The Role of Executive Function and Visual-Spatial Working Memory in the Development of Mathematics Skill

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This paper explores what we can learn from research that early cognitive processes support the development of children's mathematics skills. The role of two cognitive processes in working memory in the development of early mathematics was investigated: executive functions (EF) and visual-spatial (VS) ability. Children's mathematical skills were considered in relation to EF and VS. Recent research suggests that EF skills, which include monitoring and manipulating information in mind (working memory), suppressing distracting information and unwanted responses (inhibition) and flexible thinking (shifting), play a critical role in the development of mathematics performance. The number development also can be explained solely in terms of domain-general processes such as VS.

Most children at primary level face various problems in mathematical skills. An early step should be taken to identify the problem of Mathematics learning and this can help in providing appropriate treatment for children in the early stages. The numeracy and mathematical concepts of Mathematical concepts are an important and clear component of a very close relationship in Mathematics achievement, but there are other cognitive factors that play a significant role in contributing to the development of better Mathematics achievement (Cragg and Gilmore, 2014). Two interrelated cognitive processes in particular are critical for children's mathematics achievement: executive functions (EF) and visual-spatial (VS) skills (Bull and Lee, 2014; Cragg and Gilmore, 2014).

Among the factors that influence the problems of students' mathematical skills are cognitive factors namely executive function skills that control cognitive processes in thinking and behavior (Toll, Van der Ven, Kroesbergen and Van Luit, 2011). Furthermore, cognitive skills in maintaining and manipulating information for working memory systems are at a critical level for students in solving Mathematical problems (Raghubar, Barnes and Hecht, 2010). These cognitive skills are needed to monitor and control cognitive processes throughout complex cognitive tasks in solving problems (Miyake, Friedman, Emerson, Witzki, Howerter and Wager, 2000). Executive function skills include set-shifting ability, working memory and inhibition. These three skills are interconnected with each other and are the key executive functions in designing and solving problems (Garon, Bryson and Smith, 2008; Miyake et al., 2000).

The Role of EF in Mathematics Skill

The role of EF in the development of early numeracy is further investigated in relation to other cognitive variables that are important in this mathematics development. There was also empirical evidence suggesting that EF ability was already involved in learning early numeracy (Gathercole and Pickering, 2000) and children's VS skills also play a unique role in the development of mathematics (Meyer, Salimpoor, Wu, Geary and Menon, 2010).

2018. In Hunter, J., Perger, P., & Darragh, L. (Eds.). Making waves, opening spaces *(Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia)* pp. 424-430. Auckland: MERGA.

The EF is a skill in controlling the attention that involved in solving problems with various strategies, controlling responses, storing information in mind, preventing other distractions, compromising in a situation, reflecting on past experiences and planning in the future (Zelazo, Blair and Willoughby, 2016). EF is very important in the development of mathematics because those children with a weak memory in executive functions have difficulty in mastering mathematical skills, difficult to remember and understand instruction throughout the classroom activities (Raghubar, Barnes and Hecht, 2010).

EF is divided into three sub components. There are working memory, shifting and inhibition skills (Miyake et al., 2000). Working memory (WM) refers to the ability to store and manipulate information simultaneously. According to the influential working memory model proposed by Baddeley (1986, 2000), WM consists of (1) a central executive that coordinates incoming and outgoing information (i.e., an attentional control system), two slave systems; (2) the phonological loop for storing verbal information (verbal short-term memory) and the (3) VS sketchpad responsible for storing respectively VS information on a short-termed basis (VS short-term memory); and (4) an episodic buffer responsible for temporarily storing multimodal information from various sources (Miyake et al., 2000). According to Diamond (2013) WM is involved in storing and retaining information in mind as well as how individuals manipulate and capable of making mental changes to such information. For example working memory is required for individuals to remember information early and can link the information in the future. This skill is very important in academic development especially when performing mental calculations mentally where children can combine new information in solving mathematics problems.

According to Yeniad, Malda, Mesman, van IJzendoorn and Pieper (2013), cognitive flexibility skill (set-shifting ability) is needed in a use of various strategies to solve the mathematical problems. Shifting may be relevant for mathematical achievement, for example when switching between arithmetic operations is required. This skill refers to the individual's ability to change the mental set to something new according to the needs of the situation. The first step in cognitive flexibility (shifting) is to produce a mental representation that refers to the various strategies used in problem solving (Garon, Bryson and Smith, 2008). According to Diamond (2013) this skill is important to change the thinking approach in solving mathematics problem or to translate mathematical sentences according to their own understanding. An example is how children's perception is dealing with situations in solving mathematics problems with various ways and the individual's ability to think more than one way. Inhibition is the ability to eliminate unnecessary actions and disruptive processes (Diamond, 2013; Zelazo, Blair and Willoughby, 2016) and address the high response responses in learning (Zelazo et al., 2016). Inhibition may be relevant for math when results of intermediate solving steps need to be suppressed and used in a next step, in order to arrive at the correct answer.

