constrained the testing of MPCK were manifested in the relatively small number of items in this aspect of the scale. The test was already 3 hours in duration, and one purpose of the test was to assess a relatively broad range of middle years' mathematics with respect to MCK and diagnostic capability. Testing the provision of learning support is a time-consuming process and this was a major factor necessitating a relatively small number of items in this section of the final test. These correlational results of the study add empirical evidence informing the model-building accounting for teacher knowledge.

Conclusions

The findings have implications for the study institution in that a review of the structure of the middle years' mathematics curriculum course, indeed the entire middle years' teacher preparation program, warrants consideration. The first interpretation of the finishing MCK mean of 67% is that it is potentially encouraging. Closer examination reveals that it was easier to teach pre-service teachers whole-number and fraction computation than algebra and logarithms, suggesting these topic areas warrant special attention. What has not been clearly articulated earlier in the paper is that the focus of the intervention was relatively narrow, since the focus of the curriculum course and its subsequent assessment was on number and algebra. This focus was justified by the author because it was considered this aspect of middle school mathematics was critical. Not to address gaps in content in these concept areas could create pressure for the newly graduated teachers to teach themselves the content, as well as how to teach it, during school employment. In addition, at least half the cohort was enrolling in senior mathematics curriculum subjects. For them, middle school content consolidation would seem a worthy pre-course preparation for the teaching of senior calculus and statistics.

The relationship between knowing mathematics and knowing how to teach it is complicated. The manner in which MCK and MPCK was measured and the correlational relationship between the measures adds to the empirical data that informs these models. It is the responsibility of readers in other institutions to consider if the data from this institution have any relevance to their circumstances.

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