EF contributes to the achievement of Mathematics (Raghubar et al., 2010; Toll et al., 2011) especially in the knowledge of numbers sense (Blair and Razza, 2007; Geary, 2010). For example, children with low inhibitory control that is one component of EF may be less likely to evaluate and switch mathematical problem-solving strategies when they prove ineffective (Bull and Scerif, 2001). EF seems to be particularly important for word problems, which require students to build and manipulate models of the problems in their heads (Fuchs, Geary, Compton, Fuchs, Hamlett, Seethaler, 2010b). According to Swanson (2006), mathematical problem solving is complex because it involves multiple phases, a mental representation strategy and produces planning when information is stored in memory. Thus, these EF skills are essential to be applied to children throughout the mathematical learning process so that they are easy to do the calculation process by applying the strategies in mathematical problem solving.

The Role of Visual-Spatial in Mathematics Skill

WM is a temporary storage in the working memory model by Baddeley (1986) that has a processing system consisting of central executives, verbal working memory and visual-spatial working memory (VSWM). Verbal working memory stores sound or word structure information while VSWM maintains spatial memory and visual information and manipulates mental images. This working memory system controls the process of storing information while engaging in complex cognitive activities involving aspects of VS and language (Miyake and Shah, 1999). The VSWM is involved the manipulation of spatial information. For example, when visualizing the side of a three-dimensional object, remembering and reproducing a spatial arrangement or organizing numbers on a page to manipulate mathematically (Baddeley, 1996).

VS refers to the individual's ability to imagine an object and ability to create and manipulate the image mentally (Lerner, 2003). This skill should be mastered by students so that they can see and describe something more deeply and can relate the concept of representation in solving the given problem. Arithmetic problem solving requires VS skill to translate into a particular representational form and it requires a strategy (Imbo and LeFevre, 2010). According to Metcalfe, Ashkenazi, Rosenberg-Lee and Menon (2013) VS skill is the best contributor to mathematical problem solving. It means that when the individuals have high skill in VS, they might also be able to solve the mathematics problems very well. According to van Garderen (2006), the VS ability leads to more abstract thinking and can be used by students to solve mathematics problems more easily. The student's understanding in mathematics is more clearly by develop a visual representation of diagrammatic or symbolic representations that can represent situations in mathematical problems. Mastery in these skills can indirectly motivate students to design strategies for mathematical solutions. When associating a number to any quantity, VS skills are needed for visual representation in mathematical problem solving. Hence, the number patterns will be easier to remember when the VS skills are developed (Hoon, 2002; Krajewski and Schneider, 2009).

EFs and VS Skills in Mathematics Learning

EF and VS skills are cognitive abilities that are highly related (Miyake, Friedman, Rettinger, Shah and Hegarty, 2001). VS tasks require a complex multistep of mental manipulations that considerably engage EF to encode and analyze the problem by manipulating spatial information, store VS information temporarily, plan a strategy for solving the problem, monitor performance, shift attention (cognitive flexibility) and adapt the strategies during performance (Miyake et al., 2001). According to work with adults, experts posit that EF is needed to a greater extent in VS tasks than in phonological tasks, which may be more easily automated (Miyake et al., 2001).

EFs and VS skills are specific associations emerging for subdomains of mathematics, such as problem solving (Cragg and Gilmore, 2014). Children use EF to set goals and arrange their behaviors in a wide range of tasks, for example, to recall and apply strategies to answer math questions successfully, to switch between operations and notations, to store and retrieve the necessary parts to solve a complex multistep problem, and to inhibit or suppress any inappropriate strategies (Blair, Ursache, Greenberg, Vernon-Feagans and the Family

Life Project Investigators, 2015). For example, mathematics problem solving requires EF skills, which refer to the ability that can recall and manage information in different ways, switching attention and using working memory to follow directions (Geary, 2013). Moreover, VS skills contribute to children's mathematics performance through their ability to use adaptive strategies to solve arithmetic problems (Geary and Burlingham-Dubree, 1989) and interpret numerical information spatially (Gunderson, Ramirez, Beilock and Levine, 2012). VS skills also may be related to the understanding of part/whole relationship and units involving a dimensional geometric, which is implicated in problem-solving tasks (Verdine, Irwin, Golinkoff and Hirsh-Pasek, 2014).

EF Deficits in Children with Mathematics Difficulties

EF deficits have often been identified as potential targets of early intervention efforts designed to help young children experiencing mathematics difficulties (Bull and Scerif, 2001; Toll et al., 2011). Children with executive functioning deficits more frequently fail to complete multi-step instructions by their teachers and to finish complex tasks (Gathercole, Lamont and Alloway, 2006). For example, working memory and cognitive flexibility deficits have been reported to constrain children's counting abilities, fact retrieval (Geary, Hoard, Nugent, & Bailey, 2012), and mathematics problem solving (Andersson, 2008). This may occur due to the children struggling with storing, manipulating symbolic information and shifting between several strategies or following multistep solution procedures (Toll et al., 2011).

WM is considered important for mathematical performance because information from long-term memory must be stored and manipulated during mathematical problem solving (Andersson, 2008). Deficits in WM ability can disrupt the representation and articulation of numbers during the counting process which lead to secondary deficits in numerical processes (Zamarian, Visani, Delazer, Seppi, Mair, Diem and Benke, 2006). Deficits in WM are more strongly predictive of learning difficulties in mathematics skill during early childhood. Thus, early intervention designed to prevent learning difficulties may be more effective.

Experimental Studies

Milisavljevic and Petrovic (2008) study was conducted on students (8-16 years) of 124 people to study EF in schools for students with intellectual problems in learning. The aspect that is seen is a strategy used by students with intellectual problems with students who have no intellectual problems in solving a given mathematical problem. Unstable EF development has been identified. It is found that the problematic student fails to solve mathematical problems when given large numbers and it shows low and weak EF levels. The solution path requires regular formula planning for the given questions and will involve the cognitive process of the individual. Hence, the weak performance of the test is influenced by low working memory development, which is the bridge for EF (Cabeza and Nyberg, 2003).

Crocker, Riley and Mattson (2015) conducted a study on the relationship between mathematics and aspects of WM and VS memory of children that exposed to the effects of alcohol consumption by pregnant mothers. The two groups involved were control groups and treatment groups of 56 children. They are evaluated in mathematical achievement in terms of VS memory by using neuroscience technology tools called CANTAB and used "Spatial Recognition memory" and "Pattern Recognition Memory" tests. This finding suggests that VS skills are diminishing in mathematics achievement for children who exposed to alcohol consumption by pregnant mothers. Furthermore, the impact of alcohol on the child caused difficulty in mathematics achievement such as the weakening process in numerical skills and might interrupted the cognitive processes that focused in EF ability. This incredible disorder or illness usually occurs in the parietal part of the brain (parietal lobe) especially when it involves a calculation in mathematics function. The brain parietal part affects the mathematical function in children's cognitive development (Cantlon, Davis, Libertus, Kahane, Brannon and Pelphrey, 2011).

Geertsen, Thomas, Larsen, Dahn, Andersen, Krause-Jensen, Korup, Nielse, Wienecke, Ritz, Krustrup and Lundbye-Jensen (2016) have studied the relationship between motor skills with cognitive function domain measurements as well as academic achievement of mathematics in early childhood adolescents. A total of 423 children (9 years) were selected in this study. They used the Cambridge Neuropsychological Test Automated Battery (CANTAB) technology to measure the cognitive function domain in EF. Spatial Working Memory (SWM) test was used to evaluate how far the individual can maintained and manipulated spatial information and used strategy efficiency in mathematics problem solving. Academic achievement was tested on their ability to solve mathematics problems with 50 questions of mathematical problems (addition, rejection, multiplication, geometry and probability). The data clearly shows that the EF domain assessed by CANTAB had a significant positive relationship with the level of mathematics achievement. The students who achieved the high level scores in mathematical problem solving had a strong ability for spatial skills in manipulating information. It shows that cognitive processes can help individual to decide and control the motor in the learning process (Geertsen et al., 2016).

Implication and Conclusion

There is now increasing evidence of a strong relationship between EF and VS skills, in particular of cognitive processes in working memory and children’s mathematics achievement. Specifically, EF and VS skills may be two cognitive skills that are theoretically important for children’s early success in mathematics in elementary school. Some children may have trouble with processing numerical information and performing numerical tasks accurately because they may have problems in more general cognitive skills (EF and VS) rather than problems with understanding the information (Kolkman, Koresbergen and Leseman, 2013). This paper also provides empirical support for the evaluations of school based, multi-component interventions designed to address the early of learning difficulties through the EF deficits, particularly in working memory. Teachers need to provide frequent opportunities for children to engage with spatial activities in children’s school curricula, such as using a diagram to create a block structure in which they can practice their one-to-one correspondence as well as their spatial skills (e.g., mental rotation). Moreover, others have suggested that instruction designed to foster a deeper understanding of mathematical concepts and to build connection between numbers, words, and ideas may consequently be a better way to enhance EF processes (Clements, 2013).

References

Andersson, U. (2008). Working memory as a predictor of written arithmetical skills in children: The importance of central executive functions. *British Journal of Educational Psychology,* 78, 181–203

Baddeley AD. (1986). *Working memory*. Oxford, UK: Oxford University Press.

Baddeley, A. D. (1996). Exploring the central executive. *Quarterly Journal of Experimental Psychology*, 49, 5–28.

Baddeley AD. (2000). The episodic buffer: a new component of working memory? Trends. *Cognition Science*, 4, 417–23

Barrouillet, P., Fayol, M., & Lathuliere, E. (1997). Selecting between competitors in multiplication tasks: An explanation of the errors produced by adolescents with learning difficulties. *International Journal of Behavioral Development*, 21(2), 253–275.

Blair,C., & Razza,R.P. (2007). Relating effortful control, executive function, and false belief understanding to emerging math and literacy ability in kindergarten. *Child Development*, 78, 647–663.

Blair, C., Ursache, A., Greenberg, M., Vernon-Feagans, L., & the Family Life Project Investigators. (2015). Multiple aspects of self-regulation uniquely predict mathematics but not letter– word knowledge in the early elementary grades. *Developmental Psychology*, 51(4), 459–472.

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology*, 19(3), 273–293.

Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives*, 8(1), 36–41.

Cabeza, R., & Nyberg, L. (2003). Special issue on functional neuroimaging of memory. *Journal of Neuropsychological*, 41, 241–244.

Cantlon, J.F., Davis, S.W., Libertus, M.E., Kahane, J., Brannon, E.M., & Pelphrey, K.A. (2011). Inter-parietal white matter development predicts numerical performance in young children. Journal of *Learning and Individual Differences*, 21, 672–680.

Clements, D. H. (2013). *Building blocks*. New York, NY: McGraw-Hill.

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education*, *3*(2), 63–68.

Crocker, N., Riley, E.P., & Mattson, S.N. (2015). Visual- spatial abilities relate to mathematics achievement in children with heavy prenatal alcohol exposure. *Journal of Neuropsycology*, 29(1), 108–116.

Diamond, A. (2013). Executive functions. *Annual Review of Psychology*, 64, 135–168.

Fuchs, L. S., Geary, D. C., Compton, D. L., Fuchs, D., Hamlett, C. L., Seethaler, P. M. (2010b). Do different types of school mathematics development depend on different constellations of numerical versus general cognitive abilities? *Developmental Psychology*, 46(6), 1731–1766.

Garon, N., Bryson, S.E., & Smith, I.M. (2008). Executive function in preschoolers: A review using an integrative framework. *Psychological Bulletin*, 134(1), 31–60.

Gathercole, S. E., & Pickering, S. J. (2000). Assessment of working memory in six- and seven-year-old children. *Journal of Educational Psychology*, 92, 377–390.

Gathercole, S. E., Lamont, E., & Alloway, T. P. (2006). Working memory in the classroom. In S. J. Pickering (Ed.),*Working memory and education* (pp. 219– 240). Oxford, UK: Elsevier.

Geary, D. C., & Burlingham-Dubree, M. (1989). External validation of the strategy choice model for addition. *Journal of Experimental Child Psychology*, 47(2), 175–192.

Geary, D. C. (2010). Mathematical disabilities: Reflections on cognitive, neuropsychological, and genetic components. *Learning and Individual Differences*,20, 130–133.

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2012). Mathematical cognition deficits in children with learning disabilities and persistent low achievement: A five-year prospective study. *Journal of Educational Psychology*, 104, 206– 224.

Geary, D. C. (2013). Early foundations for mathematics learning and their relations to learning disabilities. *Current Directions in Psychological Science*, 22(1), 23– 27.

Geertsen, S.S., Thomas, R., Larsen, M.N., Dahn, I.M., Andersen, J.N., Krause-Jensen, M., Korup, V., Nielsen, C.M., Wienecke, J., Ritz, C., Krustrup, P., & Lundbye-Jensen, J. (2016). Motor skills and exercise capacity are associated with objective measures of cognitive functions and academic performance in preadolescent children. *PloS one*, 11(8).

Gooding, S. (2009). Children's difficulties with mathematical word problems. *Proceedings of the British Society for Research into Learning Mathematics*, *29*(3), 31-36.

Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: The role of the linear number line. *Developmental Psychology*, 48(5), 1229–1241.

Hoon, O. P. (2002). Neuroscience in Mathematics: An Electroencephalographic Study on Fraction Learning. *Journal of Science and Mathematics Education in Southeast Asia*, 25(2): 15–31.

Imbo, I., & LeFevre, J. (2010). The role of phonological and visual working memory in complex arithmetic for Chinese- and Canadian-educated adults. *Memory and Cognition*, 38, 176–185.

Kolkman, M. E., Koresbergen, E. H., & Leseman, P. P. M. (2013). Early numerical development and the role of non-symbolic and symbolic skills. *Learning and Instruction*, 25, 95–103.

Krajewski, K., & Schneider, W. (2009). Exploring the impact of phonological awareness, visual-spatial working memory, and preschool quantity-number competencies on mathematics achievement in elementary school: Findings from a 3-year longitudinal study. *Journal of Experimental Child Psychology,* 103(4), 516–531.

Lerner, J. (2003). *Learning Disabilities: Theories, Diagnosis, and Teaching Strategies, 9th ed*. Boston: Houghton Mifflin.

Metcalfe, A. W. S., Ashkenazi, S., Rosenberg-Lee, M., & Menon, V. (2013). Fractionating the neural correlates of individual working memory components underlying arithmetic problem solving skills in children. *Developmental Cognitive Neuroscience*, 6, 162–175.

Meyer, M. L., Salimpoor, V. N., Wu, S. S., Geary, D. C., & Menon, V. (2010). Differential contribution of specific working memory components to mathematics achievement in 2nd and 3rd graders. *Learning and Individual Differences*, 20(2), 101–109.

Meuwissen, A.S., & Zelazo, P.D. (2014). Hot and cool executive function: Foundations for learning and healthy development. *Zero to Three*, 35(2), 18– 23.

Milisavljevic, M.J., & Petrovic, D.M. (2008). Executive Functions In Children With Intellectual Disabilities. *The British Journal of Developmental Disabilities*, 54(107), 113–121.

Miyake, A. and Shah, P. (1999). Toward Unified Theories of Working Memory: Emerging General Consensus, Unresolved Theoretical Issues, and Future Research Directions. In: Miyake, A. and Shah, P., Eds., *Models of Working Memory: Mechanisms of Active Maintenance and Executive Control*, Cambridge University Press, Cambridge, 442–482.

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology*, 41, 49–100.

Miyake, A., Friedman, N. P., Rettinger, D. A., Shah, P., & Hegarty, M. (2001). How are visuospatial working memory, executive functioning, and spatial abilities related? A latent-variable analysis. *Journal of Experimental Psychology*, 130, 621–640.

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual differences, and cognitive approaches. *Learning and Individual Differences,* 20: 110–122.

Robbins, T. W., James, M., Owen, A. M., Sahakian, B. J., Lawrence, A. D., McInnes, L., & Rabbit, P. M. (1998). A study of performance on tests from the CANTAB battery sensitive to frontal lobe dysfunction in a large sample of normal volunteers: Implications for theories of executive functioning and cognitive aging. *Journal of the International Neuropsychological Society*, *4*(5), 474–490.

Swanson, H. L. (2006). Cross-sectional and incremental changes in working memory and mathematical problem solving. *Journal of Educational Psychology*, *98*(2), 265.

Toll, S. W., Van der Ven, S. H., Kroesbergen, E. H., & Van Luit, J. E. (2011). Executive functions as predictors of math learning disabilities. *Journal of Learning Disabilities*, *44*(6), 521–532.

van Garderen, D. (2006). Spatial visualization, visual imagery, and mathematical problem solving of students with varying abilities. *Journal of Learning Disabilities*, *39*(6), 496–506.

Verdine, B. N., Irwin, C. M., Golinkoff, R. M., & Hirsh-Pasek, K. (2014). Contributions of executive function and spatial skills to preschool mathematics achievement. *Journal of Experimental Child Psychology*, 126, 37–51.

Yeniad, N., Malda, M., Mesman, J., van IJzendoorn, M. H., & Pieper, S. (2013). Shifting ability predicts math and reading performance in children: A meta-analytical study. *Learning and Individual Differences, 23,* 1–9.

Zamarian, L., Visani, P., Delazer, M., Seppi, K., Mair, K. J., Diem, A., & Benke, T. (2006). Parkinson's disease and arithmetics: the role of executive functions. *Journal of the Neurological Sciences*, *248*(1), 124-130.

Zelazo, P.D., Blair, C.B., and Willoughby, M.T. (2016). *Executive function: Implications for education (NCER 2017-2000.* Washington, DC: National Center for Education Research, Institute of Education Sciences, U.S. Department of Education.